Mesh-based fourier imaging for biological and security applications

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Mesh-Based Fourier Imaging for Biological and Security Applications

by

Danielle Hayden

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Abstract

Traditional x-ray imaging provides only low contrast from low atomic number materials, like soft tissue, due to the small attenuation variations producing very small intensity changes. Higher contrast can be achieved through phase information. The phase change is obtained from the x-ray refracting in a sample, or phase object, due to the difference in refractive indexes. This causes a small angular deviation from the original path. Phase contrast imaging has not been realized in everyday practice due to the requirement for large spatial coherence width of the x-ray beam which typically requires sources on the order of 10-50 µm, the use of a grating technique or synchrotron sources. The grating-based phase imaging method depends upon multiple fine-pitched, expensive gratings and extremely precise alignment.

An alternative procedure based on a technique recently demonstrated by Bennett is mesh-based phase imaging that utilizes a single, inexpensive mesh with a coarse pitch. This considerably eases the small spot size source requirement, allowing the use of a 150 micron, micro-focus, tungsten anode source. The mesh-based phase imaging set up used to study biomedical and security screening applications consisted of a 123x123 µm stainless steel mesh and a 1200x1600 CCD detector with a pixel size of 22 microns. This mesh based approach allows for near-real-time phase extraction of the first harmonics in the Fourier domain. With the phase information and absorption information (collected at the zeroth harmonic), edge enhanced images of a mouse’s skull were optimized and several potentially dangerous liquids and powders were discriminated from water. The mesh-based phase set up resulted in high contrasts, signal-to-noise ratios and good resolution verifying the potential utility of this technique for future biomedical imaging and airport security screening.
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1. Introduction

Breast cancer is a common and deadly disease effecting 12% (1 out of 8) of American woman in their lifetime.\textsuperscript{1} Every 19 seconds someone in the world is diagnosed with breast cancer.\textsuperscript{2} It is the second most commonly diagnosed cancer in American women, with skin cancer being number one. About 30% of women diagnosed with cancer have breast cancer.\textsuperscript{1} It is also the second most deadly cancer with higher death rates than any other cancer, besides lung cancer. One woman will die of breast cancer every 13 minutes in the United States alone.\textsuperscript{1,3} These statistics will only decrease with the help of advancements in treatment and earlier detection.

Mammography is the most common screening technique used for early detection of breast cancer. However, mammograms are not perfect tests. It is possible for a mammogram to provide either false positive or false negative results.\textsuperscript{4} A false positive occurs when the mammogram finds imperfections that look like cancer but turn out to not be cancerous. This consequently generates over-diagnosis, leading to unnecessary, costly treatments like surgery and even radiation therapy.\textsuperscript{4} A false negative result occurs when a cancerous mass is completely missed by the mammogram test. False negatives normally occur when the mass is too small for the mammogram to catch. Mammograms usually work for masses on the order of one centimeter.

The number of individuals going through airport security with harmful and deadly devices such as firearms, sharp objects, explosives and incendiaries have continued to increase since 2010.\textsuperscript{5} These individuals attempt to pass through Transportation Security Administration, TSA, screenings on a daily basis. In 2015, an average of 7 firearms per day are found in carry-on luggage and from 2014 to 2015 there was a 20% increase in firearm discoveries.\textsuperscript{6} Some of the explosives and incendiaries found in 2013 include high explosive grenades, bottle containing flash powder
and M-80 fireworks, multiple containers of black powder, gunpowder, explosively viable cannonball and a live blasting cap.⁵

Part of TSA’s precautionary procedures includes sending carry-on items like purses, cell phones, brief cases, etc. and luggage through an x-ray system. The machines used throughout airports today are dual energy x-ray systems. This set-up uses tube voltage between 140 and 160 kilovolt peak (kVP).⁷ Once the x rays pass through an object, they are picked up by a detector and then passed onto a hi pass filter. The hi pass filter blocks out the lower energy x rays and carries the high energy x rays to a second detector. A computer is used to compare the original detection to the high-energy detection in order to better represent low energy objects, like organic objects.⁷

Due to high screening rates of about 1.6 billion carry-on bags a year, the x ray monitors give a color to specific items in each bag based on the amount of energy that passes through the object.⁶ The three main categories pick up organic, inorganic and metal materials. The colors to signify inorganics and metals differ from company to company but organics are most commonly represented by orange.⁷ This is because most explosives are organic. This coloring technique helps TSA screeners easily and quickly detect possible improvised explosive devices (IEDs).

Mammography and airport security use traditional x-ray imaging which measures the change in attenuation formed by the differences in thickness and density. Attenuation is the progressive loss in intensity of any kind of flux through a material. For imaging, x rays traverse an object and based on the composition of the object, the rays are partially absorbed by the material resulting in variations of intensities which are captured by the detector directly.⁸ The downfall to this method of imaging is the attenuation variations are small for materials with similar consistencies and low atomic numbers (Z).⁹ Materials that have similar consistencies do not show a great change in intensities due to the small difference in the absorption coefficient, which affects
the accurate detection of cancers embedded in breast tissue for mammography. Contrast within the x-ray images relies on the refractive index, n,

\[ n = 1 - \delta - i\beta, \]  

where \( \delta \) refers to the real decrement to the refractive index.\(^4\) This can be used to compute the total phase shift, \( \varphi(x, y) \),

\[ \varphi(x, y) = k \int_0^z \delta(x, y, z') \, dz' \]  

where \( k \) is the wavenumber,

\[ k = \frac{2\pi}{\lambda}, \]  

with the wavelength of the incident x-ray represented by \( \lambda \), and \( z \) is the distance of the propagating beam, while \( x \) and \( y \) are the transverse directions in the sample.\(^9\) The imaginary part of the refractive index, \( \beta \), describes the absorption, \( \alpha \),

\[ \alpha = \frac{4\pi}{\lambda} \int_0^z \beta(x, y, z') \, dz'. \]  

Attenuation is dependent on the change of intensity \( \Delta I \), where the intensity of the x rays traveling through a material is,

\[ I = I_0 e^{-\mu t}. \]  

\( I \) is the intensity of the photons after they pass through the material, \( I_0 \) is the intensity of the initial photons produced by the source, \( t \) is the thickness of the material and \( \mu \) is the absorption coefficient of the material.

\[ \mu \approx 2k \int_0^z \beta(x, y, z') \, dz'. \]  

Higher contrast can be achieved through phase information because the phase change, \( \Delta \varphi \), is much larger than the intensity change. This is observed by studying the imaginary part, \( \beta \), and real part, \( \delta \), of the refractive index, where \( \beta \ll \delta \). Typically, variations in intensity absorptions range from \( 10^{-9} \) to \( 10^{-11} \) and variations in phase range from \( 10^{-6} \) to \( 10^{-8} \), therefore, changes in phase are about
10$^3$ times larger. This means there is thousand-fold increase in contrast for phase contrast imaging in comparison to attenuation imaging. Several different techniques have been experimented with measuring this phase shift to develop enhanced images such as propagation-based imaging and grating-based imaging.

Propagation-based imaging (PBI) consists of a linear arrangement of a source, sample and detector. This basic set-up requires placing the detector a large distance away from the sample to obtain phase information. The refraction of incident x rays traversing the sample causes certain detector regions to collect a higher density of x rays resulting in the measurement of a higher intensity. Unfortunately, this technique requires the use of synchrotron radiation (SR) sources which are unfeasible for medical imaging or a micro-focus x ray tube source, which would demand long exposure times which are not feasible for patients due to motion blur.

Grating-based imaging consists of at least two gratings, a phase grating and an analyzer grating. For this technique, a sample is placed in front of a source and the gratings are placed a distance after the sample but before the detector. The sample modifies the incident x rays due to refraction causing a distorted wave front containing the phase occurring at the phase grating. The analyzer grating, placed directly before the detector, resolves the fringes produced by the phase grating. The disadvantage of this phase technique is that the method requires extremely precise alignment and high quality, expensive gratings. In addition, dose can also become problematic when multiple images are necessary.

To overcome the obstacles in propagation-based imaging and grating-based imaging, a mesh-based phase contrast technique was examined in this work based on the set-up requirements studied by Bennett. A wire mesh was placed between two plastic plates, to prevent bending and twisting, acting as a grid and allowed a much coarser pitch than typical gratings. The mesh was
placed a distance after the sample but before the detector. The mesh produces periodic dark bands on the detector. The fast Fourier transformation is taken of the image collected by the detector providing a zeroth order harmonic and first order horizontal and vertical harmonics in the Fourier domain. These harmonics were separated and transformed back to the spatial domain with the zeroth harmonic consisting of the attenuation and the first order harmonics consisting of both attenuation and phase information. A MATLAB code was used for the Fourier analysis to implement the image processing. The mesh-based phase imaging technique and Fourier code produced the attenuation image from the zeroth harmonic and differential phase contrast (DPC) and scatter amplitude images from the first harmonics. The DPC images were then combined to produce the two-dimensional phase images and similarly the scatter images were combined to produce the two-dimensional scatter images. Once the images were processed and combined the contrast, signal-to-noise ratio and resolution were quantified to find the best windowing function to use before the inverse transform and to optimize enhanced images containing both the attenuation and phase or scatter information. Magnification of the mouse sample was also studied. This entire procedure resulted in high resolution mouse images with enhanced edges from the phase shifts.

For the security application, many organic substances were studied to depict between the potentially hazardous powders and liquids used in explosives. Linear profiles through the air-to-powder surfaces and meniscus’ were collected from the images produced by the mesh-based phase contrast technique. The profiles from the processed attenuation and scatter images produced the most defined edges, thus, the scatter-to-absorption ratios were collected. The ratios produced by the potentially harmful substances were compared to the ratio produced by water. Select materials
produced peaks within the DPC profiles. This additional DPC information was helpful to differ materials with similar scatter-to-absorption ratios as water.

2. Theoretical Background

2.1 X-ray Tube

The x-ray tube contains a cathode that is supplied current from an electrical circuit, which causes electrons to eject from a heated coiled filament. Electrons are focused into a well-defined beam directed to the anode due to the high potential difference. Once the electrons impact the tungsten anode the electronic energy is converted into x-radiation.\textsuperscript{10}

2.1.1 Bremsstrahlung Radiation

The conversion to radiation occurs when the high-speed electron from the cathode passes by the nucleus in the anode. The moving electron is attracted to the positively charged nucleus but is prevented from entering due to the comparatively strong nuclear force. This processed is demonstrated in Figure 2-1.

![Schematic of energy transportation within an x-ray tube from reference 10.](image)

The anode also must dissipate the unwanted excess energy that was transformed into heat.
The nuclear electric field causes the electron to change direction and decelerate, resulting in a loss of kinetic energy which is converted into x rays as Bremsstrahlung radiation as shown in Figure 2-2.

![Figure 2-2: Bremsstrahlung radiation produced by high speed electrons traveling pass the nucleus of the target anode. From reference 11.](image)

### 2.1.2 Characteristic Radiation

Characteristic energy is produced when a high-speed electron from the cathode interacts with an inner level electron of the target anode. If the energy of the incoming electron exceeds the binding energy of the electron with which it collided, then the orbital electron is ejected. This causes a vacancy in the inner level orbital, which immediately gets replaced with an electron from a higher orbital. This results in a release of a specific energy which is known as characteristic radiation, as demonstrated by Figure 2-3.

![Figure 2-3: Characteristic energies produced by an incoming electron ejecting an inner electron which in turn gets replaced by a higher-level electron. The image shows where the high-level electrons potentially jump from and how the replacement electrons effect the wavelength of x ray produced. From reference 11.](image)
The type of characteristic energy, $K_{\alpha_{1&2}}, K_{\beta_{1&2}}$, etc., is based on the pair of orbitals the electron left and the one filled. A W anode tube produces bremsstrahlung and characteristic radiation with $K_{\alpha_{1&2}}$ and $K_{\beta_{1&2}}$ peaks at 59.3 keV, 58.0 keV, 69.5 keV and 67.2 keV, respectively, as represented in Figure 2-4.

![Bremsstrahlung spectrum and characteristic peaks](image)

Figure 2-4: Bremsstrahlung spectrum and characteristic peaks for tungsten anode source operated at 87 kVp from reference 12. The $K\alpha$ peak where electrons jump from the $n=2$ orbital to $n=1$ orbital and $K\beta$ peak where electrons jump from the $n=3$ orbital to $n=1$ orbital are labeled. The small peaks that are not labeled at the left side of the maximum of the Bremsstrahlung curve are the L shell peaks where electrons jump to the $n=2$ orbital.

2.1.3 X-ray Source

A source with a tungsten anode is appropriate for biological and security imaging based on its efficiency. A small fraction of the energy transported by the electrons is converted to x-radiation. The remaining energy is absorbed by the anode and converted into heat. The efficiency of x-ray production relies on the amount of radiation produced based on the anode.

$$Efficiency = V \times Z \times 10^{-6},$$

(7)

where $V$ is the voltage measured in kV and $Z$ is the atomic number. The high $Z$ makes the tungsten (W) source ideal in comparison to a molybdenum (Mo) or Rhodium (Rh) anode source, as
Tungsten has a much larger atomic number of 74 compared to Mo with an atomic number of 42 and Rhodium with an atomic number of 45. The micro-focus source used in the biological and security set up is ideal because it is favorable for high magnification and was originally developed for non-destructive inspection.\textsuperscript{13}

\section*{2.2 Elements to Consider in Image Quality}

In x-ray imaging, the quality of the image is important, along with the health of the patient. Factors that need to be considered to maintain a safe and reliable imaging session are contrast, dose and resolution, especially, for mammography. In mammography, the potential tumor to be imaged is extremely similar to the background tissue in which it is embedded. This becomes problematic when producing a good quality image requires increasing the absorbed dose.

\subsection*{2.2.1 Contrast}

For an object or anatomical structure to be visible within the x-ray image, there must be physical contrast between the structure and the tissue or other material in which it is embedded. This contrast is due to either a difference in density, thickness or chemical composition (which results in a change in average atomic number). Little contrast is produced by masses found within soft tissue because the effective atomic numbers are similar.\textsuperscript{10} Therefore, in mammography, care is necessary to maximize the contrast to detect the boundaries of abnormal soft tissues, like tumors. The contrast, $c$, can be defined as a normalized difference in intensity,

$$c = \left| \frac{l_1 - l_2}{l_1} \right| , \quad (8)$$
where \( I_2 \) is the intensity of the x rays traveling through object or anatomical structure of interest and \( I_1 \) is the intensity of the x rays traveling through the background material that the structure is embedded in. The intensities are

\[
I_1 = I_0 e^{-\mu_1 z}
\]

and

\[
I_2 = I_0 e^{-(\mu_1(z-d) e^{-\mu_2 d})}
\]

where \( \mu_1 \) is the absorption coefficient of the background material with thickness \( z \), and \( \mu_2 \) is the absorption coefficient of the embedded object of interest with thickness \( d \).\(^{14}\)

### 2.2.2 Dose

Dose is an extremely important consideration when dealing with radiology of humans. The human body is sensitive to ionizing radiation, quantified as dose. The absorbed dose, \( D \), is the amount of energy absorbed in the tissue from exposure of the ionizing radiation,

\[
D = \frac{E}{m},
\]

where \( E \) is the energy of the absorbed radiation and \( m \) is the unit mass of the target medium. Dose has the unit of Gray, Gy, where

\[
1 \text{ Gy} = 1 \text{ J/kg}.
\]

The equivalent dose, \( H \),

\[
H = D \cdot w_R
\]

is equal to the absorption dose since the radiation weighting factor, \( w_R \), for x rays is 1.\(^{15}\) Equivalent dose is used to measure the expected biological damage from the absorbed dose.

The low energy photons within the bremsstrahlung radiation do not transmit through the patient, hence these photons contribute only to the dose and not the contrast. Therefore, to limit
the dose of the patient, filters are placed between the patient and the source to absorb these unwanted photons.\textsuperscript{14}

\section*{2.2.3 Resolution}

Ideally, each point in the sample being irradiated would be represented as a point in the image. However, this is not the case. Each sample point is spread in the image and this spreading of the point is known as blur. Blur spreads the image of small detailed objects into the surrounding background which results in the reduction of visibility.\textsuperscript{10} This blur can increase from inadequate geometry caused by increasing or decreasing the magnification. Blur is also directly related to focal spot size. A small focal spot produces an increase in spatial resolution.\textsuperscript{14}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{resolution_diagram}
\caption{(a) Shows the blur effect of geometric magnification. When the object of interest is placed closer to the image receptor or detector there is less blur within the image as shown by the left most sketch. Due to the object placement close to the detector, the magnification is significantly decreased as shown by both sketches. Notice that the distance between the source and detector did not change between the two sketches. Based on reference 16. (b) Shows the blur effect of focal spot size. The left sketch shows how a larger focal spot source produces a blurred edge, also known as penumbra, at the detector. Whereas, the infinitesimally small focal spot source produces no blur, as shown by the right most sketch from reference 17.}
\end{figure}

The penumbra blur is related to both the magnification and focal spot size by,
\[ P = f \left( M - 1 \right) \]  

where \( P \) is the penumbra blur, \( f \) is the focal spot size of the source and \( M \) is the magnification represented by the distance between the source and detector divided by the distance between the source and the object.\(^{16}\)

The visibility of details within the image are strongly dependent on the object size and blur. If the blur is less than the dimension of the object or anatomical structure, then the reduced contrast will not have tremendous effect on visibility of this object or structure. If the blur value approaches the dimension of the object, the contrast is significantly reduced and hence the visibility decreases.\(^{10}\)

### 2.3 X-ray Interaction with Matter

When x rays traverse matter, the rays interact with target atoms in various ways. The interactions that occur include scattering (Compton and coherent) and photoelectric absorption. These interactions can have both positive and negative effects on the quality of the images and therefore must be taken into consideration.

#### 2.3.1 Compton Scatter

Compton scatter, also known as incoherent scatter, occurs when the initial photon strikes a charged particle, most commonly an outer shell electron. Compton scatter results in the ejected photon possessing a smaller energy and larger wavelength then the initial photon. This inelastic scattering also causes the photon to be emitted in a different direction.\(^{18}\) Due to the change in direction, the photon is detected in an unwanted area of the camera, degrading the image quality.
It is important to consider Compton scattering for imaging low atomic number materials. Attenuation is dominated by Compton scatter at energies between 100 keV and 10 MeV.\textsuperscript{14}

### 2.3.2 Coherent Scatter

For coherent scatter, also known as Rayleigh scatter, the initial photon strikes the atom, and instead of directly interacting with individual electrons, the photon excites the atom as a whole. The electric field produced by the electromagnetic wave of the initial photon expends energy causing the electrons in the target atom to oscillate in phase. The atom’s electron cloud radiates this energy by emitting a scattering photon but in a different direction.\textsuperscript{15} Therefore, the initial photon and scattered photon have the same energy but slightly different trajectories and no net energy is transferred to the target atom, as shown in Figure 2-7.\textsuperscript{19}
This why coherent scatter is also referred to as ‘unmodified scatter’. Interference from nanoscale structures results in diffraction. Bragg’s law relates the intermolecular spacing of the nanoscale structures with the angle of diffraction according to

$$\lambda = 2dsin\theta$$  \hspace{1cm} (14)

where $d$ is the inter-planar distance between the lattice planes of a crystalline solid. Different intermolecular spacings produce different angles of diffraction.\(^{20}\) The wavelength, $\lambda$, remains constant for coherent scattering. This is due to the relationship between the energy and the wavelength,

$$E = \frac{hc}{\lambda}$$  \hspace{1cm} (15)

where $E$ is the energy, $h$ is Planck’s constant with a value of about $6.626 \times 10^{-34}$ Js and $c$ is the speed of light with a value of about $3 \times 10^8$ m/s. Generally, as the x-ray energy increases, the diffraction angle decreases. Thus, this type of interaction occurs for low energy x rays, like those used in mammography which have an energy range of approximately $15 – 30$ keV.\(^{15}\)
2.3.3 Photoelectric Absorption

In photoelectric absorption, all the energy of the initial photon is absorbed by an electron, resulting in the ejection of this electron from the atom. The kinetic energy of the ejected photoelectron, \( E_{PE} \) is,

\[
E_{PE} = E_i - E_{BE}
\]

where \( E_i \) is the energy of the initial photon and \( E_{BE} \) is the binding energy of the orbital electron.\(^{15} \)

For the photoelectric effect to occur, the energy of the incident photon must be greater than the binding energy of the electron being ejected. The photoelectric interaction potentially results in the ionization of the atom with an inner core shell electron vacancy. The vacancy is quickly filled by a higher shell electron with a lower binding energy. The differences in binding energies produces characteristic x rays or Auger electrons. However, the emitted electrons or comparatively low energy photons are generally absorbed and do not degrade the imaging process.\(^{14} \) Photoelectric absorption must be considered for energies up to 500 keV and for high atomic number materials.\(^{14,21} \)

2.4 Grids

Grids consist of radiolucent or low attenuation interspace (aluminum, plastic, carbon fiber, card board, etc.) and radiopaque layers (lead, tungsten, gold, etc.).\(^{22} \) Parallel grids consist of radiopaque layers along one dimension. This way primary radiation can be transmitted, while the majority of the scattered rays are absorbed by the grid. A grid is sketched in Figure 2-8.
It can be observed that low angles, scattered rays can transmit pass the grid. The primary to scatter transmission of radiation is dependent on the grid ratio,

\[ \text{Grid Ratio} = \frac{h}{D}. \]  \hspace{1cm} (16)

A grid with a higher grid ratio will reject scatter best, due to the angle limitation permitted by the grids radiopaque layer structure.\(^{14}\) The disadvantage in using high grid ratios is that a higher dose is necessary to compensate for the additional material blocking the photons from the detector. The grid frequency is

\[ \text{Grid Frequency} = \frac{1}{t+D}. \]  \hspace{1cm} (17)

Low frequency grids range from 40 to 50 lines/cm and are commonly used for system with a moving grid that oscillates during exposure in order to blur the grid lines so that they are not visible in the image.\(^{22}\) This moving grid is known as a Bucky grid and was invented by Gustav Bucky in 1913.\(^{14}\) Medium frequency grids range from 50 to 60 lines/cm and high frequency grids range from 60 to 70 lines/cm. These grid types are conventionally used for stationary sets.\(^{22}\) High frequency grids are required for digital radiography in order to eliminate aliasing artifacts introduced from insufficient sampling of high frequency patterns.\(^{23}\) The insufficient sampling is
caused by the periodic grid itself which disguises high frequency signals as low frequency signals, as shown in Figure 2-9.

![Sampling pitch diagram](image)

*Figure 2-9: Insufficient sampling causing aliased artifacts. From reference 23.*

Cross hatched grids consist of radiopaque stripes in both horizontal and vertical directions (perpendicular to one another). This checker board style grid improves spatial resolution due to the very fine pattern and are ideally used for chest imaging and mammography.²⁴

### 2.5 Phase Contrast Imaging Methods

Phase contrast imaging is the measurement of the phase of the x rays not the intensity. It takes advantage of the refractive index of different materials to differentiate between microstructures. The refractive index is used to compute the phase shift which provides higher contrast than the differences in intensity. The change of intensity is heavily dependent on the consistency of the material, thus, structures embedded in materials with similar atomic number do not show adequate contrast. This indicates the importance of phase contrast imaging for biomedical applications, for example tumors embedded in tissues. There are several different
imaging methods of extracting phase that use different experimental set ups. Some phase contrast imaging methods include propagation-based imaging, grating-based imaging and mesh-based imaging.

2.5.1 Propagation-Based Imaging

Propagation-based imaging (PBI) is an in-line arrangement of a source, sample and detector. This technique requires displacement of the detector to obtain absorption and phase information. Placing the detector directly after the object provides an image containing absorption information. To obtain phase information, the detector is placed further from the object where both phase and absorption is collected. Refraction causes some detector regions to receive a higher density of x rays, hence, measuring higher intensity.\(^25\) The phase change occurs from the incident x rays traversing through the sample. The phase information is generated at an intermediate distance from the sample in the form of interference patterns with Fresnel fringes. These fringes lead to strong edge enhancement along the structural boundaries of the sample.\(^26\) This method of phase contrast imaging has been used for clinical implementation, including breast imaging.\(^8\) The set up for propagation-based imaging is shown in Figure 2-10.

![Figure 2-10: Setup for propagation-based imaging, where phase is obtained from interference patterns between a reference beam and beam of unknown phase.](image)
Unfortunately, spatial coherence is a strict constraint required for the formation of fringes, which forces the use of small or very distant sources. This can allow the use of small laboratory sources with sufficient spatial coherence, like micro-focus x-ray tubes, but requires long exposure times because of their low power,\textsuperscript{26} or high brilliance sources like synchrotron imaging.\textsuperscript{25} Another disadvantage is that detecting the fringes requires high resolution detectors with pixel sizes in the tens of micron range resulting in a small field of view.\textsuperscript{8} PBI provides lower contrast in comparison to other phase contrast techniques, especially when density variations in the sample are small. This technique is mainly used for its simplistic setup and edge enhancement.

\textbf{2.5.2 Grating-Based Imaging}

Grating-based phase contrast imaging uses multiple gratings to study the interference pattern. These frequently consist of two gratings, as shown in Figure 2-11. A phase grating is placed a distance after the object to act as a beam splitter for extracting phase and an amplitude grating is placed right before the detector, acting as an analyzer to evaluate the interference pattern. The phase grating creates an image at the Talbot distance with periodic fringes of the same pitch as the grating, where the Talbot distance is,

$$z = 2 \frac{d^2}{\lambda}.$$  \hspace{1cm} (18)

This requires the grating pitch, $d$, to be relatively small.\textsuperscript{14}

This method would only be applicable if a very small micro-focused source was used and placed at a significantly large distance from the phase object. This large distance limitation would result in low flux densities, hence requiring long exposure times.\textsuperscript{27} To apply this technique to conventional, high powered x-ray tubes a third grating can be added to the set up between the x-
ray source and the sample.\textsuperscript{8} This is known as the source grating and it acts as an aperture mask with transmitting slits. An example of a grating-based set up is shown in Figure 2-11.

![Grating interferometer set up](image)

**Figure 2-11: Grating interferometer set up using two different gratings, a phase grating acting as a beam splitter and an analyzing grating for evaluating the interference pattern. A source grating would be placed between the source and the sample if a high powered conventional x-ray source was being used. This source grating would act as an aperture mask with transmitting slits producing individually coherent, but mutually incoherent sources.**

However, the grating interferometer set ups are prone to flaws caused from inaccurate alignment. Thus, high quality gratings and very precise alignment are required which calls for high mechanical stability. Another disadvantage is the demand for multiple images which leads to high dose.

### 2.5.3 Mesh-Based Imaging

To overcome the flaws of grating-based imaging, grid-based methods have been recently introduced by Bennett.\textsuperscript{28} This new method is beneficial by allowing grids or wire meshes with a greater pitch to be used rather than gratings. Also, the low cost of grids in comparison to the high cost and high precision required by gratings is helpful. This technique relies on a grid that is placed between the sample object and detector, as shown in Figure 2-12.
Figure 2-12: Grid based imaging setup demonstrating the un-refracted path of the x-ray and the refracted path of the x-ray caused by traversing the phase object. From reference 14.

The grid provides periodic dark lines on the detector. The object causes the rays to refract, producing position shifts within the image.

A major hurdle for phase contrast imaging has been the small spatial coherence of conventional large spot x-ray sources. Due to the constraint of small spatial coherence, it was necessary for a 10-50 micron source to be required for phase contrast imaging. However, adding a mesh before the detector relaxes this constraint because the mesh acts as a periodic grid which allows Fourier processing. To quantify the phase shifts, the Fourier transformation of the image to the spatial frequency domain is executed providing harmonics in the Fourier domain. A cross hatched grid provides a two-dimensional domain where transformed images can be extracted from both the horizontal and vertical harmonics. This provides phase gradient information both vertically and horizontally. It is necessary for the individual harmonics to be accurately separated in the spatial frequency domain and each first-order harmonic must have sufficient amplitude. Bennett used these conditions to derive a set of design requirements:

\[ E_1 = \frac{s}{p} \frac{d_{GC}}{d_{SC}} \ll 1 \quad \text{and} \quad (19) \]
\[ E_2 = \frac{S}{p} \frac{d_{SC}}{d_{SO}} \frac{d_{SG}}{d_{SC}} > 1 \]  

where S is the size of the source, p is the grid period and as shown in Figure 2-12, \( d_{GC} \) is the grid to camera distance, \( d_{SC} \) is the source to camera distance, \( d_{SO} \) is the source to object distance and \( d_{SG} \) is the source to grid distance. Hence, these general guidelines were used to produce ideal transformed images.

The transformed images were normalized by removing the raw grid image. The normalized zeroth harmonic image is

\[ I_{(0,0)N} = \frac{I_{(0,0)}}{I_{(0,0)G}}. \]  

The normalized first order (horizontal and vertical respectively) harmonic images are

\[ I_{(1,0)N} = \frac{I_{(1,0)}}{I_{(1,0)G}} \]  

\[ I_{(0,1)N} = \frac{I_{(0,1)}}{I_{(0,1)G}}, \]

where \( I_{(0,0)} \), \( I_{(1,0)} \) and \( I_{(0,1)} \) are the processed harmonic images containing the sample object and grid and \( I_{(0,0)G} \), \( I_{(1,0)G} \) and \( I_{(0,1)G} \) are the processed harmonic grid only images. Therefore, \( I_{(0,0)N} \) is the attenuation image and \( I_{(1,0)N} \) and \( I_{(0,1)N} \) are the first order harmonics containing information from both the attenuation and the diffraction in the vertical and horizontal directions. Phase and scattering data from the first harmonics were separated as,

\[ \overline{I_{(n,m)}} = \frac{I_{(n,m)N} - I_{(0,0)N}I_{(0,0)N}}{I_{(0,0)N}} = |\overline{I_{(n,m)}}| e^{i\phi_{(n,m)}}, \]

where the magnitude of \( \overline{I_{(n,m)}} \) are the scattering amplitude images and \( \phi_{(n,m)} \) are the differential phase contrast (DPC) images of the horizontal \((n=1, m=0)\) and vertical \((n=0, m=1)\) components. The images at the first harmonics are subtracted by the attenuation image at the zeroth harmonic and then divided by the attenuation image, which was derived using the transport of intensity.
equation (TIE) for short propagation distances. The two-dimensional phase, $\phi$, is related to the periodicity of the mesh grid and the distances along the set-up,$^9$

$$
\phi = \frac{d_{SO}}{d_{SC}} d_{GC} \overline{g}(n, m) \cdot \hat{\theta}.
$$

where $d_{SO}$, $d_{SC}$ and $d_{GC}$ are the distances of the source, object, grid and camera represented in Figure 2-12 and,

$$
\overline{g}(n, m) = n \overline{g}_1 + m \overline{g}_2
$$

where $n$ and $m$ are integers and the unit vectors perpendicular to the mesh lines are,

$$
\overline{g}_1 = \frac{2\pi}{p_x} \hat{x} \quad \text{and} \quad \overline{g}_2 = \frac{2\pi}{p_y} \hat{y}
$$

where $p_x$ and $p_y$ are the periods of the mesh grid in the x and y directions. When the Fourier transform is taken of the mesh image, delta function-like spikes are produced at the mesh grid harmonics. When the sample being studied is placed within the setup, x rays are deflected due to the thickness and change in density. This causes a distorted image of the mesh grid which becomes the sample image. The angle of reflection caused by the rays passing through the sample object is, $\theta$, where

$$
\hat{\theta} \propto \frac{\lambda}{2\pi} \nabla \varphi
$$

where $\varphi$ is given by equation 2.

### 2.5.4 Fourier Processing Algorithm

A MATLAB program that was written by Professor Jon Petruccelli and Dr. Sajjad Tajir was created to take 2D Fast Fourier transforms (FFT) of a raw tiff image collected by a CCD camera. The phase shift was analyzed from the FFT producing a series of harmonics in the Fourier domain. This code implements a Fast Fourier transform providing the harmonics. These harmonics
in the Fourier Spectrum can be separated and depict the absorption and phase information, as shown in Figure 2-13.

![Diagram showing zeroth and first harmonics in the Fourier Domain](image)

**Figure 2-13**: The zeroth and first harmonics in the Fourier Domain produced by the MATLAB image processing algorithm.

The center/zeroth (0,0) harmonic contains the absorption information, while the first harmonics in the horizontal and vertical regime, (1,0) and (0,1), contain the phase and absorption information.

![Flow diagram of the Fourier Transformation image processing algorithm](image)

**Figure 2-14**: Flow diagram of the Fourier Transformation image processing algorithm, resulting in three processed images: Absorption, Scatter and Phase Contrast images. From reference 29.

The Fourier process and separation of harmonics to produce the absorption, scatter and phase information is shown in Figure 2-14.
3. Experimental Procedure

3.1 Introduction

Phase contrast imaging with Fourier analysis can provide high contrast for materials with low electron density. This type of imaging could be useful for medical imaging by producing images with better contrast of materials composed of light elements, like tissue samples. Tumors are growth masses within the body and have very similar density to the surrounding tissue. For this reason, it is hard to determine location, size and whether-or-not the tumor is harmful using traditional attenuation imaging, especially in the beginning stages of the tumor. These are difficult to study with traditional x-ray imaging due to the low attenuation contrast.

As discussed in section 2.4, if an object is placed before a periodic mesh, the phase can be extracted from the image by windowing the Fourier transform and comparing the data from the harmonics produced by the periodic mesh. A MATLAB code was required to perform the Fast Fourier transformations for image processing. Differential phase contrast (DPC) and scatter amplitudes in both the horizontal and vertical directions were computed and combined into a final enhanced image. A sketch of the mesh-based Fourier transformation imaging setup is shown in Figure 3-1.
Figure 3-1: Mesh Based Fourier Transformation Imaging Setup: The incident x-rays from the source are diffracted when passing through the phase object. These diffracted rays are represented by the dotted red lines which then travel pass the grid (in this case is a wire mesh) or are refracted by the grid. The detector then picks up these deflected/ diffracted rays and the un-deflected rays. The diffracted rays cause distortion in the grid pattern on the camera image. The distances are represented by $d$, where $d_{SO}$ is from the x-ray source to the object, $d_{OG}$ is from the object to the grid, $d_{GC}$ is from the grid to the camera and $d_{SC}$ is the total distance from the x-ray source to the camera. From reference 14.

Mesh-based Fourier imaging was tested on a mouse at an exposure that would be appropriate for humans in order to study the difference between the attenuation image and the enhanced image using phase information.

Another common issue with attenuation imaging is the inability to differentiate between dangerous liquids and powders. This is an ongoing problem for airports by misleading security measures due to inadequate or false imaging. By studying the differential phase contrast, DPC, and scatter amplitudes of several liquids and powders including, water, ethanol, peroxide, saw dust, acetone, gel methanol, flour, citric acid, polyethylene glycol and powdered sugar, differences can be observed distinguishing the various samples from water.

The experimental set up is shown in Figure 3-2.
Figure 3-2: Actual experimental set up in CXO (Center for X-ray Optics) Laboratory where the blue ovals in the figures are representing the location for the phase object. Many experiments were conducted for this thesis, therefore, the phase object changes based on the experiment at hand. a) View of set up from the right side of source, b) view from the left side of source and c) top view of set up.

In the experiments expressed within this thesis, the phase objects included a mouse submerged in formaldehyde for biological applications and a series of several liquids and powders including water, ethanol, peroxide, saw dust, acetone, gel methanol, flour, citric acid, polyethylene glycol and powdered sugar for security applications.
3.2 X-Ray Source

The x-ray source used in these experiments was a micro-focus x-ray source manufactured by Microfocus, Inc. with a spot size of 150 µm. This source included a fixed tungsten (W) anode, run at 0.40 mA (maximum current) and 25 kVp (with a maximum of 100kVp).

3.3 Detector

The detector used for these experiments was a Remote RadEye HR charge coupled device (CCD) camera manufactured by Teledyne. It had 1200x1600 pixels with a pixel size of 22 µm.

3.4 Mesh Grid

The conducted experiments used a wire mesh constructed of woven stainless steel with a period of 123x123 µm with a wire width of about 48 microns. The finer the period the higher the resolution in the image. The wire mesh was placed between two plastic sheets so that it would not bend. This constrained the mesh from bunching or bending and forced it to remain in a single 2D plane. This grid provided a periodic base within the detected image. The object then creates a phase shift in the localized harmonics found by windowing in the Fourier computation.

4. Biological Imaging

4.1 Biological Setup

The first phase object used in the biological set up was a small preserved mouse fixed in formaldehyde, as discussed in section 3.1. Effects of different dose were investigated by extending the exposure time and differences in magnification were studied by increasing the field of view. This was accomplished by changing the distance between the source and the object, $d_{SO}$, from 9.5 cm to 19.5 cm with increments of 2 cm. In doing so, the magnification decreased from about 4.5
to 2.2. The blur was also studied to compare with Bennett’s criteria, as discussed in the magnification section 2.5.3. The following sections discuss the significance of exposure time and magnification and compare the attenuated, scattered, differential phase contrast and enhanced images using the Fourier transform algorithm.

4.2 Exposure Time

To analyze the effects of dose, two images were taken with the same voltage (25 kVp), amperage (0.40 mA), distance \( d_{SO} = 9.5 \, \text{cm} \), and magnification (4.5). The only component modified was the exposure time. One image was exposed for 2 minutes, while the other was exposed for 20 minutes. This time difference was chosen to investigate how 10x the exposure time affected the quality of the processed images. An offset image, also known as a dark image, was taken with the x rays off in order to detect the dark current which can vary over time and temperature. A gain image, also known as a flat field image, was taken with nothing between the source and the camera and the x rays on. This image compensates for pixel-to-pixel variations in detector response and x ray intensity, as well as local variations or dark spots. It is extremely important for the gain image to be obtained at the same source to camera distance and same exposure time. The equation for the gain and offset corrected raw images, \( I_0 \), containing the object of interest is

\[
I_0 = \frac{I_{\text{raw}} - I_{\text{offset}}}{I_{\text{gain}} - I_{\text{offset}}},
\]

(29)

where \( I_{\text{raw}} \) is the raw image containing the object of interest that has not yet been gain and offset corrected, \( I_{\text{offset}} \) is the offset or dark image and \( I_{\text{gain}} \) is the gain image. The different exposure time images were processed using a Gaussian window with full width half max, FWHM = \( \frac{1}{2} \, \text{sw} \) where,
\[ sw = \frac{2\pi}{p}, \] (30)

with \( p \) representing the mesh period. The images are shown in Figure 4-1 and Figure 4-2.

**Figure 4-2:** Image of the mouse’s skull/jaw area for the 2-minute exposure. This had a magnification of 4.5 which contributed to the blur.

**Figure 4-1:** Image of the mouse’s skull/jaw area for the 20-minute exposure with a magnification of 4.5 as well.

To compare Figure 4-1 and Figure 4-2, a vertical linear profile was investigated at the 90th column.

The linear profile of the images and the corresponding intensity profiles are shown in Figure 4-3 and Figure 4-4.

**Figure 4-3:** (a) Linear profile of the 90th column, represented by the red line of the 2-minute exposure. The blue circle represents the edge that was used to quantify the contrast and signal to noise ratio (SNR). (b) Intensity profile of the red line in Figure 4-3(a). The red circle shows the edge of the region within the blue circle in Figure 4-3(a).
Figure 4-4: (a) Linear profile of the same column in Figure 4-3, represented by the red line of the 20-minute exposure. The blue circle represents the edge that was used for quantification. (b) Intensity profile of the red line in Figure 4-4(a). The red circle shows the edge of the region within the blue circle in Figure 4-4(a).

The vertical linear profiles of the 89th and 91st columns were also studied to record multiple measurements for the edge in this region. The contrast and signal to noise ratio (SNR) were computed for these edges to quantify the differences between the two exposure times. The attenuation contrast is,

$$C = \frac{|I_{obj} - I_{back}|}{I_{back}},$$  \hspace{1cm} (31)

where $I_{obj}$ is the maximum intensity of the sample and $I_{back}$ is the minimum intensity of the step edge. The signal to noise ratio uses the amplitude of the signal and background noise to compare the two images,

$$SNR = \frac{I_{max} - I_{min}}{\text{std}_{noise}},$$  \hspace{1cm} (32)

where $I_{max}$ is the maximum of the signal, $I_{min}$ is the minimum of the signal and std$_{noise}$ is the standard deviation of the noise within the background of the image. The standard deviation of the noise was computed using the two-dimensional ‘std2’ function within MATLAB, providing a standard deviation of a rectangular region within the background where no mouse was located.

Using the numerical values for the maximum edge height, minimum edge height and standard
deviation of the noise for the linear profiles produced by the 89th, 90th and 91st columns, the average contrast, average SNR and the corresponding uncertainties for these three columns were computed and are shown in Table 4-1.

<table>
<thead>
<tr>
<th></th>
<th>2 minutes</th>
<th>20 minutes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Avg. Max Edge Height</td>
<td>2.36 ± 0.01</td>
<td>2.17 ± 0.01</td>
</tr>
<tr>
<td>Avg. Min Edge Height</td>
<td>1.26 ± 0.01</td>
<td>1.23 ± 0.01</td>
</tr>
<tr>
<td>Std. Noise</td>
<td>0.07</td>
<td>0.06</td>
</tr>
<tr>
<td>Avg. Contrast</td>
<td>0.47 ± 0.01</td>
<td>0.43 ± 0.01</td>
</tr>
<tr>
<td>Avg. SNR</td>
<td>14.86 ± 0.29</td>
<td>16.73 ± 0.34</td>
</tr>
</tbody>
</table>

*Table 4-1: Comparison of the 2-minute and 20-minute exposures. The averages and uncertainties were obtained from the three linear profiles produced by the 89th, 90th and 91st columns. The average maximum and minimum edge heights for the 20-minute exposure time are similar to the 2-minute exposure (not 10 times larger than the 2-minute exposure) because the raw images were gain corrected.*

The uncertainties for multiple measurements were computed by finding the average values produced by the linear profiles and subtracting the average from each individual linear profile value. The differences between the average and individual linear profile values were each squared, added together, divided by the number of values measured and then square rooted. This method for finding the uncertainty for several measurements was carried out throughout chapter 4. Due to the fact that the standard deviations of the noise were obtained by taking the standard deviation of a region where there was no mouse, the noise was considered overestimated. The uncertainty in the SNR was taken as proportional to the uncertainty in the height of the signal at the edge of interest.

By observing the small change in contrast and noise from Table 4-1, it was concluded that 10 times the exposure time did not greatly improve the SNR or contrast. Therefore, it was decided that a two-minute exposure was efficient for future testing and provided a slightly higher contrast than the longer exposure. Time was saved for more images and experiments by taking shorter
duration images and this also helped preserve the health of the camera. Two minutes produces an exposure of 48 mAs, which is about half of a typical mammography exposure.

4.3 Absorption vs. Conventional Image

The absorption and conventional images of the mouse were compared. The images were gathered using a magnification of about 4.5 and an exposure of 2 minutes. The original or raw image of the mouse with the grid is shown in Figure 4-5(a). The normalized absorption image, $I_{(0,0)N}$, shown in Figure 4-5(b) was the attenuation image captured by the zeroth harmonic using a Gaussian window with a width of $\frac{1}{2}sw$. The conventional image, Figure 4-5(c), is an attenuation image without a grid, taken at the same magnification and exposure time. To compare the absorption image and the conventional image, the conventional image was processed using the Fourier transform code. The conventional original image was transformed to Fourier space and windowed about the origin using the same Gaussian window width used to process the absorption image. The log of the inverse Fourier transform of the conventional image is shown in Figure 4-5(d) to compare the contrast with the absorption image, Figure 4-5(b).

(a) Original Image  
(b) $I_{(0,0)N} = \text{Absorption Image}$
Figure 4-5: (a) Jaw image of mouse with grid. (b) Log of the absorption image from the (0,0) normalized harmonic. (c) Conventional absorption image with no grid interference. (d) Log of the processed conventional image.

The absorption image can be compared to the processed conventional image by calculating the attenuation contrast and signal to noise ratio, using equations 31 and 32. The images in Figure 4-6, show where the linear profiles were gathered and the intensity region where the maximum and minimum intensities were obtained.

Figure 4-6: (a) Absorption image with linear profiles of the 109th, 110th and 111th columns represented by the red line. The blue circle represents the region where the minimum and maximum signals were collected to compute the contrast. (b) Processed conventional image.
The numerical comparison of the absorption image and the processed conventional image are represented in Table 4-2.

<table>
<thead>
<tr>
<th></th>
<th>Absorption</th>
<th>Process Conventional</th>
</tr>
</thead>
<tbody>
<tr>
<td>Avg. Max</td>
<td>1.497 ± 0.009</td>
<td>3.477 ± 0.004</td>
</tr>
<tr>
<td>Avg. Min</td>
<td>0.643 ± 0.003</td>
<td>2.34 ± 0.01</td>
</tr>
<tr>
<td>Std. Noise</td>
<td>0.07</td>
<td>0.09</td>
</tr>
<tr>
<td>Avg. Contrast</td>
<td>0.571 ± 0.001</td>
<td>0.327 ± 0.003</td>
</tr>
<tr>
<td>Avg. SNR</td>
<td>12.47 ± 0.09</td>
<td>12.25 ± 0.09</td>
</tr>
</tbody>
</table>

Table 4-2: Comparison of the processed conventional image and the absorption image. The average and uncertainties were obtained from the three linear profiles produced by the 109th, 110th and 111th columns as represented in Figure 4-6. The method for finding the uncertainties was the same as the method used in section 4-2.

The absorption image, Figure 4-6(a), has a contrast of 0.571 ± 0.001 and SNR of 12.47 ± 0.09, whereas the processed conventional image, Figure 4-6(b), has a contrast of 0.327 ± 0.003 and SNR of 12.25 ± 0.09. This concludes that the absorption image has more contrast and a slightly higher SNR than the conventional image.

4.4 Image Optimization

The separation of harmonics was accomplished through several windowing functions. Generally, a smaller full width half max, FWHM, (thus, a smaller sigma) for a Gaussian window results in images with less resolution and fewer artifacts. Due to the fact that Bennett’s experimental set up guidelines were followed, the harmonics were efficiently separated and there were minimal artifacts. Gaussian windows with a FWHM of ½ sw were first used to study the effects of exposure time and comparing the conventional and absorption images, which provided the clearest images with low noise and no ringing or fringe effects in the phase images. However, other windowing functions were studied to optimize images, including, Gaussian windows with smaller FWHM, Tukey windows, Hann window, Hamming window and Blackman window.
The Gaussian window is,

\[ \text{window} = e^{-X^2/\sigma_x^2} \cdot e^{-Y^2/\sigma_y^2} \quad (33) \]

where \( X \) is the horizontal harmonic and \( Y \) is the vertical harmonic with the sigmas for each harmonic,

\[ \sigma_x = \frac{\text{FWHM}_x}{2\sqrt{2\log(2)}} \quad \text{and} \quad (34) \]
\[ \sigma_y = \frac{\text{FWHM}_y}{2\sqrt{2\log(2)}} \quad . \quad (35) \]

The Gaussian windows were adjusted by altering the FWHM from \( \frac{1}{2} \text{sw} \), \( \frac{1}{4} \text{sw} \), \( \frac{1}{6} \text{sw} \) and \( \frac{1}{8} \text{sw} \). The Tukey window, also known as the tapered cosine window, was another adjustable window taken into consideration where the piecewise function for this window is

\[ \text{Tukey window} = \begin{cases} 
\frac{1}{2} \left\{ 1 + \cos \left( \frac{2\pi}{r} \left[ x - \frac{r_x}{2} \right] \right) \right\}, & 0 \leq x < \frac{r_x}{2} \\
1, & \frac{r_x}{2} \leq x < 1 - \frac{r_x}{2} \\
\frac{1}{2} \left\{ 1 + \cos \left( \frac{2\pi}{r} \left[ x - 1 + \frac{r_x}{2} \right] \right) \right\}, & 1 - \frac{r_x}{2} \leq x \leq 1
\end{cases} \quad (36) \]

This was studied using the ‘tukeywin’ function for both the horizontal and vertical harmonics in MATLAB with \( r \) representing the taper ratio that was adjusted between 0 and 1. With \( r = 0 \), the Tukey window acts as a rectangular window and with \( r = 1 \), it acts as a Hann window.

For the other windowing functions considered, Hann, Hamming and Blackman the preexisting MATLAB functions were used. These windowing functions used symmetrical sequences for the fast Fourier transform analysis. This requires a symmetric window commonly with an odd length so that there is a single maximum at the center. To create symmetrical sequences, the windowing functions provided by MATLAB will automatically delete the right most coefficient of the “even” length to generate a purely real, discrete frequency spectrum with an odd length. This is accomplished by using the ‘periodic’ flag which is important for spectral
analysis like Fast Fourier Transforms (FFTs), for example, window = hann(sw,’periodic’). The Hann window has the form,

\[
window(n) = 0.5 \left[ 1 - \cos \left( \frac{2\pi n}{N-1} \right) \right]
\] (37)

where \( n \) is the variable and \( N \) is the width (N-1 ensures an odd length for a symmetric window).

The Hamming window is very similar to the Hann window, with a small change in coefficients. The Hamming window is,

\[
window(n) = \alpha - \beta \cos \left( \frac{2\pi n}{N-1} \right)
\] (38)

where \( \alpha = 0.54 \) and \( \beta = 0.46 \). The Blackman is a higher-order generalized cosine window with,

\[
window(n) = a_0 - a_1 \cos \left( \frac{2\pi n}{N-1} \right) + a_2 \cos \left( \frac{4\pi n}{N-1} \right)
\] (39)

where \( a_0 = (1-\alpha)/2, a_1 = 0.5 \) and \( a_2 = \alpha/2 \).

The windowing functions help improve the quality of the signal by exploiting the differences between the signal and corrupting noise or artifacts. By taking the Fourier transform of these windows, a series of “lobes” are created including a main lobe with the maximum amplitude at the center, and descending side lobes from the main lobes. Some window functions are shown in Figure 4-7, Figure 4-8 and Figure 4-9.

![Figure 4-7: (a) Hamming windowing function in the space domain. (b) The discrete Fourier transform of the Hamming window in the frequency domain. This shows where the main lobe is located and how the side lobes descend. Also, because the amplitude does not approach zero in the space domain, this causes the side lobe closest to the main lobe to cancel out well. However, all other side lobes have poor cancelation. From reference 30.](image-url)
Figure 4-8: (a) Hann windowing function in space domain. (b) Discrete Fourier transform of the Hann window in frequency domain. This shows the difference in how the side lobes descend in comparison to Figure 4-7. These differences occur because the amplitude in the space domain does approach zero for the Hann window. From reference 30.

Figure 4-9: (a) Blackman windowing function in the space domain. (d) Discrete Fourier transform of the Blackman window in the frequency domain. Comparing this to Figure 4-7 and Figure 4-8, it can be observed that the Blackman window is a combination of the Hann and Hamming windows, but has a wider main lobe. From reference 31.

How the side lobes descend depend on the window chosen. Using the 4.5 magnification mouse image with a two-minute exposure, several windows were compared as shown in Figure 4-10.
Figure 4-10: Comparison of Gaussian windows with different widths and the Blackman window. The linear profiles of the 105th columns were provided to show how the local peaks spread out for the smaller Gaussian window widths resulting with increased blur. The Hann, Hamming and Tukey windowing functions were not added here because they were visually too close to the Gaussian window with width $\frac{1}{2}$ sw and the Blackman window.

As observed by Figure 4-10 the optimal Gaussian window of width $\frac{1}{2}$ sw and the Blackman window produced images with similar resolution. This is the case for the Hann, Hamming and Tukey windows, as well. To compare these different windowing functions the average contrast and average SNR of the inverse Fourier images were computed at the edge formed by the mouse’s tooth as shown in Figure 4-11.

Figure 4-11: The red line represents where the linear profiles were collected, at the 104th, 105th and 106th columns and the blue circle represents the region where the edges were quantified.
The linear profiles produced by the 104\textsuperscript{th}, 105\textsuperscript{th} and 106\textsuperscript{th} columns were used to compute the averages and the corresponding uncertainties as shown in Table 4-3, Table 4-4 and Table 4-5.

<table>
<thead>
<tr>
<th>Window</th>
<th>Gaussian</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>½ sw</td>
<td>¼ sw</td>
<td>1/6 sw</td>
<td>1/8 sw</td>
</tr>
<tr>
<td>Avg. Max</td>
<td>2.36 ± 0.27</td>
<td>1.80 ± 0.09</td>
<td>1.63 ± 0.03</td>
<td>1.632 ± 0.005</td>
</tr>
<tr>
<td>Avg. Min</td>
<td>0.446 ± 0.008</td>
<td>0.85 ± 0.01</td>
<td>0.91 ± 0.02</td>
<td>0.93 ± 0.03</td>
</tr>
<tr>
<td>Std. Noise</td>
<td>0.075</td>
<td>0.023</td>
<td>0.013</td>
<td>0.009</td>
</tr>
<tr>
<td>Avg. Contrast</td>
<td>0.81 ± 0.02</td>
<td>0.53 ± 0.03</td>
<td>0.44 ± 0.02</td>
<td>0.43 ± 0.01</td>
</tr>
<tr>
<td>Avg. SNR</td>
<td>26.6 ± 3.5</td>
<td>41.4 ± 4.6</td>
<td>55.4 ± 3.3</td>
<td>76.6 ± 2.2</td>
</tr>
</tbody>
</table>

Table 4-3: Compares the contrast and signal to noise ratio of the images for several tested Gaussian window functions. The contrast decreases as the window width decreases and the SNR increases as the window width decreases.

<table>
<thead>
<tr>
<th>Window</th>
<th>Tukey</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>r = 0.5</td>
<td>r = 0.6</td>
<td>r = 0.7</td>
<td>r = 0.8</td>
</tr>
<tr>
<td>Avg. Max</td>
<td>4.17 ± 0.43</td>
<td>4.28 ± 0.18</td>
<td>4.14 ± 0.15</td>
<td>3.97 ± 0.18</td>
</tr>
<tr>
<td>Avg. Min</td>
<td>0.55 ± 0.43</td>
<td>0.66 ± 0.34</td>
<td>0.78 ± 0.26</td>
<td>0.94 ± 0.20</td>
</tr>
<tr>
<td>Std. Noise</td>
<td>0.178</td>
<td>0.157</td>
<td>0.135</td>
<td>0.112</td>
</tr>
<tr>
<td>Avg. Contrast</td>
<td>0.86 ± 0.10</td>
<td>0.84 ± 0.08</td>
<td>0.81 ± 0.07</td>
<td>0.76 ± 0.06</td>
</tr>
<tr>
<td>Avg. SNR</td>
<td>20.3 ± 4.2</td>
<td>23.1 ± 3.2</td>
<td>24.9 ± 2.9</td>
<td>26.9 ± 3.0</td>
</tr>
</tbody>
</table>

Table 4-4: Compares the contrast and signal to noise ratio of the images for several tested Tukey window functions. The contrast decreases as the taper ratio increases, as expected because as the ratio increases it acts more like a Hann window than a rectangular window. The SNR increases as the taper ratio increases, as well.

<table>
<thead>
<tr>
<th>Window</th>
<th>Blackman</th>
<th>Hann</th>
<th>Hamming</th>
</tr>
</thead>
<tbody>
<tr>
<td>Avg. Max</td>
<td>2.26 ± 0.11</td>
<td>2.40 ± 0.26</td>
<td>2.44 ± 0.27</td>
</tr>
<tr>
<td>Avg. Min</td>
<td>0.50 ± 0.05</td>
<td>0.43 ± 0.02</td>
<td>0.43 ± 0.04</td>
</tr>
<tr>
<td>Std. Noise</td>
<td>0.021</td>
<td>0.074</td>
<td>0.081</td>
</tr>
<tr>
<td>Avg. Contrast</td>
<td>0.78 ± 0.03</td>
<td>0.82 ± 0.02</td>
<td>0.82 ± 0.01</td>
</tr>
<tr>
<td>Avg. SNR</td>
<td>83.1 ± 7.3</td>
<td>26.6 ± 3.4</td>
<td>24.8 ± 2.9</td>
</tr>
</tbody>
</table>

Table 4-5: Compares the contrast and signal to noise ratio of the images for the Blackman, Hann and Hamming window functions. The Hann and Hamming windows have a slightly higher contrast, but the Blackman window has a significantly larger SNR.
As shown in Figure 4-10, the blur increased as the width of the Gaussian window decreased. This is supported by Table 4-3, which shows the gradual decrease of contrast as the widths decrease. As observed by Table 4-3, Table 4-4 and Table 4-5, the uncertainties in the signal-to-noise ratio were relatively large. This large uncertainty was caused from the large 22 µm pixel size of the detector. The peak or edge height region occurs within 1 to 3 pixels, therefore the intensity of the signal is averaged over a pixel. This could cause the maximum of the peak to be realistically larger than the average detected by the camera and could cause the minimum to be smaller, therefore, the height of the signal may be underestimated. Also, the region of the edge being measured is slanted as shown in Figure 4-11 by the blue circle, causing the peak to slightly shift for neighboring columns. Depending on how far the signal shifts and what pixel of the camera detects and averages the intensity of the signal, the signal height is affected. This is a reoccurring problem for the SNR uncertainties throughout chapter 4.

Overall, the windows produced similar contrast, apart from the Gaussian windows with smaller window widths, but provided vast differences in SNR. Taking both these imaging quantifications into consideration, it was concluded that the Blackman window created similar contrast with a comparatively large SNR than the competing windowing functions. Hence, the Blackman window provided the most detailed image without additional artifacts or fringing effects.

After the Blackman window was applied, the inverse Fourier transforms were performed to return the zeroth and first harmonics back to the spatial domain resulting in the processed images. To enhance the two-dimensional phase images, the DPC images from the horizontal and vertical harmonics were combined using the Pythagorean sum and a ratio factor,

\[
I_{2D\ phase} = \sqrt{\varphi_v^2 + r \cdot \varphi_h^2},
\]

(40)
where $I_{2D\text{phase}}$ is the two-dimensional phase contrast image, $\varphi_h$ and $\varphi_v$ are the DPC images at the first horizontal and vertical harmonics consecutively and $r$ is the horizontal-to-vertical DPC ratio used to combine the two DPC images to maximize edge enhancement. Vertical linear profiles were studied of the $104^{th}$, $105^{th}$ and $106^{th}$ columns to quantify the differences between different DPC ratio factors as shown in Table 4-6.

<table>
<thead>
<tr>
<th>DPC Ratio</th>
<th>Avg. Max</th>
<th>Avg. Min</th>
<th>Std. Noise</th>
<th>Contrast</th>
<th>SNR</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.49 ± 0.02</td>
<td>0.26 ± 0.02</td>
<td>0.010</td>
<td>0.46 ± 0.03</td>
<td>22.5 ± 2.2</td>
</tr>
<tr>
<td>2</td>
<td>0.58 ± 0.03</td>
<td>0.28 ± 0.03</td>
<td>0.012</td>
<td>0.51 ± 0.06</td>
<td>24.9 ± 3.4</td>
</tr>
<tr>
<td>2.5</td>
<td>0.62 ± 0.03</td>
<td>0.29 ± 0.04</td>
<td>0.013</td>
<td>0.53 ± 0.06</td>
<td>25.2 ± 3.4</td>
</tr>
<tr>
<td>2.7</td>
<td>0.63 ± 0.03</td>
<td>0.29 ± 0.04</td>
<td>0.013</td>
<td>0.54 ± 0.06</td>
<td>25.2 ± 3.3</td>
</tr>
<tr>
<td>2.8</td>
<td>0.63 ± 0.03</td>
<td>0.29 ± 0.04</td>
<td>0.014</td>
<td>0.54 ± 0.06</td>
<td>25.25 ± 3.3</td>
</tr>
<tr>
<td>2.9</td>
<td>0.64 ± 0.03</td>
<td>0.29 ± 0.04</td>
<td>0.014</td>
<td>0.54 ± 0.06</td>
<td>25.26 ± 3.2</td>
</tr>
<tr>
<td>3</td>
<td>0.65 ± 0.03</td>
<td>0.29 ± 0.04</td>
<td>0.014</td>
<td>0.55 ± 0.06</td>
<td>25.1 ± 3.1</td>
</tr>
<tr>
<td>3.5</td>
<td>0.67 ± 0.03</td>
<td>0.29 ± 0.04</td>
<td>0.015</td>
<td>0.56 ± 0.05</td>
<td>24.9 ± 2.9</td>
</tr>
<tr>
<td>4</td>
<td>0.69 ± 0.03</td>
<td>0.30 ± 0.03</td>
<td>0.016</td>
<td>0.57 ± 0.05</td>
<td>24.5 ± 2.7</td>
</tr>
<tr>
<td>5</td>
<td>0.73 ± 0.04</td>
<td>0.30 ± 0.03</td>
<td>0.018</td>
<td>0.59 ± 0.04</td>
<td>23.7 ± 2.4</td>
</tr>
<tr>
<td>6</td>
<td>0.77 ± 0.04</td>
<td>0.30 ± 0.03</td>
<td>0.020</td>
<td>0.60 ± 0.04</td>
<td>22.7 ± 2.1</td>
</tr>
</tbody>
</table>

Table 4-6: Comparison of the horizontal-to-vertical ratio used to combine the DPC images into a single two-dimensional image. The contrast increases as the ratio increases. This shows that a horizontal-to-vertical DPC ratio of 2.9 is optimal and produces the largest SNR.

Therefore, a ratio of 2.9 is optimal, resulting in the highest SNR of 25.26 ± 3.2. The horizontal-to-vertical ratio is larger than 1 because of the edge being focused on. The mouse’s tooth edge is slanted but is more horizontal than vertical which is why the optimal 2D phase images require more horizontal information than vertical information, thus giving a ratio greater than 1. Also, as suggested by Table 4-6, both the contrast and SNRs were extremely similar to one another, hence it was hard to depict which ratio was better from visually observing the images. Figure 4-12 shows
the visual comparison and profiles of the ratios that have the greatest differences in SNR from the ideal 2.9 ratio.

Figure 4-12: The top images are the two-dimensional phase contrast image. The bottom images are the corresponding linear profiles at the 105th column. This shows the phase contrast with a horizontal-to-vertical ratio of (a) 2, (b) 2.9 and (c) 6.

As the ratio increases the noise increases, which is why the signal to noise ratio decreases.

Once the horizontal-to-vertical DPC ratio was optimized, the final DPC enhanced image including both the absorption image and the two-dimensional phase gradient image required improvement. The DPC enhanced image was refined by increasing the 2D gradient of the phase images by a factor before adding the absorption image,

$$I_{DP_{C\text{\,}}\text{\,enhanced}} = I_{abs} + (\text{factor} \cdot I_{2D \text{\,phase \,gradient}}),$$

(41)

where $I_{DP_{C\text{\,\,enhanced}}}$ is the DPC enhanced final image, $I_{abs}$ is the absorption image at the (0,0) harmonic, $I_{2D \text{\,phase \,gradient}}$ is the gradient of the phase images collected by the first harmonics added together and factor is simply the multiplication scalar. The gradient of the DPC provided
increased edge enhancement as the DPC factor increased, but also increased the noise, as shown in Table 4-7.

<table>
<thead>
<tr>
<th>DPC Factor</th>
<th>DPC Enhanced Image</th>
<th>Linear Profile</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td><img src="image1.jpg" alt="Image 1" /></td>
<td><img src="profile1.png" alt="Profile 1" /></td>
</tr>
<tr>
<td>1</td>
<td><img src="image2.jpg" alt="Image 2" /></td>
<td><img src="profile2.png" alt="Profile 2" /></td>
</tr>
<tr>
<td>2</td>
<td><img src="image3.jpg" alt="Image 3" /></td>
<td><img src="profile3.png" alt="Profile 3" /></td>
</tr>
<tr>
<td>3</td>
<td><img src="image4.jpg" alt="Image 4" /></td>
<td><img src="profile4.png" alt="Profile 4" /></td>
</tr>
</tbody>
</table>
Table 4-7: This shows the DPC enhanced images for each DPC factor studied. The red line represents where the linear profile was collected, at the 105<sup>th</sup> column. The linear profile corresponds to the red line for each enhanced image using the selected DPC factor.

Due to the increase in noise, the SNR ratios decreased as the DPC factor, edge enhancement and contrast increased. Therefore, to optimize the DPC factor the resolution was examined by computing the FWHM of a sigmoidal curve fitted to a peak formed by the mouse’s tooth edge. The edge being referred to is the area represented by the blue circle in Figure 4-13, along the 105<sup>th</sup> column.

Figure 4-13: The red line represents where the linear profile was taken and the blue circle represents the edge used for computing the contrast, SNR and FWHM.
Excel was used to plot the intensity at the 105\textsuperscript{th} column within the region of interest (the blue circle in Figure 4-13). These scatter points were then fitted to the Gaussian error function,

\[ F(x) = A' \int_{x_0}^{x} e^{\frac{(x'-\mu)^2}{2\sigma^2}} \, dx' = A e^{r f \left( \frac{x-\mu}{\sqrt{2}\sigma} \right)} + d \]  \hspace{1cm} (42)

where \( A \) is the amplitude of the curve, \( x \) is the pixel within the region of interest, \( \mu \) is the x position of the center of the peak, \( \sigma \) is the standard deviation which controls the curves width and \( d \) is just an offset. The parameters \( A, \mu, \sigma \) and \( d \) were adjusted using excel solver so that the root sum squared was minimized. Where the error between the fitting function and the intensity was represented by the root sum squared,

\[ RSS = \sqrt{[y(x) - F(x)]^2} , \hspace{1cm} (43) \]

with \( y(x) \) equal to the intensity from the linear profile at pixel, \( x \) and \( F(x) \) equal to the Gaussian error function at pixel, \( x \). The root sum squared was computed for each pixel within the tooth edge region. By minimizing the error, the Gaussian error function was fit to the intensities collected by the linear profile providing an adjusted standard deviation, \( \sigma \) as shown in Figure 4-14.
Figure 4-14: This shows the Gaussian error function fit to the intensity values from the linear profiles within the tooth edge region. The blue scatter points refer to the intensities from the peak region of interest and the orange line represents the Gaussian error function fit to the intensity scatter points by minimizing the root sum square at each pixel.

Due to the fact that the images were windowed out during the Fourier transformations, the number of ‘pixels’ in the windowed image is different from the numbers of pixels in the original raw images or detector pixels. The windowed images had a total pixel count of 129 x 129 pixels and the original raw image had a total vertical pixel count of 1200 pixels and horizontal count of 1600 pixels. Due to the fact that the linear profiles are vertical, the effective pixel size for the resolution will be computed using the N = 1200 pixels from the original raw image or detector and
\[ n_w = 127 \text{ pixels from the windowed image. However, the grid is being magnified due to geometrical magnification within the setup which will be discussed in section 4.5, which must also be considered when computing the effective pixel size. This is the equation to compute the number of pixels with respect to the mesh grid and detector, } n. \]

\[ n = \left( \frac{1}{g} \right) \left/ \left( \frac{1}{\pi \rho} \right) \right. \]  

(44)

Where \( g \) is the mesh period of about 123 microns and \( \rho \) is the pixel size on the detector of about 22 microns. Therefore, by plugging in all these values we get \( n = 214.6 \) pixels. Magnification is computed through dividing \( n \) by the number of ‘pixels’ in the windowed image, where \( n / n_w \) produces a magnification of about 1.66. By using this magnification, the final pixel, \( p_f \) size is computed where 129 pixels times 1.66 magnification is about 214.14 microns. Hence, the final pixel size depends on the magnified mesh period, not the original pixel size and the effective pixel size, \( p_{eff} \), is given by

\[ p_{eff} = \frac{p_f}{M}, \]  

(45)

where \( M \) is the geometrical magnification of the mouse, in this case \( M = 4.5 \). Therefore, the effective pixel size would be 47.6 pixels. The effective pixel size is then used to compute the resolution by taking the value of the standard deviation that the Solver application provides and multiplying it by the effective pixel size. The resolution can be further quantified by computing the FWHM,

\[ FWHM = 2 \cdot \sqrt{2 \ln(2) \cdot \sigma}. \]  

(46)

Due to the fact that sigma was strongly dependent on the initial guess of the varying parameters, \( A, \mu, \sigma \) and \( d \) the initial guesses for the parameters were slightly changed in order to get three different values for the parameters. Using the different values for the amplitude and...
standard deviation, the averages and uncertainties were obtained. \( \mu \) did not vary because it denotes the center of the peak and the root sum squared did not vary because the solver application found the local minimum value for this parameter. The root sum squared (RSS), average sigma \((\sigma)\), average amplitude \((A)\) and FWHM values were recorded in Table 4-8 for DPC factors of 0.5, 1, 2, 3, 4, 5, 6, 7, 8 and 10.

<table>
<thead>
<tr>
<th>DPC Factor</th>
<th>Amplitude</th>
<th>Sigma (pixels)</th>
<th>FWHM (pixels)</th>
<th>RSS</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>1.31 ± 0.14</td>
<td>79.8 ± 8.7</td>
<td>187.9 ± 20.6</td>
<td>0.07</td>
</tr>
<tr>
<td>1</td>
<td>1.34 ± 0.13</td>
<td>71.3 ± 6.7</td>
<td>167.9 ± 15.7</td>
<td>0.06</td>
</tr>
<tr>
<td>2</td>
<td>1.34 ± 0.14</td>
<td>54.5 ± 5.6</td>
<td>128.3 ± 13.2</td>
<td>0.04</td>
</tr>
<tr>
<td>3</td>
<td>1.65 ± 0.25</td>
<td>53.7 ± 8.0</td>
<td>126.4 ± 18.8</td>
<td>0.05</td>
</tr>
<tr>
<td>4</td>
<td>1.74 ± 0.28</td>
<td>47.2 ± 7.6</td>
<td>111.2 ± 17.9</td>
<td>0.07</td>
</tr>
<tr>
<td>5</td>
<td>1.77 ± 0.11</td>
<td>41.2 ± 2.6</td>
<td>97.0 ± 6.1</td>
<td>0.09</td>
</tr>
<tr>
<td>6</td>
<td>1.78 ± 0.07</td>
<td>36.3 ± 1.4</td>
<td>85.5 ± 3.3</td>
<td>0.11</td>
</tr>
<tr>
<td>7</td>
<td>2.12 ± 0.14</td>
<td>38.5 ± 2.6</td>
<td>90.7 ± 6.0</td>
<td>0.14</td>
</tr>
<tr>
<td>8</td>
<td>2.38 ± 0.14</td>
<td>38.9 ± 2.3</td>
<td>91.7 ± 5.3</td>
<td>0.16</td>
</tr>
<tr>
<td>10</td>
<td>3.04 ± 0.12</td>
<td>41.4 ± 1.6</td>
<td>97.4 ± 3.7</td>
<td>0.21</td>
</tr>
</tbody>
</table>

*Table 4-8: Table of the error between the actual intensities and the Gaussian error fit line (RSS), standard deviation, amplitude and full width half max for each DPC factor studied. Generally, as the DPC factor increases the error increases as well, apart from a factor of 0.5 and 1. The table shows that the optimal DPC factor with the smallest FWHM, hence producing the best resolution, is a factor of 6.*

By observing Table 4-8, the image that has the smallest FWHM of 85.5 ± 3.3 pixels and therefore the best resolution uses a DPC factor of 6.

The final scatter enhanced image consisting of the absorption and scatter images was also optimized. The same optimal 2.9 horizontal-to-vertical ratio that resulted from observing the horizontal-to-vertical DPC ratio in Table 4-6 was used to compute an optimal two-dimensional scatter image, \( I_{2D\text{scatter}} \), which is the Pythagorean sum of the scatter images,
\[ I_{2D\ scatter} = \sqrt{S_v + r \cdot S_h}. \]  

(47)

\(S_v\) is the scatter image at the first vertical harmonic, \(S_h\) is the scatter image at the first horizontal harmonic and \(r\) is the horizontal-to-vertical ratio with \(r = 2.9\). This 2D scatter image was then multiplied by a factor and added to the absorption image to provide a final scatter enhanced image as represented by,

\[ I_{scatter\ enhanced} = I_{abs} + (factor \cdot I_{2D\ scatter}). \]

(48)

The same linear profile at the 105\textsuperscript{th} horizontal column and region has shown in Figure 4-13 for each trial was studied to compare different scatter factors. This scatter factor followed the same pattern as the DPC factor, where the noise, edge enhancement and contrast increased as the scatter factor increased, but the SNR decreased as shown in Table 4-9.

<table>
<thead>
<tr>
<th>Scatter Factor</th>
<th>Scatter Enhanced Image</th>
<th>Linear Profile</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td><img src="image1.png" alt="Image" /></td>
<td><img src="image2.png" alt="Image" /></td>
</tr>
<tr>
<td>1</td>
<td><img src="image3.png" alt="Image" /></td>
<td><img src="image4.png" alt="Image" /></td>
</tr>
</tbody>
</table>
Table 4-9: This table compares different scatter factors by visually observing the noise increase as the scatter factor increases in the enhanced scatter images and comparing the peaks within the corresponding vertical linear profiles taken by the 105th column. The edge that was fit using the Gaussian error function was the mouse’s tooth as shown in Figure 4-13.
Therefore, similar to the DPC factor, the resolution was investigated to compute the optimal scatter factor. By applying the same fitting process using the Gaussian error function and excel solver as demonstrated when optimizing the DPC factor, the following graphs were created.
Figure 4-15: This shows the Gaussian error function fit to the intensity values from the linear profiles within the tooth edge region. The blue scatter points refer to the intensities from each pixel within the peak region of interest and the red line represents the Gaussian error function fit to the intensity scatter points by minimizing the root sum square at each pixel.

Using equations 42, 43 and 46, the root sum squared, average sigma, average amplitude and FWHM were computed using the excel solver application as exhibited when optimizing the DPC factor. The following variables are compared in Table 4-10, where the effective pixel size was used to compute the adjusted sigma and amplitude values with units of pixels.
<table>
<thead>
<tr>
<th>Scatter Factor</th>
<th>Amplitude</th>
<th>Sigma (pixels)</th>
<th>FWHM (pixels)</th>
<th>RSS</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>3.65 ± 0.13</td>
<td>184.3 ± 6.5</td>
<td>434.0 ± 15.2</td>
<td>0.10</td>
</tr>
<tr>
<td>1</td>
<td>4.47 ± 0.13</td>
<td>174.1 ± 5.0</td>
<td>409.9 ± 11.8</td>
<td>0.11</td>
</tr>
<tr>
<td>2</td>
<td>6.03 ± 0.12</td>
<td>161.3 ± 3.3</td>
<td>379.7 ± 7.8</td>
<td>0.13</td>
</tr>
<tr>
<td>3</td>
<td>7.74 ± 0.09</td>
<td>157.4 ± 1.7</td>
<td>370.5 ± 4.1</td>
<td>0.17</td>
</tr>
<tr>
<td>4</td>
<td>8.98 ± 0.15</td>
<td>147.3 ± 2.4</td>
<td>347.0 ± 5.7</td>
<td>0.21</td>
</tr>
<tr>
<td>5</td>
<td>10.58 ± 0.30</td>
<td>145.5 ± 4.1</td>
<td>342.7 ± 9.8</td>
<td>0.26</td>
</tr>
<tr>
<td>6</td>
<td>12.51 ± 0.17</td>
<td>148.1 ± 2.0</td>
<td>348.7 ± 4.8</td>
<td>0.31</td>
</tr>
<tr>
<td>7</td>
<td>14.65 ± 0.13</td>
<td>152.3 ± 1.4</td>
<td>358.5 ± 3.3</td>
<td>0.36</td>
</tr>
<tr>
<td>8</td>
<td>16.87 ± 0.27</td>
<td>156.2 ± 2.5</td>
<td>367.9 ± 5.9</td>
<td>0.41</td>
</tr>
<tr>
<td>10</td>
<td>21.16 ± 0.63</td>
<td>160.8 ± 4.8</td>
<td>378.7 ± 11.2</td>
<td>0.52</td>
</tr>
</tbody>
</table>

Table 4-10: This table compares different scatter factors and shows that the optimal resolution occurs with a factor of 5. Similar to Table 6, the error increases as the factor increases.

The enhanced scatter image with the best resolution consisting of the smallest FWHM of $342.7 ± 9.8$ pixels is produced with a scatter factor of 5 as shown in Table 4-10. Typically, scatter is used to describe microstructures smaller than the camera resolution and is not used to enhance edges. To enhance edges the phase information is studied. Furthermore, the enhanced DPC images with the best resolution were computed using a horizontal-to-vertical DPC ratio of 2.9 and a DPC factor of 6, while the enhanced scatter images were computed using a horizontal-to-vertical scatter ratio of 2.9 and a scatter factor of 5.

### 4.5 Magnification

Object magnification is dependent on the field of view within the set up. The geometric magnification is sketched in Figure 4-16.
By similar triangles, the image size is related to the object size by

$$\frac{h}{d_{so}} = \frac{H}{d_{sc}}.$$  \hspace{1cm} (49)

Therefore, the magnification of the object at the camera is,

$$M = \frac{d_{sc}}{d_{so}}.$$  \hspace{1cm} (50)

Magnification can be investigated by increasing the distance between the source and the mouse. The width of the mouse, connected to a ring stand, was a constraint of how close the object could get to the grid, therefore, limiting the maximum source-to-object distance to 19.5 cm.

Furthermore, it is required that the individual harmonics are cleanly separated in the spatial frequency domain and the first order harmonics in the x and y direction have sufficient amplitudes. The requirements from Bennett, equations 19 and 20, allow images to be obtained from each harmonic. In addition to Bennett’s criteria for phase imaging, even in conventional imaging the images appear to be “smudged” due to the fact that the x-rays emerge from a spot on the anode, not an infinitesimal point. This blur can increase from magnification. Blur is

$$Blur = \frac{(d_{sc} + d_{GC}) \times S}{d_{so}}.$$  \hspace{1cm} (51)
The results of Bennett’s rules, demagnification distances, equation 50 and equation 51, are given in Table 4-11.

<table>
<thead>
<tr>
<th>dGC (cm)</th>
<th>dSC (cm)</th>
<th>dSO (cm)</th>
<th>dSG (cm)</th>
<th>E₁ &lt;&lt; 1</th>
<th>E₂ &gt; 1</th>
<th>M</th>
<th>Blur</th>
</tr>
</thead>
<tbody>
<tr>
<td>17</td>
<td>43</td>
<td>9.5</td>
<td>26</td>
<td>0.48</td>
<td>2.6</td>
<td>4.5</td>
<td>0.068</td>
</tr>
<tr>
<td>17</td>
<td>43</td>
<td>11.5</td>
<td>26</td>
<td>0.48</td>
<td>2.0</td>
<td>3.7</td>
<td>0.056</td>
</tr>
<tr>
<td>17</td>
<td>43</td>
<td>13.5</td>
<td>26</td>
<td>0.48</td>
<td>1.6</td>
<td>3.2</td>
<td>0.048</td>
</tr>
<tr>
<td>17</td>
<td>43</td>
<td>15.5</td>
<td>26</td>
<td>0.48</td>
<td>1.3</td>
<td>2.8</td>
<td>0.042</td>
</tr>
<tr>
<td>17</td>
<td>43</td>
<td>17.5</td>
<td>26</td>
<td>0.48</td>
<td>1.1</td>
<td>2.5</td>
<td>0.037</td>
</tr>
<tr>
<td>17</td>
<td>43</td>
<td>19.5</td>
<td>26</td>
<td>0.48</td>
<td>0.9</td>
<td>2.2</td>
<td>0.033</td>
</tr>
</tbody>
</table>

The images were processed using the Fourier MATLAB algorithm. With decreasing magnification, more of the mouse’s skull is viewed. Also, because the mouse had to be moved in order to increase the distance between the source and the object, slightly different portions of the skull are shown. As observed in Table 4-11 the requirement E₂ was broken for a magnification of 2.2. However, requirement E₁ was held and E₂ was not far off from Bennett’s requirement, so it was decided to test this magnification anyway. This allowed more of the mouse to be viewed with less blur, without causing problematic disturbances when separating the individual harmonics.
Figure 4-17: Compares the traditional, phase gradient, processed attenuation and enhanced images of the mouse at different magnifications.
Figure 4-17 shows how magnification affected the 2D phase gradient which in turn affected the enhanced image. To specifically compare the phase gradients at magnifications, 3.2, 2.8, 2.5 and 2.2, linear profiles were collected around the same region of the mouse as shown in Figure 4-18.

<table>
<thead>
<tr>
<th>M</th>
<th>Phase Gradient Image</th>
<th>Linear Profile</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.2</td>
<td><img src="image" alt="Phase Gradient Image" /></td>
<td><img src="image" alt="Linear Profile" /></td>
</tr>
<tr>
<td>2.8</td>
<td><img src="image" alt="Phase Gradient Image" /></td>
<td><img src="image" alt="Linear Profile" /></td>
</tr>
<tr>
<td>2.5</td>
<td><img src="image" alt="Phase Gradient Image" /></td>
<td><img src="image" alt="Linear Profile" /></td>
</tr>
<tr>
<td>2.2</td>
<td><img src="image" alt="Phase Gradient Image" /></td>
<td><img src="image" alt="Linear Profile" /></td>
</tr>
</tbody>
</table>

*Figure 4-18: The red line in each phase gradient image shows where the linear profiles were collected to study the same region of the mouse’s skull. The red circles in the linear profiles show the edge of the skull under investigation.*
The contrast and signal-to-noise ratios were computed for the edges within the red circles in each linear profile. The averages were obtained by computing the contrast and SNRs from the linear profiles shifted by one row up and down from the original linear profile denoted by the red line in Figure 4-18. This data was compiled into Table 4-12.

<table>
<thead>
<tr>
<th>M</th>
<th>3.2</th>
<th>2.8</th>
<th>2.5</th>
<th>2.2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Avg. Max</td>
<td>0.225 ± 0.003</td>
<td>0.18 ± 0.02</td>
<td>0.22 ± 0.02</td>
<td>0.222 ± 0.002</td>
</tr>
<tr>
<td>Avg. Min</td>
<td>-0.15 ± 0.10</td>
<td>-0.15 ± 0.10</td>
<td>-0.11 ± 0.07</td>
<td>-0.14 ± 0.09</td>
</tr>
<tr>
<td>Std. Noise</td>
<td>0.014</td>
<td>0.0114</td>
<td>0.0115</td>
<td>0.01</td>
</tr>
<tr>
<td>Avg. Contrast</td>
<td>1.67 ± 0.07</td>
<td>1.87 ± 0.09</td>
<td>1.52 ± 0.07</td>
<td>1.62 ± 0.03</td>
</tr>
<tr>
<td>Avg. SNR</td>
<td>26.9 ± 0.8</td>
<td>28.9 ± 2.2</td>
<td>29.3 ± 1.5</td>
<td>35.9 ± 0.8</td>
</tr>
</tbody>
</table>

Table 4-12: Compares the average contrast and average SNR between magnifications of 3.2, 2.8, 2.5 and 2.2. All magnifications have similar contrast, but the magnification of 2.2 has the largest SNR, thus consists of the ideal distances.

Notice that the contrast is greater than one, which is not normally the case. This is occurring from taking the gradient of the phase images which produces a positive maximum and negative minimum, hence, using equation 31 a contrast value greater than one is computed.

According to Table 4-12, the magnification of 2.2 followed closely to Bennett’s criteria and produced the highest average SNR of 35.9 ± 0.8 without fringing and unwanted artifacts. This image provided sufficient phase contrast, a larger imaging window, and edge enhancement. After analyzing Bennett’s predictions in Table 4-11, the magnification images in Figure 4-18 and the quantifications in Table 4-12, it was concluded that the distances d_{SO}, d_{SG}, d_{GC}, and d_{SC} at respectively, 19.5 cm, 26 cm, 17 cm and 43 cm were optimal for the enhanced image as shown in Figure 4-19.
Figure 4-19: Enhanced image of mouse by summing the attenuation image from the zeroth harmonic to the 2D sum of the gradient of the differential phase contrast for the first harmonics in the horizontal and vertical directions. This image had a magnification of 2.2 and was retrieved using a Blackman window.

The image represented by Figure 4-19 went through the same DPC enhanced optimization process as discussed in section 4.4. The contrast for this 2.2 magnification image followed the same pattern as the 4.5 magnification image by increasing as the DPC factor increased. The SNR had a similar repetition as the 4.5 magnification image by decreasing as the DPC factor increased. The resolution was quantified by using the solver application in excel and minimizing the root sum square of the Gaussian error function and actual intensity scatter points. Therefore, to optimize the DPC enhanced image, one needs to compare the contrast, SNR and resolution. Depending on what is more pressing, contrast or resolution, the DPC factor will differ. Due to the fact that the enhanced image has a smaller magnification of 2.2, the DPC factor differs from that of the 4.5 magnified image. For the DPC enhanced 2.2 magnified mouse image, the resolution was optimized providing an optimal DPC factor of 1.2.
4.6 Phase Gradient Image

The original mouse image of magnification 2.2 was further analyzed to obtain the differential phase contrast (DPC) images of horizontal (1,0) and vertical (0,1) harmonics. The same Blackman window was used for separating the harmonics and a two-minute exposure time was continued. The zeroth harmonic does not contain any phase contrast information, therefore the DPC is studied at the first harmonics. The DPC for the horizontal and vertical harmonics is shown in Figure 4-20.

![Horizontal (1,0) DPC](image1.jpg) ![Vertical (0,1) DPC](image2.jpg)

(a) Horizontal (1,0) DPC  (b) Vertical (0,1) DPC

*Figure 4-20: (a) Differential phase contrast of the horizontal first harmonic showing an increase in edge enhancement along the x direction. (b) Differential phase contrast of the vertical first harmonic showing edge enhancement along the y direction.*

These images show an increase in edge enhancement along the x direction for Figure 4-20(a) and edge enhancement along the y direction for Figure 4-20(b). The DPC images were combined by taking the gradient of the two images separately and then adding the horizontal and vertical
components together to produce the two-dimensional gradient of the phase contrast ($I_{2D \, phase \, gradient}$) as shown in Figure 4-21.

![Image](image_url)

**Figure 4-21:** Two-dimensional gradient of the phase contrast gathered from the gradients of the horizontal and vertical first harmonics separately and then adding them together.

This shows that there is more edge enhancement for phase contrast imaging than traditional imaging.

### 4.7 Scattering Amplitude Image

Further analysis was continued for the mouse image of magnification 2.2. This consisted of computing the scattering amplitude images in the horizontal and vertical first harmonics represented by the magnitudes of $T_{(n,m)}$ with $n=1$ and $m=0$ or $n=0$ and $m=1$, as given in equation 24. The scattering images for the first harmonics are shown in Figure 4-22. These scattering images show the additional edge enhancement that can be obtained.
Comparing the two-dimensional gradients of the differential phase contrast (DPC) and the scatter, it can be observed which one shows more information. The two-dimensional gradient of the scatter was computed by taking the gradient of scatter image at each first harmonic and then adding these gradients together. This comparison is demonstrated in Figure 4-23.

Figure 4-22: Scattering amplitude images from the first horizontal and vertical harmonics.

Figure 4-23: a) The 2D gradient of the scatter images from the horizontal and vertical first harmonics combined. b) The 2D gradient of the DPC images from the horizontal and vertical first harmonics combined, which shows more edge enhancement.
From these images, it can be concluded that the 2D gradient of the DPC has better edge enhancement than the scatter image.

5. Material Discrimination for Security

5.1 Security Setup

In the second experiment a variety of oxidants and combustible materials were tested for the ability to be distinguished from water, including ethanol, peroxide, saw dust, acetone, gel methanol, flour, citric acid, polyethylene glycol and powdered sugar. The distances where slightly adjusted to 21.5 cm ($d_{SO}$), 26 cm ($d_{SG}$), 17 cm ($d_{GC}$) and 43 cm ($d_{SC}$) in order to capture the small samples. An addition to the MATLAB code was added to study the profile vertically down the meniscus of the liquids and center of the powders. An edge height was computed at the interface where the air met the liquids and powders. Studying these edge heights, or intensities, of the scattered images, attenuated image and DPC images provided information that could be used to distinguish between each of the materials.

5.2 Phase Contrast Images

The liquids and powders were distributed into small plastic centrifuge vials. Due to the corrosive nature of acetone, extra caution and quick imaging was necessary when experimenting with this liquid. The plastic vials were not made of polypropylene plastic and therefore, the acetone would eat through the vials over time. The intensity variation between the air and the sample within the vial was studied. By observing the edge height of the sample and air, the combination of the absorption image collected from the (0,0) harmonic and the scatter image collected from the first harmonics provided sufficient contrast to compare most of the materials, even though attenuation is dependent primarily on atomic number. The DPC profile spikes provide additional information.
which is useful when discriminating between materials with similar absorption-to-scatter edge heights. The attenuation image is the absorption image gathered at the zeroth harmonic of the Fourier spectrum. The attenuation image is shown in Figure 5-1(b) and was obtained using the Blackman window.

Phase contrast imaging depends on electron density which allows improvement in the detection of different materials. Due to the fact that phase changes more rapidly at the edges of materials, there is a significant increase in edge enhancement. The edge enhancement was visible in the intensity profiles of the materials. The phase contrast and scatter amplitude of ethanol is shown in Figure 5-1 (d) and (c).

![Image](a.png) ![Image](b.png) ![Image](c.png) ![Image](d.png)

*Figure 5-1: Images of ethanol. (a) Original image of ethanol with grid. (b) Processed attenuation image gathered from the zeroth harmonic. (c) Scatter amplitude image from the (0,1) harmonic, with edge enhancement for the vial and strong meniscus. (d) Differential phase contrast image from the (0,1) harmonic with some edge enhancement from vial and meniscus. All images were obtained using the Blackman window.*

The processed attenuation image (from the zeroth harmonic) shows strong contrast between the sample and air as anticipated. The scatter amplitude and DPC images for the first harmonic, (0,1), show some edge enhancement to differentiate between substances of similar attenuation.
5.3 Averaged Linear Profiles

Vertical linear profiles through each of the samples were obtained using MATLAB. The linear profiles traveled longitudinally through the meniscus of the liquids and edges of the powders, so that the edge height, or intensity, of the sample-to-air could be computed. To reduce noise, five neighboring linear profiles were measured for each sample and averaged, producing a single profile. These profiles were gathered from the attenuation image, scatter images, and differential phase contrast images. These vertical line segments used for the linear profiles are demonstrated for the attenuation image, scatter image from the vertical first (0,1) harmonic, and the DPC image from the vertical first (0,1) harmonic of ethanol in Figure 5-2.

![Graphs](image)

*Figure 5-2: (a) Average vertical line profile through processed attenuation image, (b) scatter amplitude image for (0,1) harmonic, and (c) DPC image for (0,1) harmonic.*

5.4 Material Discrimination Results

The linear profiles of the processed attenuation (absorption) images, scatter images and differential phase contrast (DPC) images for each sample were collected. These linear profiles are demonstrated in Figure 5-3.
As shown in Figure 5-3 the absorption image produces a very clean edge height. The absolute edge height however, depends on the thickness of the sample and the intensity of the beam. The size or thickness of the sample may not be known, for example in security screening applications. The scatter images show strong edge height as well, but there is more noise through the sample itself. Very little information can be obtained from the majority of the DPC images, however, peroxide and citric acid show unique intensity changes where the sample meets air in the vial which can be used as additional information to distinguish these two substances from water.

In order to compare the liquid and powder samples to water, the edge heights in the linear profiles for each sample were used to assess the potential additional information from the scatter to distinguish between the materials. The ratio of scatter edge height to absorption edge height is independent of the thickness of the substance at hand. Therefore, these ratios could be used to detect the presence of a particular liquid or powder without the knowledge of the size of the
container that the substance is being carried in. The average absorption edge height and average scatter edge height at each first harmonic obtained by three separate trials and the scatter to absorption ratios for each first harmonic are shown in Table 5-1.

<table>
<thead>
<tr>
<th>Material</th>
<th>Avg. Absorption</th>
<th>Avg. Scatter for (0,1) Harmonic</th>
<th>Avg. Scatter for (1,0) Harmonic</th>
<th>Scatter to Absorption for (0,1) Harmonic</th>
<th>Scatter to Absorption for (1,0) Harmonic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ethanol</td>
<td>0.39 ± 0.03</td>
<td>0.093 ± 0.004</td>
<td>0.11 ± 0.03</td>
<td>0.24 ± 0.03</td>
<td>0.29 ± 0.10</td>
</tr>
<tr>
<td>Peroxide</td>
<td>0.84 ± 0.07</td>
<td>0.26 ± 0.05</td>
<td>0.31 ± 0.03</td>
<td>0.31 ± 0.08</td>
<td>0.37 ± 0.10</td>
</tr>
<tr>
<td>Acetone</td>
<td>0.36 ± 0.05</td>
<td>0.12 ± 0.02</td>
<td>0.135 ± 0.003</td>
<td>0.33 ± 0.10</td>
<td>0.37 ± 0.08</td>
</tr>
<tr>
<td>Gel Methanol</td>
<td>0.51 ± 0.03</td>
<td>0.182 ± 0.002</td>
<td>0.18 ± 0.01</td>
<td>0.36 ± 0.03</td>
<td>0.35 ± 0.02</td>
</tr>
<tr>
<td>Flour</td>
<td>0.36 ± 0.03</td>
<td>0.145 ± 0.003</td>
<td>0.146 ± 0.002</td>
<td>0.40 ± 0.04</td>
<td>0.40 ± 0.01</td>
</tr>
<tr>
<td>Citric Acid</td>
<td>0.62 ± 0.04</td>
<td>0.20 ± 0.02</td>
<td>0.2426 ± 0004</td>
<td>0.32 ± 0.06</td>
<td>0.39 ± 0.5</td>
</tr>
<tr>
<td>Sawdust</td>
<td>0.1288 ± 0.0002</td>
<td>0.072 ± 0.001</td>
<td>0.09781 ± 00004</td>
<td>0.562 ± 0.008</td>
<td>0.76 ± 0.01</td>
</tr>
<tr>
<td>Polyethylene Glycol</td>
<td>0.37 ± 0.01</td>
<td>0.134 ± 0.002</td>
<td>0.161 ± 0.001</td>
<td>0.36 ± 0.02</td>
<td>0.43 ± 0.01</td>
</tr>
<tr>
<td>Powdered Sugar</td>
<td>0.25 ± 0.02</td>
<td>0.12 ± 0.01</td>
<td>0.14 ± 0.02</td>
<td>0.47 ± 0.08</td>
<td>0.57 ± 0.11</td>
</tr>
</tbody>
</table>

Table 5-1: The averages were obtained from the linear profiles of three different images. The (0,1) harmonic shows the scatter at the first vertical harmonic and the (1,0) represents the scatter at the first horizontal harmonic. The error bars of the absorption and both scatter edge heights were obtained by taking the standard deviation of the three trials. The error bars for the scatter-to-absorption ratios were obtained by multiplying the scatter to absorption ratio to the absorption uncertainty divided by the absorption mean added to the scatter uncertainty divided by the scatter mean. It is just a coincidence that the error bars for some of the measurements were so small, like the average scatters for citric acid, sawdust and polyethylene glycol. This was because the edge height for each trial did not vary much.

The averages of the edge heights were collected by three separate images taken on different days.

The scatter to absorption ratios of each material were then compared to water by dividing the scatter to absorption ratio of water to each of these ratios. This resulted in the scatter to absorption ratios relative to water, as shown in Table 5-2.
<table>
<thead>
<tr>
<th>Material</th>
<th>Scatter/Absorption Relative to Water for (1,0) Harmonic</th>
<th>Scatter/Absorption Relative to Water for (0,1) Harmonic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ethanol</td>
<td>0.91 ± 0.48</td>
<td>1.02 ± 0.21</td>
</tr>
<tr>
<td>Peroxide</td>
<td>1.17 ± 0.54</td>
<td>1.31 ± 0.45</td>
</tr>
<tr>
<td>Acetone</td>
<td>1.16 ± 0.45</td>
<td>1.41 ± 0.55</td>
</tr>
<tr>
<td>Gel Methanol</td>
<td>1.09 ± 0.27</td>
<td>1.51 ± 0.22</td>
</tr>
<tr>
<td>Flour</td>
<td>1.27 ± 0.27</td>
<td>1.71 ± 0.31</td>
</tr>
<tr>
<td>Citric Acid</td>
<td>1.21 ± 0.38</td>
<td>1.36 ± 0.35</td>
</tr>
<tr>
<td>Sawdust</td>
<td>2.37 ± 0.47</td>
<td>2.38 ± 0.21</td>
</tr>
<tr>
<td>Polyethylene Glycol</td>
<td>1.36 ± 0.28</td>
<td>1.53 ± 0.19</td>
</tr>
<tr>
<td>Powdered Sugar</td>
<td>1.77 ± 0.68</td>
<td>1.98 ± 0.48</td>
</tr>
</tbody>
</table>

Table 5-2: Compare the scatter-to-absorption ratio relative to water for the first vertical and horizontal harmonics. The scatter-to-absorption ratios were computed by dividing the absorption by the scatter for each material including water. To obtain the scatter-to-absorption ratios relative to water, the scatter-to-absorption ratio for each potentially combustible material was divided by the scatter-to-absorption ratio of water. The uncertainties for these ratios relative to water were computed by dividing the uncertainty of water by the scatter-to-absorption ratio of the water and adding this to the uncertainty of the material divided by the scatter-to-absorption ratio of the material.

This was accomplished by taking the ratio of the scatter edge height to the absorption edge height and normalizing this ratio to the scatter-to-absorption ratio of water. The results of these ratios for each sample are shown in Figure 5-4.
Figure 5-4: Ratio of the edge height in the scatter harmonic image to the edge height for the absorption image of the zeroth harmonic for each material, normalized to the scatter-to-absorption ratio for water. One component of the scatter/absorption ratios for ethanol was 0.1 less than the scatter from water and one component of the scatter/absorption ratios for acetone was 0.4 more than the scatter from water. For the other materials except citric acid and peroxide, at least one of the scatter components was 0.5-1.5 higher than for water, which might lead to good detectability.

Materials with the scatter-to-absorption ratio significantly different than unity (whether it be higher or lower) could potentially be distinguished as dangerous materials from water.

5.5 Peroxide and Citric Acid

As shown in Figure 5-4, the scatter-to-absorption ratio of peroxide is close to that of water, thus failing to produce enough information to distinguish between the two. However, peroxide provides a unique edge height in the DPC linear profile that differs from the other samples studied, including water. The edge height was constructed from the bright band that peroxide forms from the phase peak at the surface of the liquid, as shown in Figure 5-5.
Figure 5-5: (a) Differential phase contrast image at the (1,0) harmonic. This shows the bright band formed at the surface of the liquid. (b) 2D phase gradient which consists of the gradient of each DPC at both harmonics and then adding them together in order to get 2D. These harmonics were added together using the same 2.9 horizontal-to-vertical DPC ratio. This shows the bright band as well and distinguishes where the vial and peroxide are due to the vertical harmonic.

This detectable phase peak provides additional information about the material and therefore can be used to distinguish peroxide from ethanol or water. The ratio of the DPC edge height to the edge height formed by absorption was computed with a result of 0.32 ± 0.04. The linear profile for the absorption, scatter and DPC edge heights are shown in Figure 5-6.

Figure 5-6: Edge height of peroxide (a) from the processed absorption at the zeroth harmonic (b) from the scatter at the (1,0) harmonic and (c) from the DPC at the (1,0) harmonic.
It was observed that peroxide must be imaged in a timely manner and with the cap off the vial to reduce the chance of bubbling and pressure changes. This could have potentially caused movement of the liquid within the image frames.

The scatter to absorption ratio of citric acid was also relatively close to that of water, as shown in Figure 5-4. Similar to peroxide, it produces a distinguished DPC spike due to the bright band that forms at the edge of the powder’s surface, as shown in Figure 5-7.

![Figure 5-7](image)

*Figure 5-7: (a) The differential phase contrast of the horizontal harmonic. (b) The two-dimensional gradient of the DPC using the same 2.9 horizontal-to-vertical ratio.*

As observed in Figure 5-7, the band is not as crisp and bright as the peroxide band. This is probably due to fact that peroxide is in liquid form and the citric acid is in powder form. Therefore, the peroxide has a smoother more definite surface than the citric acid. The ratio of the DPC edge height to the edge height formed by absorption was computed with a result of $0.25 \pm 0.04$ which is less than peroxide, as expected because the edge height of the DPC for citric acid in comparison to the absorption edge height was smaller. The difference in DPC spike size can be observed in Figure 5-4. The linear profile for the absorption, scatter and DPC edge heights are shown in Figure 5-8.
Figure 5-8: Edge height of citric acid (a) from the processed absorption at the zeroth harmonic (b) from the scatter at the (1,0) harmonic and (c) from the DPC at the (1,0) harmonic.

6. Conclusions

Mesh-based phase contrast imaging was exploited to study under what conditions this technique produces superior images compared to traditional attenuation imaging. The mesh-based imaging method is simpler and more practical than a competing phase contrast imaging method known as grating-based imaging. Grating-based imaging requires at least two expensive and high precision gratings with a pitch on the order of 7-10 µm. Mesh-based imaging only requires a single wire mesh with a coarser pitch with a period on the order of 100 µm. The wire mesh used in the biological and security set-ups had a period of 123 x 123 µm. A micro-focus x-ray source consisting of a fixed tungsten anode with a spot size of 150 µm was used to take images of the mouse, liquids and powders with the wire mesh acting as a grid. The images were processed in the Fourier domain and each harmonic was separated. The zeroth harmonic contained the absorption information and the first harmonics included the scatter information and phase information which increases the edge enhancement due to the large phase shifts occurring at the edges. After performing the inverse Fourier transform, the images produced by the imaginary part of the first...
horizontal and vertical harmonics were summed to create a two-dimensional differential phase contrast image. A final enhanced image was constructed by adding the absorption image (produced by the zeroth harmonic) to the two-dimensional phase image.

The images were optimized by quantifying the contrast, signal-to-noise ratio (SNR) and resolution (FWHM) of the detailed mouse images when varying different variables and parameters. Numerical values from the processed images were collected by taking linear profiles at the same columns or rows and studying the same edge within the linear profiles for each image. This provided a maximum and minimum height used to compute the contrast. The SNRs were obtained by taking the standard deviation of the same region in the background of the image and using the signal heights obtained by the linear profiles.

Several windowing functions were investigated such as Gaussian windows of widths $\frac{1}{2}$ sw, $\frac{1}{4}$ sw, $\frac{1}{6}$ sw and $\frac{1}{8}$ sw, the Hann window, the Hamming Window, Tukey windows with taper ratios of 0.5, 0.6, 0.7, 0.8, and 0.9 and the Blackman window. There was larger variation in the SNRs, with small fluctuations of contrast between each windowing function. It was concluded by optimizing the 4.5 magnification mouse image, that the Blackman window with a respectively high contrast of $0.78 \pm 0.03$ and a much larger SNR ratio of $83.1 \pm 7.3$ produced the sharpest images. After the windowing function was optimized, the DPC ratio factor used to optimize the two-dimensional phase images through a combination of the DPC images from the horizontal and vertical harmonics, was investigated to produce a phase image with increased edge enhancement. The different ratio variables studied included, 1, 2, 2.5, 2.7, 2.8, 2.9, 3, 3.5, 4, 5 and 6. The two-dimensional phase ratio factor that produced the highest SNR of $25.26 \pm 3.2$ for the 4.5 magnification was 2.9. This same 2.9 factor was used for the horizontal-to-vertical scatter ratio. Once the horizontal-to-vertical ratio factors were optimized, the resolution of the final DPC
enhanced images and final scatter enhanced images were investigated. The resolution was quantized by fitting a sigmoidal curve with an initial sigma to each edge peak formed by the absorption image added to the 2D phase gradient image. The DPC factors investigated included 0.5, 1, 2, 3, 4, 5, 6, 7, 8 and 10. The root sum square was minimized for each curve by adjusting the sigma and other parameters, mu and amplitude, using the Solver application in Excel. The adjusted sigma was then used to compute the full width half max. The fitted curve with the lowest FWHM holds the highest resolution. The DPC factor that produced the lowest FWHM of $85.5 \pm 3.3$ pixels for the 4.5 magnification mouse image was a factor of 6. The resolution quantification of the scatter enhanced image, which was formed by adding the absorption image to the 2D scatter image optimized by a scatter ratio of 2.9, produced a scatter factor of 5 with the lowest FWHM of $342.7 \pm 9.8$ pixels.

Once the final DPC enhanced images and final scatter enhanced images were optimized for the 4.5 magnification mouse image, the effects of demagnification on blur and SNR were studied using the guidelines suggested by Bennett based on the experimental set-up distances and period of the mesh. To decrease magnification of the mouse, the distance between the source and the mouse was increased. Bennett’s theoretical guideline limited the magnifications to 4.5, 3.7, 3.2, 2.8, 2.5 and 2.2 and as the magnification decreased so did the blur. By studying the two-dimensional phase gradient images of all magnifications, the SNR was quantified through linear profiles of the same skull region. Even though a magnification of 2.2 broke Bennett’s theoretical requirement number two, $E_2$, it produced the smallest blur of 0.03 and the highest signal-to-noise ratio of $35.9 \pm 0.8$.

Using the same technique as the biological set up, phase and scatter images were collected of several potentially hazardous substances and water. This was investigated to help airport
security better distinguish between powders and liquids of similar composition. The magnification was slightly changed to 2 and the materials that were compared to water included oxidants such as peroxide and combustible materials such as ethanol, saw dust, acetone, gel methanol, flour, citric acid, polyethylene glycol and powdered sugar. The Blackman window was used to optimize the images and linear profiles were collected through the meniscus or powder-to-air surface producing large edge heights. The edge heights provided by the absorption and scatter images were used to produce scatter-to-absorption ratios for each substance, including water. The ratio is independent of the thickness, size or shape of the container holding the materials and therefore can be used to detect the presence of a particular powder or liquid. The scatter-to-absorption ratios for each substance were then divided by the scatter-to-absorption ratio of water to compare each material to water with a ratio of one. The ratios relative to water were 0.91 ± 0.48 for ethanol, 1.16 ± 0.54 for peroxide, 1.41 ± 0.55 for acetone, 1.51 ± 0.22 for gel methanol, 1.71 ± 0.31 for flour, 1.36 ± 0.35 for citric acid, 2.38 ± 0.21 for sawdust, 1.53 ± 0.19 for polyethylene glycol and 1.98 ± 0.48 for powdered sugar. Additional information from the peak produced at the substance-to-air surface in the differential phase images were used to better distinguish between water and peroxide or citric acid. This extra information was useful because the scatter-to-absorption ratios of peroxide and citric acid were relatively close to that of water.

Mesh-based phase contrast imaging provides an adjustable set up with a relatively inexpensive mesh grid that can be used for diverse applications. The short, 48 mAs, single shot, raw images of the object of interest with a superimposed mesh were processed in the Fourier domain. The images produced from the normalized first order harmonics provide pure phase information. Due to the fact that the absorption, high contrast DPC and scatter images with
pronounced edge enhancement can all be retrieved from a single raw image, the mesh-based imaging method is promising for future medical and security scanning uses.

7. References

http://www.breastcancer.org/about_us/press_room/press_kit/facts_figures


