Information-Based Physics: An Intelligent Embedded Agent's Guide to the Universe

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Information-Based Physics: An Intelligent Embedded Agent's Guide to the Universe

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Santa Fe Institute, 26 March 2013
Influence

I know about the universe because it influences me

In fact, **everything** I know about the universe is conveyed via such influences.

Moreover, I cannot come to know about what does not influence me.
Agent-Centric View

Everything I can know is completely describable in terms of how it influences me.
Information

Information acts to constrain our beliefs

You can believe anything you want… until you obtain information
Physical Laws are shaped by three factors:

- The nature of influence
- Constraints on the quantification of such influences
- Inferences that can be made from the information obtained via influences
### Physical Laws are shaped by three factors:

- The nature of influence
- Constraints on the quantification of such influences
- Inferences that can be made from the information obtained via influences

**Information-Based Physics**
Progress

Derivation of Probability Theory
as a quantification of the Boolean algebra of statements

Derivation of the Feynman Path Integral Formulation of Quantum Mechanics
as a quantification of measurement sequences
Quantification

“Measure what is measurable, and make measurable that which is not so.”

Galileo Galilei

Map order-theoretic or algebraic entities to sets of real numbers, and encode operations on those entities in terms of operations on numbers (*Laws*).

The challenge is to do this in an apt and consistent manner.

The utility lies in the fact that many problems possess similar symmetries, which lead to identical laws.
Inference
States and Statements

States of a piece of fruit

Statements about a piece of fruit

Statements describe potential states

powerset

\{ a, b \} \quad \{ a, c \} \quad \{ b, c \}

\{ a \} \quad \{ b \} \quad \{ c \}

subset inclusion

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Implication

statements about a piece of fruit

ordering encodes implication
Inference

Quantify to what degree knowing that the system is in one of three states \{a, b, c\} implies knowing that it is in some other set of states \{ a, b \} \{ a, c \} \{ b, c \} \{ a \} \{ b \} \{ c \} \{ a, b, c \}

Bi-Valuation: \( p(\{c\} \mid \{a,b,c\})\)

inference works backwards
Assume $v(a \lor b) = v(a) \oplus v(b)$

Then

\[(v(a) \oplus v(b)) \oplus v(c) = v(a) \oplus (v(b) \oplus v(c))\]
The Associativity Equation

The algebraic symmetry of associativity along with a concept of ordering

\[(a \lor b) \lor c = a \lor (b \lor c)\]

must be preserved by our quantification

\[(v(a) \oplus v(b)) \oplus v(c) = v(a) \oplus (v(b) \oplus v(c))\]

This means that the operation \( \oplus \) is a transform of addition (Aczel 1966, Knuth & Skilling 2012):

\[h(v(a \lor b)) = h(v(a) \oplus v(b)) = h(v(a)) + h(v(b))\]
General Case

\[ v(y) = v(x \land y) + v(z) \]

\[ v(x \lor y) = v(x) + v(z) \]

\[ v(x \lor y) = v(x) + v(y) - v(x \land y) \]
Probability

Associativity of Join
\[ p(a \lor b \mid i) = p(a \mid i) + p(b \mid i) - p(a \land b \mid i) \]

Associativity of Direct Product of Hypothesis Spaces
\[ p(a, b \mid i, j) = p(a \mid i) p(b \mid j) \]

Associativity of Context
\[ p(a \mid c) = p(a \mid b) p(b \mid c) \]

which can be used to derive Bayes theorem

Why Sums Rule

\[ p(x \lor y \mid i) = p(x \mid i) + p(y \mid i) - p(x \land y \mid i) \]

\[ I(X;Y) = H(X) + H(Y) - H(X,Y) \]

\[ \max( x, y ) = x + y - \min(x, y) \]

\[ \chi = V - E + F \]

\[ \log(\gcd( x, y )) = \log( x ) + \log( y ) - \log(\lcm(x, y)) \]
Quantum Measurement Sequences
Quantify a quantum mechanical measurement sequence [m1, m2, m3] with a pair of real numbers \((a_1, a_2)\).
Parallel Combination of Measurements

source
M₁
M₂
M₃
Parallel Combination of Measurements

\[ m_2 \lor m_2' = m_{2, m_2'} \]

Associativity Again!

\[
\begin{pmatrix}
  a_1 \\
  a_2
\end{pmatrix}
\oplus
\begin{pmatrix}
  b_1 \\
  b_2
\end{pmatrix}
= \begin{pmatrix}
  a_1 + b_1 \\
  a_2 + b_2
\end{pmatrix}
\]

Serial Combination of Measurements

\[
\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \otimes \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} a_1 b_1 - a_2 b_2 \\ a_1 b_2 + a_2 b_1 \end{pmatrix}
\]

Distributivity, Reciprocity, Agreement with probability


Quantum vs. Classical States

The Classical State Space is an Antichain.

Whereas the QM space of measurement sequences is a partially ordered set.

QM Slit Experiments

Fruit

a  b  c

The Classical State Space is an Antichain.

Whereas the QM space of measurement sequences is a partially ordered set.
Quantum Amplitudes and Probabilities

Measurement Sequences

Complex amplitudes quantify relationships among sequences

Statements about Sequences

Complex amplitudes are used to compute probabilities

\[ Z_{LR} = Z_L + Z_R \]

\[ |Z_L|^2 + |Z_R|^2 \]

\[ |Z_L|^2 \quad |Z_R|^2 \quad |Z_{LR}|^2 \]
Space-Time Relationships
from a quantification of causal sets of events

Derivation of the Dirac Equation in 1+1 Dimensions
as a quantification of direct particle-particle influence
Influence
Influence

Look only at the most basic property of influence and see what physics we get.

A influences B
B is influenced by A

Influence-Induced Order
Define a pair of Events as the Boundary of Influence

**Event A:** $A$ influences $B$

**Event B:** $B$ is influenced

$A \leq B$

The direction of the ordering relation is arbitrary.

(could write $A \geq B$)

This will be relevant later.
A set of influences can be described by a partially-ordered set (poset) of events.

Chains, which are totally ordered, represent a sequence of events.
Quantification
Rather than endowing the poset with additional properties, our goal is simply to identify a consistent means by which events in the poset can be aptly quantified.
Chains are easily quantified by a **monotonic valuation** assigning to each element a real number.
Chain Projection

\[ P_x = p_x \]

\[ p_i \geq x \quad \text{for all} \quad p_i \geq p_x \]

\[ p_i || x \quad \text{for all} \quad p_i < p_x \]

\[ \overline{P}_x = \overline{p}_x \]

\[ p_i \leq x \quad \text{for all} \quad p_i \leq \overline{p}_x \]

\[ p_i || x \quad \text{for all} \quad \overline{p}_x < p_i < p_x \]

\[ p_i \geq x \quad \text{for all} \quad p_i \geq p_x \]

\[ p_i \leq x \quad \text{for all} \quad p_i \leq \overline{p}_x \]

\[ p_i || x \quad \text{for all} \quad p_i > \overline{p}_x \]
Quantification with Pairs

Quantification can be extended by relating poset elements to the embedded chain via \textbf{chain projection}.

For an element \( x \), there is the potential to be quantified by a pair of numbers

\[ (p_x, \bar{p}_x) \]
Quantification by Chain Projection
Intervals
Closed intervals can be quantified by pairs: \((p_6, p_4)\)

Or by scalars (theorem): \(p_6 - p_4\)
Generalized intervals are defined by their endpoint elements.

They can be quantified by:

4-tuples: \((p_b, \bar{p}_b; \ p_a, \bar{p}_a)\)

Pairs: \((p_b - p_a, \bar{p}_b - \bar{p}_a)\)

 Scalars (theorem):
\[(p_b - p_a)(\bar{p}_b - \bar{p}_a)\]
Generalized Intervals

4-tuple: \((p_b, \bar{p}_b; \ p_a, \bar{p}_a)\)

Pair: \((\Delta p, \Delta \bar{p})\)

Scalar (theorem): \(\Delta p \Delta \bar{p}\)
Quantifying Intervals

Quadruple
(6, 3; 5, 1)

Pair
(6-5, 3-1) = (1, 2)

Scalar
(6-5)(3-1) = 2
Quantifying Intervals

Quadruple
\((6, 3; 3, 3)\)

Pair
\((6-3, 3-3) = (3, 0)\)

Scalar
\((6-3)(3-3) = 0\)
Quantifying Intervals

Quadruple
(6, 3; 4, 4)

Pair
(6-4, 3-4) = (2, -1)

Scalar
(6-4)(3-4) = -2
Induced Subspaces
An element $x$ is **collinear** with finite chains $P$ and $Q$, iff the projections $P_x$ and $\bar{P}_x$, can be found by first projecting $x$ onto $Q$ and then onto $P$, and vice versa.
Betweenness

P-side

Case I

\[ P_x = \overline{P}Q_x \]
\[ Q_x = \overline{Q}P_x \]

Case II

\[ P_x = \overline{P}Q_x \]
\[ Q_x = \overline{Q}P_x \]

Case III

\[ P_x = \overline{P}Q_x \]
\[ Q_x = \overline{Q}P_x \]

Q-side

\[ \overline{P}_x = \overline{P}Q_x \]
\[ \overline{Q}_x = \overline{Q}P_x \]

Between

\[ P_x = \overline{P}Q_x \]
\[ Q_x = \overline{Q}P_x \]
Induced Subspaces

Every pair of chains induces a subspace in a poset

\[ x \in \overline{PQ} \]

\[ x \notin \overline{PQ} \]
Collinear chains can be ordered.

This induced subspace brings with it an additional dimension.

There can be many induced subspaces.

Ax = ABCDx
Coordinated Chains
Coordinated Chains

are two chains that agree on lengths of each others intervals

This construct will allow us to explore quantification using only the fact that they are influenced
<table>
<thead>
<tr>
<th>CONSISTENCY PRINCIPLE</th>
</tr>
</thead>
<tbody>
<tr>
<td>If two chains (agents) agree on the quantification of each others’ closed intervals, then they must agree on the quantification of every interval they both observe.</td>
</tr>
</tbody>
</table>
Coordinated Chains
Coordinated Chains
Coordinated Chains
Quantification via Coordinated Chains

$\Delta p$ (\(p_b - p_a\), \(q_b - q_a\))

$\Delta q$ (\(p_a - p_a\), \(q_b - q_a\))

Interval Pair: \((\Delta p, \Delta q) = (p_b - p_a, q_b - q_a)\)

Interval Scalar: \(\Delta p \Delta q = (p_a - p_a)(q_b - q_a)\)
Distance
Interval Classes

- antichain-like
- chain-like
- projection-like
Closed Intervals Reside on Chains

$[p_6, p_4] = \{p_6, p_5, p_4\}$

Closed intervals can be quantified by pairs: $(p_6, p_4)$

Or by scalars (theorem): $p_6 - p_4$
Distance Along Chains

The length of a purely chain-like interval can be written as:

\[ d(a, b) = \frac{\Delta p + \Delta q}{2} \]
Collinear chains can be ordered.

Intervals defined between chains when joined obey associativity.

This implies that the quantification of the distance between coordinated chains must be additive (linear).
Distance Between Chains

The distance cannot depend on which elements are used.

\[ D[P, Q] = a\Delta p + b\Delta q = a\Delta p' + b\Delta q' \]

Solution (setting arb constant to 1/2):

\[ D[P, Q] = \frac{\Delta p - \Delta q}{2} \]
Symmetric-Antisymmetric Decomposition
Decomposition

\[ \Delta p = p_b - p_a \]

\[ \Delta q = q_b - q_a \]

\[ (\Delta p, \Delta q) = \left( \frac{\Delta p + \Delta q}{2}, \frac{\Delta p + \Delta q}{2} \right) + \left( \frac{\Delta p - \Delta q}{2}, -\frac{\Delta p - \Delta q}{2} \right) \]

symmetric \hspace{2cm} \text{antisymmetric}
Minkowskian Form

THEOREM: Minkowskian Form

The pair when decomposed into symmetric and anti-symmetric pairs

\[(\Delta p, \Delta q) = \left(\frac{\Delta p + \Delta q}{2}, \frac{\Delta p + \Delta q}{2}\right) + \left(\frac{(\Delta p - \Delta q)}{2}, -\frac{(\Delta p - \Delta q)}{2}\right)\]

defines the scalar

\[\Delta p\Delta q = \left(\frac{\Delta p + \Delta q}{2}\right)^2 - \left(\frac{\Delta p - \Delta q}{2}\right)^2\]

which is the sum of the scalars defined by the pairs resulting from the decomposition.
Transformation
Linearly Related Chains

Consider two chains that project onto one another in a constant fashion:

\[(ak, bk)_P = (am, bn)_{P'}\]

so that the scalar

\[k^2 = mn\]
THEOREM: Generalized Lorentz Transformation

A pair quantifying an interval in the frame PQ is related to the pair quantifying an interval in the frame P’Q’ by

\[ L_{PQ \rightarrow P'Q'} (\Delta p, \Delta q)_{PQ} = \left( \Delta p \sqrt{m \over n}, \Delta q \sqrt{n \over m} \right)_{P'Q'} \]

where the chain-like interval in PQ quantified by \((\sqrt{mn}, \sqrt{mn})\) projects to \((m, n)\) in P’Q’
SPACE-TIME PICTURE

Spacetime
By Kyle Haller
Time reflects the fact that everything does not happen at once

Space reflects the fact that everything does not happen to you

Susan Sontag
Space-Time Picture

\[ \Delta t = \frac{\Delta p + \Delta q}{2} \]

\[ \Delta x = \frac{\Delta p - \Delta q}{2} \]
Minkowski Metric

\[ \Delta p \Delta q = \left( \frac{\Delta p + \Delta q}{2} \right)^2 - \left( \frac{\Delta p - \Delta q}{2} \right)^2 \]

\[ \Delta s^2 = \Delta t^2 - \Delta x^2 \]
Lorentz Transformation

The pair transformation

\[(\Delta p', \Delta q') = (\Delta p \sqrt{\frac{m}{n}}, \Delta q \sqrt{\frac{n}{m}})\]

Can be rewritten as

\[\left(\frac{\Delta t' + \Delta x'}{2}, \frac{\Delta t' - \Delta x'}{2}\right) = \left(\frac{\Delta t + \Delta x}{2} \sqrt{\frac{m}{n}}, \frac{\Delta t - \Delta x}{2} \sqrt{\frac{n}{m}}\right)\]
Lorentz Transformation

Solving for $\Delta t'$ and $\Delta x'$

By defining

$$\beta = \frac{m - n}{m + n} = \frac{\Delta x}{\Delta t}$$

we have

$$\Delta t' = \frac{1}{\sqrt{1 - \beta^2}} \Delta t + \frac{-\beta}{\sqrt{1 - \beta^2}} \Delta x$$

$$\Delta x' = \frac{-\beta}{\sqrt{1 - \beta^2}} \Delta t + \frac{1}{\sqrt{1 - \beta^2}} \Delta x$$
Natural Speed Limit

Maximum speed occurs when \( n = \Delta q = 0 \)

\[ \beta = \frac{m-n}{m+n} = +1 \]

or when \( m = \Delta p = 0 \) so that

\[ \beta = \frac{m-n}{m+n} = -1 \]

Such intervals have the same speed \( \beta = \pm 1 \) in all frames

\[ (\Delta p', \Delta q') = \left( 0 \frac{\sqrt{m}}{\sqrt{n}}, \Delta q \frac{\sqrt{n}}{\sqrt{m}} \right) = (0, \Delta q') \]
The Free Particle
A free particle influences but is not influenced

Coordinated chains define a 1+1 dimensional subspace

\( P \) and \( Q \) : Observer Chains
\( \Pi \) : “Particle”

Poset connections represent direct particle-particle interactions where \( \Pi \) can influence one observer at a time.
Observer Detections

Observer chain P is influenced at p1, p3, p4, p6

Observer chain Q is influenced at q2, q5, q7
Incomplete Information

Observer chain P is influenced at
p1, p3, p4, p6

Observer chain Q is influenced at
q2, q5, q7

However, even by combining detections, the particle interaction pattern cannot be uniquely reconstructed
Reconstruction Attempts

There are \( \binom{3+4}{3} = \binom{3+4}{4} = 35 \) possible reconstructions!
The detected interactions result in 35 possible bit strings.

Poset Picture: 35 possible interaction patterns
Space-Time Picture: 35 possible space-time paths
Zitterbewegung

The particle is interpreted as zig-zagging back-and-forth at the maximum speed, $\beta = \pm 1$

It acts as if making bishop-moves on a chessboard

$\beta = \frac{\Delta p - \Delta q}{\Delta p + \Delta q}$
Nothing moves in this picture. Particles “transition” based on their interactions.
Space-Time Picture (IT from BIT)

No continuous motion. Only transition defined by interaction.

Positions (and velocities) are not well-defined.

All possible paths must be considered
Space-Time Picture (IT from BIT)

Feynman Checkerboard Model of the Dirac Eqn.  
Feynman & Hibbs, 1965

Investigated by many others  
eg: Gaveau, Schulman, McKeon, Ord,  
Gersch, Plavchan, Earle
Inference
Entropy of a Bit Sequence

\[ \text{Prob}(P) = \frac{\Delta P}{\Delta P + \Delta Q} \]
\[ \text{Prob}(Q) = \frac{\Delta Q}{\Delta P + \Delta Q} \]
\[ \Delta P = \Delta t + \Delta x \]
\[ \Delta Q = \Delta t - \Delta x \]
\[ \Delta P + \Delta Q = 2 \Delta t \]

\[ S = \frac{\Delta P}{\Delta P + \Delta Q} \log \frac{\Delta P}{\Delta P + \Delta Q} + \frac{\Delta Q}{\Delta P + \Delta Q} \log \frac{\Delta Q}{\Delta P + \Delta Q} \]
\[ S = \frac{1}{2} (1 + \beta) \log \frac{1}{2} (1 + \beta) + \frac{1}{2} (1 - \beta) \log \frac{1}{2} (1 - \beta) \]
\[ \ldots \]
\[ \ldots \]
Entropy of a Bit Sequence

\[ S = -\log \frac{1}{2} + \log \gamma - \beta \log (z + 1) \]

- Lorentz factor!
- Velocity!
- Red-shift!
Inferences about Sequences

Look at PQ and QP sequences

Wavefunctions are not things! They are pairs of numbers assigned to perform inference.

Uncertain Sequence Order Coarse-Grained Measurement
Initial State is Uncertain!

\[ \varphi(r,t+\Delta t) \]

- P
- Q
- \( \varphi(r,t) \)
- ?

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Two Components

\( \varphi(r, t) = \begin{pmatrix} \varphi_P \\ \varphi_Q \end{pmatrix} \)

Must sum over four sequences!

Handled by summing over two paths per component
Moves = Matrices

\[ Q = \begin{pmatrix} 0 & 0 \\ y & x \end{pmatrix} \]

\[ P = \begin{pmatrix} x & y \\ 0 & 0 \end{pmatrix} \]

\[ P \left( \begin{pmatrix} \varphi_P \\ \varphi_Q \end{pmatrix} \right) = \begin{pmatrix} x & y \\ 0 & 0 \end{pmatrix} \left( \begin{pmatrix} \varphi_P \\ \varphi_Q \end{pmatrix} \right) = \begin{pmatrix} x\varphi_P + y\varphi_Q \\ 0 \end{pmatrix} \]

\[ Q \left( \begin{pmatrix} \varphi_P \\ \varphi_Q \end{pmatrix} \right) = \begin{pmatrix} 0 & 0 \\ y & x \end{pmatrix} \left( \begin{pmatrix} \varphi_P \\ \varphi_Q \end{pmatrix} \right) = \begin{pmatrix} 0 \\ y\varphi_P + x\varphi_Q \end{pmatrix} \]
Only Two Possibilities

\[ \text{Prob} \left( \left( r - \frac{\Delta r}{2}, t + \frac{\Delta t}{2} \right) \bigg| (r, t) \right) + \text{Prob} \left( \left( r + \frac{\Delta r}{2}, t + \frac{\Delta t}{2} \right) \bigg| (r, t) \right) = 1 \]
Matrix Constraints

\[
Prob \left( \left( r - \frac{\Delta r}{2}, t + \frac{\Delta t}{2} \right) \mid (r, t) \right) + Prob \left( \left( r + \frac{\Delta r}{2}, t + \frac{\Delta t}{2} \right) \mid (r, t) \right) = 1
\]

\[
(Q\varphi)^\dagger (Q\varphi) + (P\varphi)^\dagger (P\varphi) = 1
\]

\[
\varphi^\dagger (Q^\dagger Q + P^\dagger P)\varphi = 1
\]

\[
Q^\dagger Q + P^\dagger P = I
\]
Matrix Constraints

Since

\[ P = \begin{pmatrix} x & y \\ 0 & 0 \end{pmatrix}, \quad Q = \begin{pmatrix} 0 & 0 \\ y & x \end{pmatrix} \]

\[ Q^\dagger Q + P^\dagger P = I \]

is

\[ \begin{pmatrix} 0 & y^* \\ 0 & x^* \end{pmatrix} \begin{pmatrix} 0 & 0 \\ y & x \end{pmatrix} + \begin{pmatrix} x^* & 0 \\ y^* & 0 \end{pmatrix} \begin{pmatrix} x & y \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \]

which implies

\[ x^*x + y^*y = 1 \]

\[ x^*y + y^*x = 0 \]
Solving Constraints

Write \( x = ae^{i\alpha} \quad y = be^{i\beta} \)

the constraints \( x^*x + y^*y = 1 \)
\( x^*y + y^*x = 0 \)

become \( a^*a + b^*b = 1 \)
\( e^{i\theta} + e^{-i\theta} = 0 \)

Where \( \theta = \alpha - \beta \)
Solution

\[ a^*a + b^*b = 1 \]

\[ e^{i\theta} + e^{-i\theta} = 0 \]

The relative phase angle \( \theta \) must be \( \frac{\pi}{2} \) or \( \frac{3\pi}{2} \)
(need complex numbers)

The amplitudes describe the relative probability of changing direction.

Consider the case where these are equal:

\[ a = b = \frac{1}{\sqrt{2}} \]
Choosing $x$ to be real, we have

\[
P = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ 0 & 0 \end{pmatrix} \quad Q = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 \\ i & 1 \end{pmatrix}
\]

So that

\[
P \begin{pmatrix} \varphi_P \\ 0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \varphi_P \\ 0 \end{pmatrix}
\]

\[
P \begin{pmatrix} 0 \\ \varphi_Q \end{pmatrix} = \frac{i}{\sqrt{2}} \begin{pmatrix} \varphi_Q \\ 0 \end{pmatrix}
\]

\[
Q \begin{pmatrix} \varphi_P \\ 0 \end{pmatrix} = \frac{i}{\sqrt{2}} \begin{pmatrix} 0 \\ \varphi_P \end{pmatrix}
\]

\[
Q \begin{pmatrix} 0 \\ \varphi_Q \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \varphi_Q \end{pmatrix}
\]
Transfer Matrices

Choosing $x$ to be real, we have

\[
P = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ 0 & 0 \end{pmatrix} \quad Q = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 \\ i & 1 \end{pmatrix}
\]

So that

\[
P \left( \varphi_P \right) = \frac{1}{\sqrt{2}} \begin{pmatrix} \varphi_P \\ 0 \end{pmatrix} \quad Q \left( \varphi_P \right) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \varphi_P \end{pmatrix}
\]

\[
P \left( \varphi_Q \right) = \frac{i}{\sqrt{2}} \begin{pmatrix} \varphi_Q \\ 0 \end{pmatrix} \quad Q \left( \varphi_Q \right) = \frac{i}{\sqrt{2}} \begin{pmatrix} 0 \\ \varphi_Q \end{pmatrix}
\]

Factor of $i$ on reversal

Factor of $i$ on reversal
Assign an $i\epsilon$ for every reversal.
Sum over all possible paths.
Yields Dirac Equation for 1+1 dimensions

**Feynman Checkerboard Model of the Dirac Eqn.**
Feynman & Hibbs, 1965

Investigated by many others eg: Gaveau, Schulman, McKeon, Ord, Gersch, Plavchan, Earle
THANK YOU

Templeton Foundation
“Quantifying Relations as a Foundation for Physics”

Thanks To:
Keith Earle
Newshaw Bahreyni
Ariel Caticha
Seth Chaiken
Philip Goyal
Oleg Lunin
Margaret May
John Skilling


