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Grid-based fourier transform phase contrast imaging

Sajjad Tahir
University at Albany, State University of New York, sajjadtahir@gmail.com

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Grid-Based Fourier Transform Phase

Contrast Imaging

by

Sajjad Tahir

A Dissertation
Submitted to the University at Albany, State University of New York
in partial fulfillment of
the requirements for the degree of
Doctor of Philosophy

College of Arts & Sciences
Department of Physics
2015
Grid-Based Fourier Transform Phase Contrast Imaging

by

Sajjad Tahir

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& MY GRAND MOM (LATE)
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Abstract

Low contrast in x-ray attenuation imaging between different materials of low electron density is a limitation of traditional x-ray radiography. Phase contrast imaging offers the potential to improve the contrast between such materials, but due to the requirements on the spatial coherence of the x-ray beam, practical implementation of such systems with tabletop (i.e. non-synchrotron) sources has been limited. One recently developed phase imaging technique employs multiple fine-pitched gratings. However, the strict manufacturing tolerances and precise alignment requirements have limited the widespread adoption of grating-based techniques. In this work, we have investigated a technique recently demonstrated by Bennett et al.\textsuperscript{1} that utilizes a single grid of much coarser pitch. Our system consisted of a low power 100 µm spot Mo source, a CCD with 22 µm pixel pitch, and either a focused mammography linear grid or a stainless steel woven mesh. Phase is extracted from a single image by windowing and comparing data localized about harmonics of the grid in the Fourier domain. A Matlab code was written to perform the image processing. For the first time, the effects on the diffraction phase contrast and scattering amplitude images of varying grid types and periods, and of varying the window function type used to separate the harmonics, and the window widths, were investigated. Using the wire mesh, derivatives of the phase along two orthogonal directions were obtained and new methods investigated to form improved phase contrast images.
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1 Introduction

Breast cancer is a random and deadly disease. It is the most frequently diagnosed cancer and is the leading cause of cancer death among women worldwide. Every 20 seconds, a case of breast cancer is diagnosed among women around the world, and every 74 seconds, someone dies from breast cancer. More than 1.6 million new cases of breast cancer were diagnosed around the world in 2010 and approximately 0.45 million died from the disease.[1] In the United States alone, a new case of breast cancer is diagnosed every 2 minutes and deaths occur from the disease every 13 minutes.

Imaging has an important role in the diagnosis of breast cancer, covering from whole-body imaging of the patient down to atomic and molecular structures. Current probes in diagnostic radiology range from Magnetic Resonance Imaging (MRI) and ultrasonography (US) through laser and x rays to neutrons and radioisotopes. Some methods, such as CT scans and MRI are very expensive and other such as molecular screening are not yet available. Currently, x-ray mammography is the most widely used and the only efficacious method of diagnostic radiology for early detection and diagnosis of breast cancer.

Unfortunately, attenuation contrast in mammography is very low. This leads to missed cancers and to false positives and unnecessary biopsies. X-ray phase contrast is inherently higher for low-density materials such as tissue. Phase contrast
imaging techniques uses the phase change of the x-ray beam when passing through the sample, rather than the attenuation change used by standard radiographic imaging or computed tomography (CT). The phase shift cannot be measured directly, but is transformed into intensity variations recorded by the detector. Several techniques have been developed to measure the phase shift, such as crystal-based diffraction enhanced imaging (DEI), grating-based, and Propagation-based techniques.

Propagation-based phase contrast imaging uses a simple in-line arrangement of x-ray source, the sample, and an x-ray detector. The detector is placed far from the sample to detect the interference between the refracted x rays through the sample and the un-deflected beam. This technique has low temporal stability requirements, which provides advantages over other phase imaging methods. However, the technique originally used spatially coherent synchrotron radiation (SR) sources that are clearly unsuitable for clinical use. Conventional micro-focus x-ray tubes can provide the spatially coherent x ray beam but require long exposure times.[2]

A number of researchers have been studying grating-based techniques. Grating-based imaging also utilizes differences in refractive index but uses pair of gratings to recover the phase information. An object placed in path of the incident beam modifies the original wavefront and this distorted wave contains the phase information. An analyzer grating is placed in front of the detector to resolve the fringes. In this way the quantitative information about the phase gradient within the
object can be obtained at the detector plane. The disadvantage of this method is that it utilizes high quality gratings of few µm periods. Grating alignment requires high stability and precision, and multiple exposures lead to higher dose.

To overcome the issues with the propagation-based and grating-based techniques, an alternative, grid-based, technique was investigated, based on the demonstration by Bennett et al.[1] This method uses single conventional grids typically used for radiography to image gradients in refractive index. The experimental setup for this technique is simple and easy to align. The wave front of the x-rays while passing through the object is distorted by the phase shift of the object. Phase information can be retrieved by observing the change of the fringe position at the detector plane. Images of the objects were acquired with the superimposed grid and transformed to the Fourier domain to obtain a series of harmonic spectra. The zeroth order harmonic and first order harmonics were separated and then transformed back to the spatial domain to retrieve the attenuation and the phase information. The Fourier analysis was carried out using the fast Fourier transform method. A Matlab code was written to perform the image processing. Gaussian windowing was found to be superior to square and rectangular spectral windows for separating the harmonics. Grid imaging provides three signals, the attenuation, differential phase contrast (DPC) related to the refractive index, and a scatter strength image.[5] The background noise and the grid effects were removed by normalizing the individual object image signals with the respective signals of the image taken without the object. Chapter 3 discusses this grid-based imaging technique in detail.
2 Theory and Background of Phase Imaging

2.1 X-ray Tube

A simple x-ray tube has two parts. The first part is the cathode, which is a negatively charged filament wire that generates the electrons to produce x rays. The high temperature of the filament wire of the cathode causes electrons to be emitted. The second part is the anode, which is positively charged. Electrons are emitted from the heated cathode by thermionic emission. The electrons are accelerated by applying a high voltage between the cathode and the anode that allows them to collide into a small area of the target called the focal spot. Approximately 99% of the energy of the bombarding electrons is converted into heat and only 1% into x-ray production. The number of x-ray photons produced is proportional to the electron current.

The voltage applied between the anode and the cathode controls the energy of the x-ray photons. It is measured in kilovolts peak potential (kVp). As the kVp is increased, the energy of the x rays increases. The highest possible energy of an x-ray photon is equal to the kinetic energy of the bombarding electron provided by the applied voltage between the anode and cathode.

The tube current controls only number of x-ray photons produced while the tube voltage also affects the energy of the x-ray photons. Therefore, it influences how well the x-ray photons penetrate through tissue. In addition, tube voltage also has
an effect on the number of x-ray photons produced. In fact, the x-ray intensity increases as the tube voltage squared but only linearly in tube current.

2.2 Production of X rays

The x-ray tube encloses a vacuum in which a high voltage is used to accelerate the electrons emitted from the hot cathode. When these fast moving electrons interact with the target material like copper or tungsten, they transfer part or all of their energy to the production of electromagnetic radiation. This is called bremsstrahlung radiation. For electrons that slow down and stop in a given material, the bremsstrahlung energy spectrum is a continuum with maximum photon energy equal to the electron energy. The emission of low energy photons predominates and the average photon energy is less than the incident electron energy. Additionally, a fast electron can knock out electrons from an inner shell leaving an atom in an excited state. The atom can then return to the ground state by emitting a

Fig. 2-1: Production of characteristics x-rays. Image from, C. MacDonald, “X-ray Optics, Analysis & Imaging”, to be published, Princeton University Press.
characteristic x-ray photon of energy equal to the difference in energies of the excited and the ground state as shown in Fig. 2-1. A typical x-ray spectrum measured from a tube source displays the bremsstrahlung continuum and characteristic x rays is shown in Fig. 2-2.

![Image](characteristic-x-ray-spectrum.png)

**Fig. 2-2**: Typical x-ray spectrum from a tube source. Image from, C. MacDonald, “X-ray Optics, Analysis & Imaging”, to be published, Princeton University Press.

### 2.3 X-ray Source Factors in Image Quality

#### 2.3.1 Contrast

In mammography, the requirements for the image contrast and resolution are very high. High contrast is necessary to provide improved detection of the borders of soft
tissue abnormalities such as tumor masses. Contrast is defined as a normalized difference in intensity,

\[ C = \frac{I_2 - I_1}{I_1} \]  \hspace{1cm} (2-1)

where \( I_1 \) and \( I_2 \) are intensities through an area of interest and a neighboring area. The intensities are

\[ I_1 = I_0 e^{-\mu_1 z}, \]  \hspace{1cm} (2-2)

and

\[ I_2 = I_0 e^{-\mu_1 (z-d)} e^{-\mu_2 d}, \]  \hspace{1cm} (2-3)

where \( z \) is the thickness of the patient with average absorption coefficient \( \mu_1 \) and \( d \) is the thickness of an embedded tumor with coefficient \( \mu_2 \). Because both the coefficients and their differences decrease with increasing photon energy, low photon energies give better contrast. In the early days of mammography, sealed tungsten (W) anode tubes operating at 30-35 kV were used. The x-ray spectrum was relatively hard (~keV), resulting in poor contrast. Since the mid-1960s, the use of molybdenum (Mo) anodes has led to improved image quality.

### 2.3.2 Dose

Absorbed dose is the measure of amount of energy deposited in tissue upon exposure to the ionizing radiation. It is measured in milligray (mGy), where 1 Gy = 1 Joule/kg. Equivalent dose measured in milliSievert (mSv) is used to measure the expected biological damage from the absorbed dose. It is the absorbed dose times a
quality factor. For x rays, the quality factor is one, so the equivalent dose, in mSv is equal to the absorbed dose in mGy. The lowest energy bremsstrahlung photons do not transmit through the patient and so contribute only to the dose, not to contrast. Filters are placed between the tube and the patient to absorb these low energy photons. Higher tube voltages increase the average photon energy, and so reduce the dose if the same intensity is used, but at the expense of contrast. If the contrast is kept constant, higher tube voltages would result in higher dose.

2.3.3 Blur

Blur is defined as the spreading of the image of small objects into the surrounding background area that reduces the visibility. One of the factors that has an impact on image blur is the focal spot size. A small focal spot produces less image blur that would otherwise reduce the ability to visualize small objects (i.e., increases the spatial resolution). A small focal spot can only be obtained at low tube power,

---

Fig. 2-3: Geometrical depiction of rotating anode x ray source target and beam.
otherwise the anode would melt. However low tube power requires long exposure times, which can lead to motion blur. Originally, mammography used sealed tubes with relatively large focal spots. Later, the introduction of dedicated mammography units with rotating anode tubes and small foci resulted in substantial reduction in geometric blurring, and moreover enabled use of the magnification technique. The main difference between them is the current and the electron beam shape for x ray photon production. The formation of the source spot for rotating anode (high power) is illustrated in Fig. 2-3.[7] Blur formation is illustrated in Fig. 2-4 where x rays intensity at the edges will increase/decrease, causes to reduce the sharpness. This blurry region at the edges depends upon the source spot size and the magnification of the image.

![Fig. 2-4: Illustration of blur formation.](image)

The Center for X-ray Optics (CXO) at UAlbany has tungsten (W), rhodium (Rh) and molybdenum (Mo) sealed tube sources and copper (Cu) and Mo rotating anode sources. In this work, Rh and Mo target sources from Oxford instruments were used for grid-based imaging.

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2.4 X-ray Imaging Detectors

Roentgen discovered x rays by noticing that some invisible rays emanating from his Crookes tube were exciting a glow on a nearby phosphor screen. Within days, he was recording x-ray images on photographic film. Until the past two decades, film was still the most prevalent imaging detector for x rays. Advantages of digital imaging systems over the film are immediate preview and availability, no processing costs and the ability to apply computational or electronic processing techniques to increase contrast and reduce noise.

2.4.1 Phosphor Screen

Most detectors are more sensitive to light than to x rays. Thus, they often utilize a thin (1-10 µm) layer of phosphor to absorb the x-ray energy and create photoelectrons. The electrons are then trapped into a metastable state and emit a photon upon de-excitation to the ground state. Thus, x rays cause the screen to emit light in the visible range that can be viewed by using CCD camera, film or other light sensitive detector.

2.4.2 Image Plates

An alternative technology is the computed radiography (CR). This was the first digital radiography system introduced to the market by Fuji in the early 1980s.[3] It consists of an image plate that has barium-fluoro-bromide laminated on a special magnetized fiber sheet.[8] In this luminescent material, electrons excited by ionizing
radiation are trapped at metastable, intra-band states with lifetimes on the scale of days. The material thereby stores an image in the distribution of trapped states. The image is read by irradiating with laser light which excites the traps and leads to recombination of electrons and holes and the release of visible light, as shown in Fig. 2-5.[32] The image reader Fuji BAS-1800 is used to read the image plate at 50, 100

or 200 µm laser pixel size with a reading time of approximately 3 minutes.[8] The main advantages of CR technology are that it is well established, robust, and relatively inexpensive and has good reproducibility. The image stored on these image plates will degrade approximately 25% within first 8 hours. Because of their sensitivity to background and scatter radiation, the image plates should be erased before use if stored for more than 72 hours. In addition, the photo-stimulated

Fig. 2-5: Energy diagram of photo-stimulated luminescence mechanism. [From Paul Leblans, “Storage Phosphors for Medical Imaging”, Materials 2011, 4, 1034-1086; doi: 10.3390/ma4061034].

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phosphor plates are reusable, have linear response over a wide range of x-ray intensities, and are completely erased by exposing to a white light.

2.4.3 Real Time Digital Detectors

Newer digital radiography technology was developed to improve the image quality, and allow for real time read out. The systems demonstrate higher detection efficiency and this can be used either to reduce the acquisition dose or to reduce the noise at the same dose. Noise in an image is due to Poisson statistics from the relatively small number of detected photons per pixel. The signal-to-noise ratio (SNR), is usually measured as the ratio of the average pixel value to the standard deviation of the background pixels of an image. It decreases with increasing incident intensity or detection efficiency. A digital x-ray imaging system can use either a direct or an indirect process for converting X-rays into electric charges [10]. Indirect detection requires a phosphor, sometimes directly deposited on the detector. Direct conversion requires thicker or higher density material. The stored charge is created from photoelectric absorption of photons. In CCD detectors, when proper voltage differences are applied on the electrodes, the charge is transferred from well to well under the electrodes. The charge is shifted out of the array via vertical and horizontal charge coupling, converted to voltage via a simple follower amplifier, then serially read out, and converted to digital number via an analog-to-digital convertor (ADC) [11]. CCD detectors have high sensitivity and high quantum efficiency but the high production cost limits their active area to 2-5 cm².
2.5 X-ray Interactions with Matter

While passing through the material, x rays interact with the atoms of the target material in different ways, including coherent scattering, photoelectric absorption, incoherent scattering, pair production, and photodisintegration.

In *coherent scattering*, also known as the Thompson or Rayleigh scattering, an x ray excites an atom but a photon is re-emitted without any net energy transfer to the atom while the direction of the photon is changed. The probability of coherent scattering is significant only for photon energies below a few hundred keV and is most prominent in high-Z materials. The average deflection angle increases with decreasing energy.

*Photoelectric absorption* occurs when the x-ray photon is absorbed by the target atom and results in the ejection of electrons. If the ejected electron is from inner core level, the atom can return to the ground state with the emission of an Auger electron or x-ray characteristic of the atom. This subsequent emission of electrons or lower energy photons is generally absorbed and does not contribute to the image formation process. Photoelectron absorption is dominant up to energies of about 500 keV as well as for high atomic Z materials.

*Incoherent scattering*, also known as Compton scattering, occurs when the incident x-ray photon ejects an outermost electron from an atom. A photon is then emitted with reduced energy. Energy and momentum are conserved in this process. The scattered x-ray photon has less energy and therefore longer wavelength than the
incident photon. Compton scattering is important for low atomic number specimens. At energies of 100 keV-10 MeV the attenuation of radiation is mainly due to Compton scattering.

*Pair Production* can occur only when the x-ray photon energy is greater than 1.02 MeV and so is unimportant for imaging. While passing near a heavy nucleus, an x ray photon annihilates into an electron and positron pair. Positrons are very short lived in the presence of electrons and annihilate with the formation of two photons of energy 0.51 MeV each. Pair production is of particular importance when very high-energy photons pass through materials of a high atomic number.

![Graph showing photon mass attenuation coefficient versus photon energy for iron.](image)

*Fig. 2-6: Photon mass attenuation coefficient versus photon energy for iron.* [From National Institute of Standards and Technology]
Photodisintegration is the process by which the x-ray photon is captured by the nucleus of the atom with the ejection of a particle from the nucleus when all the energy of the x-ray is given to the nucleus. Because of the enormously high energies involved, this process may be neglected for the energies of x-rays used in radiography.

The relative probability of the five interactions for iron is shown in Fig. 2-6. It is seen that in the diagnostic imaging range, near 0.1 MeV, photoelectric absorption, coherent and incoherent scattering are the most important.

2.6 Bragg’s Law and Monochromatization

Coherent scatter can interfere constructively to create a diffracted beam. Diffraction of x rays from a crystalline material is governed by Bragg's law,

\[ 2dsin(\theta) = n\lambda \]  \hspace{1cm} (2-4)

where \( n \) is an integer (the order of diffraction), \( \lambda \) is the wavelength of the incident x rays, \( d \) is the spacing of the atomic planes of the crystal and \( \theta \) is the angle between incident ray and the scattering plane (Bragg’s angle). The incident angle does not need to precisely equal the Bragg angle \( \theta \) but must lie within the range of angles given by the Darwin width of the crystal (typically a few arc second). The Darwin width, the width of the total reflective profile of a nearly perfect monochromator crystal is [14]
\[ \omega_n = 2.12r_e \left( \frac{\lambda}{n+1} \right)^2 \left( \frac{\rho_a F_h}{\pi \sin (2\theta)} \right), \]  

(2-5)

where \( r_e \) is the classical electron radius, \( \lambda \) is the incident photon wavelength, \( \rho_a \) is the atomic density, \( n \) is the order of harmonic present in the beam, and \( F_h \) is the structure factor,

\[ F_h = F(\sin \lambda/d)/(h^2 + k^2 + l^2), \]  

(2-6)

where \( h, k, l \) are the Miller indices of the reflection plane. The Darwin width varies with the energy of the x rays. As the wavelength is reduced the Bragg angle and the Darwin width become smaller. For silicon crystals, the bandwidth is 0.02 mrad at 8.1 keV and 0.01 mrad at 17.5 keV, respectively [15].

![Graphs showing reflectivity and FWHM for different Si reflection planes.](image)

**Fig. 2-7:** a) Single crystal reflectivity curve for Si(111) at 10 keV, b) Angular divergence of sources and diffraction width for different Si reflection planes. [Figure from *X-ray Absorption – Principles, Applications, Techniques of EXAFS, SEXAFS and XANES*, ed. by D. C. Koningsberger and R. Prins, John Wiley (1988)]

The rocking curve obtained by rotating the crystal about the Bragg’s reflection angle for Si(111) is shown **Fig. 2-7** (a) and the energy dependence of the Darwin width (or
rocking curve width) for different reflection planes in Fig. 2-7(b). If the crystal is used as a monochromator, the energy resolution is

$$\frac{\Delta E}{E} = \frac{\omega}{\tan \theta}. \quad (2-7)$$

### 2.7 Index of Refraction

Absorption and coherent scatter can be described by the refractive index $n$ of the material,

$$n = \sqrt{\frac{\varepsilon}{\varepsilon_0}} = 1 - \delta - i \beta. \quad (2-8)$$

$\beta$ is the absorption term which is proportional to the cross section for absorption. The real part of the refractive index decrement, $\delta$, is proportional to the cross section for coherent scatter. The real part of the index is approximately

$$n = 1 - \delta = \sqrt{1 - \frac{\omega_p^2}{\omega^2}} \quad (2-9)$$

where $\omega_p$ is the plasma frequency of the material,

$$\omega_p^2 = \frac{N_e e^2}{m \varepsilon_0}, \quad (2-10)$$

$N_e$ is the electron density of the material, $e$ and $m$ are the charge and mass of the electron, and $\omega$ is the x-ray photon angular frequency. In the x-ray energy range, $\omega_p \ll \omega$ [13].

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2.8 Introduction to Phase contrast Imaging

Conventional imaging techniques rely on absorption and scatter to create intensity variations in the beam. However, these intensity variations arise from absorption (the $\beta$ term in Eq. 2 – 8), and are quite small for soft tissues. Differences in the real part of the refractive indices result in phase differences which are a much larger potential source of image contrast. Because of the small angle of refraction of x rays and the difficulties in the fabrication of appropriate and efficient x-ray optics, it is much more challenging to implement phase-contrast techniques in the x-ray regime compared to visible light imaging. A variety of techniques have been developed to obtain phase contrast for low Z materials, including grating-based and propagation-based imaging [22]. Three of them, diffraction-enhanced, propagation-based, and grating-based imaging techniques are currently being widely pursued for the application of phase contrast in medical imaging.

2.8.1 Propagation-Based Phase Imaging (PBPI)

This method is based on observation of the interference pattern between the refracted and un-deflected waves produced by a spatially varying phase shift. As a spatially coherent x-ray beam propagates through an object, the refracted (phase-shifted) beam in the forward direction interferes with the un-refracted beam. After further propagation beyond the object, the change in intensity in the transmitted beam can be observed. At the edges of features in the object, i.e. when gradient of the phase $\phi$, $|\nabla \phi(x)|$ and Laplacian $|\nabla^2 \phi(x)|$ are large, strong interference patterns
can be seen. Resolving the interference pattern depends upon the characteristics of
the source and detector. For short propagation distances, the image contrast is
proportional to both the phase change $\nabla \phi(x)$, and the Laplacian of the phase change,
$\nabla^2 \phi(x)$, which is proportional to the Laplacian of the projected electron density,
$\nabla^2 \rho_e(x,y)$.[16] As the second derivative becomes significant at the edges, this
method is very useful for edge enhancement.

Conventional absorption images can be obtained by placing the detector
immediately after the object. If the detector distance is increased from the object,
diffraction effects make edge enhancement prominent. By taking several images of
the sample at different sample-detector distances, the propagation of the x-ray wave
front is sampled in the spatial domain, allowing the separation of phase and
attenuation contrast. This principle is shown in Fig. 2-8.

![Fig. 2-8: Illustration of propagation-based phase imaging technique.][30]
The advantage of the propagation based imaging method over the other methods of phase contrast is that it is easy to implement because of its close resemblance to present radiography systems.[16] Another advantage is the reduced contribution of incoherent scattering provided by the air gap between the object and image planes.[17] However, the method is very sensitive to the quality of in-beam components such as windows or filters.[16] This technique originally used expensive and huge synchrotron radiation (SR) sources that are clearly unsuitable for clinical use. Conventional micro-focus x-ray tubes can be used but require long exposure times.[16] Broadband radiation may also be used which facilitates the use of lab sources.

2.8.2 Crystal based phase imaging

An alternative approach to retrieve phase-contrast information utilizes the angular deflection of the radiation transmitted through an object with the help of crystal analyzer. Diffraction enhanced imaging (DEI) is an x-ray imaging technique that is capable of revealing the contrast generated not only from absorption, but also from the refractive and diffractive properties of the object examined [18]. In comparison with the absorption contrast obtained by conventional radiography, the contrast generated by DEI technique enhances the visibility of features in the object. The schematic of the DEI setup is shown in Fig. 2-9.

A silicon monochromator crystal is used to create a collimated monochromatic beam according to Bragg’s law of diffraction. This monochromatized beam is then allowed to pass through the sample, where it is refracted. Refraction is a very small
angular deflection of the order of few $\mu$-radians. An analyzer crystal placed between the sample and the detector can differentiate this small angular variation. This crystal acts as an angular selection device that allows an analysis of the angular spectrum of the x rays passing through the sample by converting the angular information to an intensity change.

In addition to refraction contrast, this system has a unique ability to provide essentially scatter free images of the objects, resulting in improvement in the signal-to-noise ratio (SNR). Conventional radiography images include coherent scatter, incoherent scatter, as well as the desired primary beam. The contrast is reduced because of the scattered intensity, which creates a background fog. In addition, the scatter intensity increases the noise in the image.

There are some issues with implementation of DEI for clinical purposes. Because the analyzer crystal has micro radian angular acceptance, it is very sensitive to

![Fig. 2-9: Schematic of the diffraction enhanced imaging DEI setup.](image)
alignment. It can lose its alignment with time because of unavoidable thermal vibrations within the system. The main disadvantage of the DEI system is that the low angular acceptance requires a high intensity monochromatic x-ray beam. Such beams are readily available at synchrotron facilities for research and rarely available for clinical purposes. The use of laboratory-based sources is necessary for clinical use of DEI, but achieving the required intensity after the analyzer crystal is difficult. Using suitable optics like polycapillary optics after the x ray source can help mitigate the intensity problem.

The disadvantage of this technique is that even with polycapillary optics, long exposure may be required, and the optics adds additional beam non-uniformity.

2.8.2.1 Polycapillary X-ray Optics

Polycapillary optics are made of thousands of thin walled hollow glass tubes that guide the x rays along the tube using the total external reflections. Total external reflection of x ray is governed by Snell's law[12]

\[ n_{air} \sin \left( \frac{\pi}{2} - \theta_c \right) = n_{glass} \sin \left( \frac{\pi}{2} \right) \]  

(2-11)

At the critical angle, \( \theta_c \), the angle in the medium becomes zero, so, making a small angle approximation and setting \( n_{air} = 1 \), gives

\[ \sin \left( \frac{\pi}{2} - \theta_c \right) = \cos(\theta_c) \approx 1 - \frac{\theta_c^2}{2} = n_{glass}. \]  

(2-12)

From Eq. (2-9) & (2-12), the critical angle \( \theta_c \) becomes
\[ \theta_c = \sqrt{2 \delta B} \frac{\omega_p \text{glass}}{\omega}. \]  

(2-13)

For borosilicate glass, the critical angle is approximated by [12],

\[ \theta_c = \frac{30 \text{ (mrad-keV)}}{E \text{ (keV)}}, \]  

(2-14)

where \( E \) is the x ray photon energy. At \( E = 10 \text{ KeV} \), the critical angle is about 3 mrad.

Transmission of x rays passing through polycapillary depends upon the incident angle. If the incidence angle is greater than the critical angle (\( \theta > \theta_c \)), the x ray will not reflect down the channel. If the incident angle is less than the critical angle (\( \theta < \theta_c \)), the x rays are guided down the channel by total internal reflection as shown in Fig. 2-10.

Fig. 2-10: Sketch of the x rays passing through a single polycapillary channel. A ray entering at an angle greater than critical angle will pass through while striking at an angle less than the critical angle is blocked.
A polycapillary fiber and a large fiber shaped into a collimating optic as shown in Fig. 2-11(b). If the optic is aligned with a diverging x ray source on the left, a collimated output beam of high intensity can be obtained on the right.

Fig. 2-11: a) SEM monograph of a polycapillary fiber having approximately 250 channels each of 50 μm and a total diameter of 500 μm, b) Sketch of a collimating lens that takes an input on the left from a diverging source and delivers the collimated x ray beam to the right [15].

2.8.3 Grating-Based Phase Imaging (GBPI)

Momose *et al.* initially introduced grating-based differential phase contrast imaging

Fig. 2-12: Experimental setup for grating-based phase contrast imaging with a phase grating to split the beam and an analyzer grating absorption grating to collect the interference pattern at the detector surface.
at a synchrotron facility in 2003 [19]. Weitkamp et al. [20] and Momose et al. [21] published the first tomographic grating-based phase contrast images in 2005. The experiments employed two gratings, a phase grating as a beam splitter, and an absorption grating to analyze the interference pattern. Pfeiffer et al. achieved the transition to lab-based incoherent sources in 2006 [22] with the addition of a third grating near the source, which acted as an aperture mask with transmitting slits. An experimental arrangement for a typical grating-based imaging setup using a tabletop conventional x-ray source is shown in Fig. 2-12. It consists of a source grating placed after the x-ray source, a phase grating placed after the object, and an analyzer absorption grating placed just before the detector. The source grating is usually placed close to the source to effectively create an array of small line sources.[25]

The phase grating generates an image at the Talbot distance consisting of periodic fringes of the same pitch as the phase grating. An extremely fine pitch d (on the order of microns) is necessary because the Talbot distance is \( z = 2d^2/\lambda \), which would be unreasonably far without small d. The object placed in the x-ray beam path will impart a phase, which alters the fringe position. Since a fine grating generates fringes that are smaller than the detector pixels, an analyzer grating placed immediately before the detector is used to measure fringe displacement. By scanning the analyzer grating along the transverse direction \( x_g \), the intensity in each pixel \((m, n)\) oscillates as a function of \( x_g \).
\[ I(m, n, x_g) = \sum_i a_i(m, n) \cos \left( \frac{2\pi x_g}{p} + \psi_i(m, n) \right) \]

\[ \approx a_0(m, n) + a_1(m, n) \cos \left( \frac{2\pi x_g}{p} + \psi_i(m, n) \right), \tag{2-15} \]

where \( a_0 \) is the x-ray absorption contrast signal, \( a_1 \) is related to the small-angle scatter signal, \( \psi_i \) represents the differential phase contrast signal, and \( p \) is the phase grid period.

Grating-based phase-contrast imaging offers the advantage over the other methods that it is not only feasible with polychromatic beams, but also with high-powered large spot laboratory sources. This method is capable of delivering a contrast which is related to the internal structure of the traversed object.[24] In comparison to the propagation technique that yields the second derivative of the phase, the grating technique measures deflections and thus offers the first derivative of the wave front, which may make the reconstruction of the phase less prone to noise in the image although it does create a phase wrapping issue. The grating-based technique has some disadvantages. It requires high quality gratings that are very difficult and expensive to fabricate. Another disadvantage is the requirement of very precise alignment, which further requires high mechanical stability. In addition, the requirement of multiple images leads to high exposure to radiation.

**2.8.4 Grid-based Imaging**

To overcome these practical problems with propagation, crystal- and grating-based techniques, an alternative, grid-based, technique was recently proposed by Bennett
This method utilizes the conventional grids typically used for radiography, which are readily available and low cost. The experimental setup for this technique requires only a single grid, which does not need precise alignment as shown in Fig. 2-13. When an x-ray beam from the source is allowed to pass through the phase object, it refracts at a small angle from its original path because of the difference in the refractive indexes. The transmitted waveform is consequently deformed.

The Fourier transformation of the image to the spatial frequency domain produces a series of harmonics at integer multiples of the grid frequency,

\[ g = \frac{2\pi d_{SG}}{P d_{SC}} \]  

(2-16)

where \( d_{SG} = d_{SO} + d_{OG} \) and \( d_{SC} = d_{SO} + d_{OG} + d_{GS} \) are the grid and camera distances from the source and \( P \) is the grid period. To obtain the images from individual harmonic spectra, it is necessary that the harmonics are separated by more than the spatial

---

**Fig. 2-13: Schematic of Experimental Setup.** Distances \( d_{SO}, d_{OG} \) and \( d_{GC} \) are source-to-object, object-to-grid and grid-to-camera respectively.
bandwidth of the object.\[1\] The bandwidth in the direction perpendicular to the grid lines of each harmonic is

\[
\frac{2\pi}{MS},
\]

where \(S\) is the x-ray source spot size and \(M\) is the magnification factor,

\[
M = \frac{d_{GC}}{d_{SC}}.
\]

This bandwidth should be less than the separation of the harmonic peaks

\[
\frac{2\pi}{P},
\]

The source size is responsible for blurring in the image that acts as a low pass filter in the Fourier domain. The blur destroys information above a certain spatial frequency in the Fourier domain. Thus the first grid harmonic \((-1/P)\) must be less than the low-pass cutoff frequency \((-1/S)\) due to source size, which leads to the criteria

\[
C_1 = \frac{S}{P} \frac{d_{GC}}{d_{SC}} \ll 1.
\]

It is also necessary that there is an adequate intensity in the higher harmonics. Therefore, the object must be placed close to the source to increase the incident intensity. That leads to another criteria
\[ C_2 = \frac{S d_{OC} d_{SG}}{P d_{SO} d_{SC}} > 1. \]  

Grid-based phase contrast imaging is the subject of this thesis and will be developed and analyzed in more detail in Chapter 3 and 4.

## 3 Grid Based Phase Imaging

### 3.1 Introduction

Conventional x-ray imaging utilizes differential attenuation that arises from variations in thickness, composition, and density of the imaged objects to generate contrast. However, attenuation variations are often quite small for low atomic number (Z) materials such as soft tissue. Higher contrast can often be obtained from phase information because the relative phase change of x rays passing through an object is much larger than the relative change in intensity due to the absorption.\[30\] The contrast is generated due to the difference in refractive index \( n \) given in Eq. 2-2.\[31\] The real part, \( 1 - \delta \), describes the relative phase delay \( \Delta \phi \) of x-rays, through

\[ \Delta \phi(x, y) = -k \int \delta(x, y, z) \, dz, \]  

(3-1)

where \( z \) is the beam direction and \( x \) and \( y \) the transverse directions in the patient or sample. The wave number, \( k = \frac{2\pi}{\lambda} \) where \( \lambda \) is the wavelength of x rays. For an x-ray energy range of 10 to 100 keV and low Z materials, the refractive index decrement \( \delta \)
ranges from approximately $10^{-6}$ to $10^{-8}$, roughly 1000-fold higher than absorption term $\beta$, which ranges from $10^{-9}$ to $10^{-11}$.

As discussed in chapter 2, to overcome the practical problems with grating based techniques, an alternative, grid-based method was recently proposed by Bennett et al.[1], which allows grids of much coarser pitch to be used, such as those typically used in radiography, which are readily available at low cost. The experimental setup for this technique is shown in Fig. 3-1.

![Grid-Based Imaging Setup](image)

**Fig. 3-1: Grid-Based Imaging Setup.** Incident x-rays diffract upon passing through the phase object. The distorted wave front is shown by red dotted lines. The grid pattern on the camera is distorted by the refraction of the rays through the object. Distances $d_{SO}$, $d_{OG}$ and $d_{GC}$ are source-to-object, object-to-grid and grid-to-camera respectively. The source-to-camera distance $d_{SC}$ is then the sum of $d_{SO}$, $d_{OG}$ and $d_{GC}$.

### 3.2 Experimental Procedure

The experimental imaging setup is illustrated in Fig. 3-1. Two Oxford Apogee x-ray sources of the same model XTF5011 and maximum 50W, operated at 35 KVp and 0.5 mA were used in the experiment. The first source had a 55 $\mu$m focal spot with Rh
anode and the second source had a 100 μm spot and Mo anode. A grid was placed between the object and the camera at a distance $d_{OG}$ of about 18 cm from the object. The detector was a 1200×1600 pixel Remote RadEye HR CCD camera manufactured by Teledyne with a pixel size of 22 μm. It was placed a distance $d_{GC}$ from the grid, which was varied from 10 to 20 cm. These distances were chosen to satisfy Bennett’s criteria of Eq. 2-17 and 2-18. Design parameter comparison is given in **Table 3-1**. Images of different size objects with and without grids were acquired at small varying detector $d_{GC}$ and grid $d_{OG}$ distances with an exposure time of about 3 s. The magnification of the object image was approximately 1.5.

![Experimental test setup](image)

**Fig. 3-2:** Top view of experimental test setup used to acquire the images.

<table>
<thead>
<tr>
<th>Design Parameters</th>
<th>$S$ μm</th>
<th>$P$ μm</th>
<th>$d_{GC}$ cm</th>
<th>$d_{SC}$ cm</th>
<th>$d_{SO}$ cm</th>
<th>$d_{SG}$ cm</th>
<th>$C_1 &lt;&lt; 1$</th>
<th>$C_2 &gt; 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Experimental values</strong></td>
<td>100</td>
<td>100</td>
<td>33.5</td>
<td>43</td>
<td>9.5</td>
<td>26</td>
<td>0.4</td>
<td>2.2</td>
</tr>
<tr>
<td><strong>Bennett et al.</strong></td>
<td>50</td>
<td>127</td>
<td>50</td>
<td>100</td>
<td>12</td>
<td>50</td>
<td>0.19</td>
<td>1.45</td>
</tr>
</tbody>
</table>

**Table 3-1:** Experimental design parameters comparison with Bennett *et al.*
3.2.1 Grid Types

In conventional radiography, grids are mainly used to reject Compton scatter so that it does not reach the detector. They were invented by Gustav Bucky in 1913 and so are sometimes called Bucky grids.[38] Grids are composed of radiolucent (transparent) interspace material (aluminum, plastic or carbon fiber) with alternating radiopaque stripes (lead, tungsten, gold etc), as shown in Fig. 3-3. Transmission of x-rays through the grid depends upon the grid ratio,

$$\text{Grid ratio} = \frac{h}{D}$$ \hspace{1cm} (3-2)

![Diagram of grid ratio influence on x-ray beam acceptance angle.]

Fig. 3-3: Grid ratio influence on x-ray beam acceptance angle.

The Fig. 3-3 illustrates the influence of grid ratio on the angle of acceptance of the x-ray beam. The angle is a minimum for high grid ratios, which is best for scatter rejection, but high grid ratio grids usually have poor primary transmission. Grids can be parallel or focused, and can be linear or crossed.
In a parallel grid, the radiopaque stripes are parallel to each other when viewed in the cross-section, as illustrated in Fig. 3-6. Focused grids are similar to the parallel grids except that the radiopaque stripes are slightly angled towards the focal spot, as shown in Fig. 3-6. Therefore, these grids can only be used at a specified focal distance. Cross grids are constructed by superimposing two grids with the lead strips perpendicular to one another, as shown in Fig. 3-6. If the two grids are focused, then their focal distances should be similar.

### 3.2.2 Grids Used in the Experiment

In this work, two different grids were used to obtain the images of objects as illustrated in Fig. 3-7. The first grid (G1) was a linear focused grid composed of

- **a)** Linear Grid, G1, (220 μm)

- **b)** Mesh Grid, G3, (75×97 μm)

**Fig. 3-7:** Two different types of grids used in the experiment.
alternating tungsten (W) and low density material strips having grid ratio of 10:1, grid period of 217 μm and a focal distance of 58 cm. It is shown in Fig. 3-7(a). The second linear grid (G2) was similar to G1 with alternating lead (Pb) and Aluminum (Al) strips, a grid ratio of 12:1, and a period of 300 μm. The higher grid ratio gives a more limited acceptance angle for the incoming x-ray beam as compared to G1.

The linear grids were then replaced by a woven stainless steel wire mesh grid, (G3), normally used for filtration processes. It is shown in Fig. 3-7(b). It has a grid period of 75×97 μm. The finer period helps to achieve better resolution images.

3.3 Rh X-ray Source Spot Profile

An image of the x-ray source spot was obtained using a Fuji image plate placed at a distance of 60 cm from the source. The intensity profile across the spot, shown in Fig. 3-8(b) is plotted along the line in (a). The intensity profile across the spot was flat to 5% over 5.1 cm.

![Fig. 3-8: a) Rhodium (Rh) target x-ray source spot image and b) its intensity profile, taken with 35 kVp, 0.3 mA and 2 s exposure time. The cut edge at the top left corner in the image (a) is the anode edge shadow of the x-ray tube.](image)
3.4 Source Spot Size Measurement

After some initial image measurements, the source spot size was measured using a 100 μm pinhole aligned with the source at a distance of 13 cm. An image was acquired at a distance of 35 cm from the pinhole, as shown in Fig. 3-9. It is clear from the image that the source spot is strongly elliptical rather than round. The source spot size was estimated from the image and the geometric magnification to be about 90 μm long by 50 μm wide, so roughly twice as large as expected along the longer axis. The longer axis corresponds to the direction of the arrow in Fig. 3-8.

To verify this effect, further images were taken with another source of the same type, with a Mo target and 100 μm focal spot size. The images were acquired and processed to obtain high contrast phase images discussed in chapter 5 in more detail.

![Fig. 3-9: Schematic of pinhole aligned with the source and the image acquired.](image)

Later, Oxford Inc., provided information on the required bias voltage of the power supply. Another power supply having a bias voltage output connection was tested with the Rh target source. The bias voltage was adjusted to 70V. The pinhole image, shown in Fig. 3-10, was still a bit elliptical but a significant decrease in longer axis
was observed. The source spot size was estimated from the image and the geometric magnification to be about 63 µm long by 50 µm wide.

![Figure 3-10: Pinhole image obtained with corrected bias voltage using different power supply.](image)

3.4.1 Image Processing Algorithm

If $I_0(x)$ is the intensity variation after the object, and introducing a grid before or after the object acts like as a mask with strong intensity variation $G(x)$, then the total intensity after the combined object and grid is $I_0(x)G(x)$. Use of a periodic grid makes it straightforward to isolate $I_0$ and $\psi_0$ from a single raw image taken at a distance $z_d$. A Matlab code was written to perform the image processing. For a grid period $p$, the mask transmittance $G$ can be expanded in Fourier series

$$G(x) = \sum_{m,n} c_{m,n} \exp(\pi i g \cdot x), \quad (3-3)$$

where $g = \frac{2\pi}{p} g$. and $g$ is a unit vector perpendicular to the grid lines. For a 2D mesh, the differential phase contrast image is taken as

$$\psi_{m,n} = \frac{d_{SO}}{d_{SC}} d_{GC} g(m,n) \cdot \theta, \quad (3-4)$$
where

\[ g(m, n) = m \tilde{g}_1 + n \tilde{g}_2. \] \tag{3-5}

and

\[ \tilde{g}_1 = \frac{2\pi}{P_x} \hat{x} \text{ and } \tilde{g}_2 = \frac{2\pi}{P_y} \hat{y}, \] \tag{3-6}

where \( P_x \) and \( P_y \) are the mesh periods along \( x \) and \( y \)-axis, distances \( d_{SO} \), \( d_{SC} \) and \( d_{GC} \) are given in Fig. 3-1 and \( \theta \), the angle of deflection through the object is

\[ \theta \propto \frac{\lambda}{2\pi} \nabla \phi, \] \tag{3-7}

where \( \phi \) is given in Eq. 3-1.

Thus \( \psi \) is proportional to the phase gradient in the direction of the grids. A schematic illustration of the image processing is shown in Fig. 3-11. The analysis of the phase shift was carried out in the Fourier domain. The Fourier spectra of the raw images were obtained by a 2D Fast Fourier transform (FFT) and a series of grid harmonics were observed, as shown in Fig. 3-12.
The Fast Fourier transform of the grid yields peaks with a separation \( s_w = \frac{2\pi}{p} \), as shown in Fig. 3-12(b). The peak at the center (zeroth harmonic) contains only absorption information while the higher order peaks (including the first harmonic) contain both absorption and phase information. To isolate these terms, the separation of the individual harmonics must be greater than their spatial bandwidth. The bandwidth of a signal is the spread of the frequency components of significant energy present in the signal. Thus, the feature sizes of the order of grid period or greater can be observed and features smaller than the grid period are blurred out and cannot be observed in the image.

Fig. 3-12: a) Raw image of a glass bead with superimposed linear grid of 220 \( \mu \text{m} \) period, b) 2D Fourier image in (a), c) Bead raw image with 2D mesh grid of 75 \( \mu \text{m} \) pitch, and d) 2D Fourier image in (c). Both the raw images and the Fourier images are negative.
A variety of windowing functions were tested to isolate the individual harmonics. The harmonics were then shifted to center of the Fourier domain and transformed back to the spatial domain by taking the inverse 2D FFT. The processed images with the objects were then normalized by the bare grid images as

$$I_{m,n} = \frac{I_{m,n}}{I_{m,n,BG}}, \quad (3-8)$$

where $I_{m,n}$ and $I_{m,n,BG}$ are the $m^{th} \times n^{th}$ harmonic object and bare grid images, respectively. The image $I_{0,0}$ is the retrieved absorption image from the central harmonic and $I_{m,n}$ is the normalized $m,n$ order harmonic images, affected by both attenuation and diffraction. To separate out the phase and scattering information, the ratio of the first harmonic image and the attenuation image is taken as

$$(I_{\text{diff}})_{m,n} = \frac{I_{m,n}}{I_{0,0}} = S_{m,n} e^{i\psi_{m,n}}, \quad (3-9)$$

where

$$S_{m,n} = |I_{\text{diff}}|_{m,n}. \quad (3-10)$$

The $m,n^{th}$ harmonic image is [1]

$$I_{m,n} = I_0 S_{m,n} e^{i\psi_{m,n}}, \quad (3-11)$$

where $I_0$ is the contact image. The magnitude of $I_{\text{diff}}$ calculated for each pixel, gives the scattering amplitude image, $S_{m,n}$. For the 1D grid, there is only one index, n, which is written here as $S_{0,n}$. The argument $\psi_{m,n}$ is a directional derivative of the phase in the direction $mg_1 + ng_2$ as can be seen from Eq. (3-4) and (3-5). The
differential phase contrast (DPC) image $\psi_{m,n}$ shows phase contrast most strongly along the direction of the derivative. To construct an image that represents phase contrast more symmetrically, DPC images presenting orthogonal directional derivatives can be combined. These DPC images can be obtained by taking, for example, the $\psi_{1,0}$ and $\psi_{0,1}$ when using a 2D grid. By taking a first derivative along the DPC derivative direction on each image and summing the results, the Laplacian image $\zeta$ is

$$\zeta = \frac{d}{dx}(\psi_{1,0}) + \frac{d}{dy}(\psi_{0,1}).$$  \hspace{1cm} (3-12)

As an alternative, a more symmetric phase contrast image $\alpha$ can be obtained by the Pythagorean sum of the two DPC images

$$\alpha = \sqrt{P_x^2 \psi_{(1,0)}^2 + P_y^2 \psi_{(0,1)}^2}.$$  \hspace{1cm} (3-13)

Because of the different period of the mesh grid along x- and y-axis, the DPC images were weighted by multiplying them with the respective grid period $P_x$ and $P_y$.

### 3.4.2 Phase Image Retrieval for Elliptical Source

The raw images obtained in Fig. 3-12, taken with the 220 $\mu$m grid period, were processed to obtain the absorption image $I_0$, scattering amplitude image $S_{0,1}$ and diffraction phase contrast image $\psi_{0,1}$, as shown in Fig. 3-13. It was observed that the phase image $\psi_{0,1}$ had the edge enhancement tilted at an angle of about $40^\circ$, although
it was expected to be in the direction perpendicular to the grid lines. The grid was then rotated by 90°, and the processed images were observed to have the edge enhancement surprisingly at the same angle, rather than rotated by 90°. Also, analyzing absorption and scattering amplitude images carefully, it was further observed that the absorption image had blurring at the same angle while the scattering amplitude image has edge enhancement at 40°.

Another image of the glass bead was obtained, this time with the 300 μm grid period and processed to retrieve $I_0$ and $\psi_{0,1}$ as shown in Fig. 3-14 (a) and (c) respectively. In addition, a conventional image of the same bead was obtained to analyze shown in Fig. 3-14 (b). Blurred edges were observed at the same angle. This led to the conclusion that the x-ray source spot had changed and was no longer round, which was confirmed by the source spot measurement of section 3.4. The intensity along the longitudinal axis of the source spot leads to edge enhancement of the processed images. Because the algorithm associates blurring of the edges with differential phase, the gradient due to blur, which was due to the elliptical source spot, shows

Fig. 3-13: Retrieved images from a raw image in Fig. 4.9(a). a) Absorption image $\log(T_0)$, b) Scattering image $S_{0,1}$, and c) DPC image $\psi_{0,1}$.
up as an apparent phase gradient in the images. It is therefore important to confirm that the phase effects move appropriately with the grid directions.

Fig. 3-14: Retrieved images from a single raw image of bead with grid of 300 μm period, a) Absorption image log($\tilde{I}_0$), b) Conventional image, c) DPC image $\psi_{1,0}$, d) Intensity profiles along lines in (c).
4 Results

4.1 Absorption Vs Conventional Image

Because the Rh source demonstrated edge blurring, the remainder of the images were taken with the Mo source. A 2.5 mm diameter glass bead was imaged using the Mo source and the linear grid (G1) of pitch 220 μm placed 18 cm from the object. The raw image is shown in Fig. 4-1(a). The grid-to-camera distance was $d_{GC} = 10$ cm with an exposure of 3 s. The absorption image $I_{0,0}$ shown in Fig. 4-1(b) was retrieved from the central (0,0) harmonic using a Gaussian window with a width of FWHM $\sim \frac{1}{4} \text{sw}$. The conventional magnification image without the grid was obtained at the same distance, as shown in Fig. 4-1(c). To allow for a more direct comparison,

![Fig. 4-1: a) Bead image of 2.5 mm with a superimposed grid of period 220 μm, b) Log of the absorption image obtained from the central harmonic (0,0), c) Conventional absorption raw image, and d) Log of the processed conventional image.](image)
the conventional raw image was transformed to Fourier space and windowed at (at the origin) using a Gaussian window of the same width used to obtain the absorption image, $I_{0,0}$ in (b). The log of the inverse Fourier transform was then displayed as shown in (d) to compare the contrast with image in (b). Attenuation contrast $C_{\text{Att}}$, as discussed in Chapter 2, was computed as

$$C_{\text{Att}} = \frac{|I_{\text{obj}} - I_{\text{Bkg}}|}{I_{\text{Bkg}}}, \quad (4-1)$$

where $I_{\text{obj}}$ is intensity with the object present image and $I_{\text{Bkg}}$ is intensity of image without the object. The absorption image (b) contrast, 1.06±0.08 was comparable to the processed conventional image (d) contrast, 0.99±0.06.

### 4.2 Phase Contrast Image Retrieval

The raw image in Fig. 4-1(a) was further analyzed to obtain the phase contrast image $\psi_{0,1}$ from the first order harmonic (0,1). A Gaussian window of width FWHM $\sim \frac{1}{4}sw$ was used to separate the harmonic. Differential phase contrast (DPC) image $\psi_{0,1}$ was retrieved as shown in Fig. 4-2. The edge enhancement is observed along the vertical axis as expected due to the horizontal grid lines.

Another image of the bead was also taken with the woven mesh grid of 75×97 μm pitch. The raw image is shown in Fig. 4-3(a). The distances were similar, $d_{50} = 10$ cm,
$d_{OG} = 18.5$ cm and $d_{GC} = 15$ cm with an exposure of 2 s. Individual harmonics shown in Fig. 3-12(d) were separated using a Gaussian window width of $sw = \frac{\pi}{2p}$.

Differential phase contrast images $\psi_{1,0}, \psi_{0,1}$ and $\psi_{1,1}$ obtained from their corresponding harmonics are shown in Fig. 4-3(b), (c) & (d). The edge enhancement can be seen oriented along x-axis in Fig. 4-3(b), along y-axis in (c) and along diagonal in (d) as were expected. Harmonic strength decreases with increase in $m,n$ as shown in image $\psi_{1,1}$ in Fig. 4-3(d), so that the image becomes noisier. An intensity profile taken along the line indicated in (c) is shown in (e).

The edge enhancement signal peak height ($P_h$), as shown in Fig. 4-3(e) is significantly large to be distinguished from the background noise. The background noise $N$ was computed by windowing a region, about 50 pixels wide, sufficiently far from the

Fig. 4-3: Phase images retrieved from a raw image of the glass bead: a) Raw image, b) DPC $\psi_{1,0}$, c) DPC image $\psi_{0,1}$, d) DPC image $\psi_{1,1}$, and e) Intensity profile along the line indicated in (c) showing edge enhancement for boundaries as well as small internal defect.
object and computing its standard deviation. The peak height $P_h$ shown in Fig. 4-3(e), at the edge of an object was computed and peak-to-noise ratio (PNR) was $15 \pm 1$.

A similar image, this time of a 2.5 mm outer diameter polyvinyl chloride tube, was obtained with the superimposed mesh grid of $75 \times 97 \, \mu m$. The object, grid and detector distances were similar to the bead image and are given in Fig. 4-4.

---

**Fig. 4-5:** Image of 2.5 mm outer diameter tube with a 75 $\mu$ m period mesh grid having distances $d_{SO} = 9.5$ cm, $d_{OG} = 16.5$ cm and $d_{GC} = 17$ cm, and retrieved DPC images $\psi_{m,n}$: a) Raw image b) Absorption image $I_{0,0}$, c) Phase image $\psi_{1,0}$, d) Phase image $\psi_{0,1}$, e) Intensity profile along the lines indicated in (c), f) Intensity profile along the lines indicated in (d).

---

**Fig. 4-4:** Distances of the imaging setup for Fig. 4-5. The face indicates the object location.
Absorption and phase images $I_{0,0}$, $\psi_{1,0}$ and $\psi_{0,1}$ shown in Fig. 4-5(b), (c) and (d) were retrieved from zero and first order harmonics. As before the (1,0) harmonic produced a phase derivative along the vertical axis as shown in Fig. 4-5(c) and peaks in the intensity profiles along the vertical line in (e), while the (0,1) harmonic produced a phase derivative along the horizontal axis as shown in Fig. 4-5(d) and peaks in the intensity profiles along the horizontal line in (f).

### 4.3 Effects of Window Function Types

A 2.5 mm outer diameter polyvinyl chloride tube was imaged with a superimposed mesh grid of period 75×97 μm, Fig. 4-7: Distances shown for imaging setup.

![Fig. 4-7: Distances shown for imaging setup.](image)

**Fig. 4-7: Distances shown for imaging setup.**

![Fig. 4-6: Window styles from top to bottom, rectangular, Gaussian and square.](image)

**Fig. 4-6: Window styles from top to bottom, rectangular, Gaussian and square.**

Fig. 4-8: Phase images obtained using different window functions of same width of FWHM= $\frac{sw}{4}$ pixel.

![Fig. 4-8: Phase images obtained using different window functions of same width of FWHM= $\frac{sw}{4}$ pixel.](image)
at distances shown in Fig. 4-7 with an exposure of 2 s. Different windowing functions of same FWHM = \( \frac{sw}{4} \), which is the height in case of square and rectangular window, shown in Fig. 4-6 were tested to separate the harmonics. For square and rectangular windows, the desired harmonic is selected by selecting only those pixels within the given window and inverse Fourier transforming only that region, rather than simply setting the pixels outside that region to zero. This reduces ringing artifacts. The image was processed to obtain the DPC images shown in Fig. 4-8.

Peak-to-noise ratios (PNR) for the different window functions were computed, and given in Table 4-1. The PNR for the Gaussian window is higher than for the rectangular and square windows. The ringing artifacts visible in the images for the square and rectangular images in Fig. 4-8 may be responsible for higher noise.

<table>
<thead>
<tr>
<th>Window type of width FWHM=30</th>
<th>Bkg. Stand. Dev. (N)</th>
<th>DPC signal peak height (P_h)</th>
<th>PNR (P_h/ N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rectangular</td>
<td>0.021</td>
<td>0.24</td>
<td>11</td>
</tr>
<tr>
<td>Square</td>
<td>0.011</td>
<td>0.24</td>
<td>22</td>
</tr>
<tr>
<td>Gaussian</td>
<td>0.008</td>
<td>0.24</td>
<td>30</td>
</tr>
</tbody>
</table>

Table 4-1: PNR of phase images with different window functions.
4.4 Window Width Effects

Adequate separation of the windowed data of one harmonic from the next is necessary to avoid fringe artifacts in the phase image. A Gaussian window of FWHM greater than 1/4\textsuperscript{th} of the $sw=\frac{2\pi}{p}$, produced these artifacts, as shown in Fig. 4-9(a) and its intensity profile in (b).

![Image](a) $\psi_{0,1}, \sigma=SW/4$

![Image](b) Intensity profile in (a)

**Fig. 4-9**: a) Phase image $\psi_{0,1}$ with $\sigma \approx \frac{1}{3}$ of SW, b) Intensity profile along line indicated in (a).

Because the windowing function serves as a low pass filter in Fourier space, the resolution of the recovered phase and attenuation images decrease along with the window width, as illustrated in Fig. 4-10(a), obtained using windows from the widest, $\sim 1/4$\textsuperscript{th} of the $sw$ to the narrowest at $\sim 1/10$\textsuperscript{th} of the $sw$, (which corresponds to minimum resolvable feature size of almost 1 mm in detector plane). Comparing the images in Fig. 4-10(a) through (d), a significant decrease in detailed contrast and increase in edge blurring were observed with the decrease in the window width.
Table 4-2: Comparison of peak-to-noise ratio (PNR) by varying window width.

<table>
<thead>
<tr>
<th>Window width (SW ≡ ( \frac{2\pi}{p} ))</th>
<th>Std. Dev. of Bkg. (N)</th>
<th>Phase image peak height</th>
<th>(P/N) PNR</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/4th of SW</td>
<td>0.007</td>
<td>0.32</td>
<td>46</td>
</tr>
<tr>
<td>1/6th of SW</td>
<td>0.008</td>
<td>0.32</td>
<td>40</td>
</tr>
<tr>
<td>1/8th of SW</td>
<td>0.011</td>
<td>0.32</td>
<td>29</td>
</tr>
<tr>
<td>1/10th of SW</td>
<td>0.016</td>
<td>0.32</td>
<td>20</td>
</tr>
</tbody>
</table>

Fig. 4-10: Phase image \( \psi_{0,1} \) retrieved using window widths from 1/4\(^{th}\) to 1/10\(^{th}\) of \( SW \equiv \frac{2\pi}{p} \) in (a) through (d). (e) - (h) are the intensity profiles along the lines in (a) through (d).

Table 4-2 shows the comparison of peak-height to noise ratio (PNR) for the varying window widths from 1/4\(^{th}\) to 1/10\(^{th}\) of \( Sw \). PNR decreases with decreasing window width, as expected.
4.5 Effects of Grid Period

![Fig. 4-11: \( \psi_{0,1} \) obtained using different period grids: a) Phase image using a linear grid of period 300 \( \mu \text{m} \), b) Phase image using a linear grid of period 220 \( \mu \text{m} \), c) Phase image using a wire mesh grid of period 75×97 \( \mu \text{m} \), \( \psi_{0,1} \) along the direction of 97 \( \mu \text{m} \) period.](image)

Phase images were obtained using variety of grids with periods ranging from 300 to 75×97 \( \mu \text{m} \) as shown in Fig. 4-11. A significant increase in PNR, edge sharpness and reduced noise were observed in the image obtained with mesh grid due to higher grid frequency of 75×97 \( \mu \text{m} \) in Fig. 4-11(c), which compared to period of 220 \( \mu \text{m} \) and 300 \( \mu \text{m} \), allowed wider windowing functions to be used in the Fourier plane. PNR values for images in Fig. 4-11 were computed, given in Table 4-3.

<table>
<thead>
<tr>
<th>Grid type/period (µm)</th>
<th>Bkg. Stand. Dev. (N)</th>
<th>DPC signal peak height (Pₚ)</th>
<th>PNR (Pₚ/ N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mesh (75×97)</td>
<td>0.0047</td>
<td>0.24</td>
<td>51</td>
</tr>
<tr>
<td>Linear (220)</td>
<td>0.0037</td>
<td>0.12</td>
<td>32</td>
</tr>
<tr>
<td>Linear (300)</td>
<td>0.0023</td>
<td>0.07</td>
<td>30</td>
</tr>
</tbody>
</table>

Table 4-3: PNR comparison for different grid periods.
4.6 Scattering Amplitude Image Retrieval

In addition to the phase images, the scattering amplitude images, $S_{m,n}$, as given in Eq. 3-10 were also computed. Edge enhancement was observed in both images $S_{1,0}$ and $S_{0,1}$, as shown in Fig. 4-12(a) & (b). The images were then added to obtain an image having edge enhancement all around the object as shown in Fig. 4-12(c) and the intensity profile in (d). As shown in the intensity profile, the “scatter” image includes effects of absorption as well as edge deflection. The edge enhancement peak height-to-noise ratio (PNR) for phase image $\psi_{0,1}$ is higher than for the scatter amplitude image $S_{m,n}$ as given in Table 4-4.

<table>
<thead>
<tr>
<th>Image Type</th>
<th>Bkg. Stand. Dev. (N)</th>
<th>Edge signal peak height ($P_h$)</th>
<th>PNR ($P_h/N$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Phase Image, $\psi_{0,1}$</td>
<td>0.016</td>
<td>0.24</td>
<td>15</td>
</tr>
<tr>
<td>Scatter Image, $S_{0,1}$</td>
<td>0.019</td>
<td>0.1</td>
<td>5.3</td>
</tr>
</tbody>
</table>

Table 4-4: Edge enhancement to noise ratio for phase and scatter images.

Scattering amplitude images $S_{1,0}$ and $S_{0,1}$ of the tube of Fig. 4-5(a) are shown in Fig. 4-13(a) and (b). Again, these show the absorption and phase effects along the vertical and horizontal directions, as seen in the intensity profile in Fig. 4-13(d) along line in Fig. 4-13(c). The sum of $S_{1,0}$ and $S_{0,1}$ gives the image shown in Fig. 4-13(c).
Fig. 4-12: Scattering amplitude images retrieved from the raw image of the bead in Fig. 3(c): a) Scattering amplitude image $S_{1,0}$, b) Scattering amplitude image $S_{0,1}$, c) Sum of $S_{1,0}$ & $S_{0,1}$, d) Intensity profile along line indicated in (c).

Fig. 4-13: Polyvinyl chloride tube of 2.5 mm outer diameter: a) Scattering amplitude image $S_{1,0}$, b) $S_{0,1}$, c) $S_{1,0} + S_{0,1}$, d) Intensity profile along line in (c).
4.7 2D Phase Image Construction

Symmetric combinations of the two phase images $\psi_{1,0}$ and $\psi_{0,1}$ of the glass bead and the polyvinyl chloride tube were done by comparing the constructed images $\zeta$ & $\alpha$, given in Eq. 3-12 & 3-13, as shown in Fig. 4-14(a) and (b). Simple addition of the diffraction phase images, shown in Fig. 4-14(c) is also compared to $\zeta$ & $\alpha$. These reconstructed images provide two dimensional edge enhancements in a single image. The signal peak height of the edge enhancement, shown in Table 4-5, is more prominent for the square sum image $\alpha$ as compared to the Laplacian image $\zeta$ or the simple sum.

Fig. 4-14: Combination of phase images to construct 2D phase image, a) Sum of the Laplacian of $\psi_{1,0}$ and $\psi_{0,1}$ and its intensity profile of the bead image, b) Square root of the sum of the squares of $\psi_{1,0}$ and $\psi_{0,1}$ of the bead image, c) Sum of $\psi_{1,0}$ and $\psi_{0,1}$, d) Sum of the Laplacian of $\psi_{1,0}$ and $\psi_{0,1}$ of the tube image, e) Square root of the sum of the squares of $\psi_{1,0}$ and $\psi_{0,1}$ of the tube image weighted by their respective period.
<table>
<thead>
<tr>
<th>Images in Fig 11</th>
<th>Image</th>
<th>Bkg. Stand. Dev. (N)</th>
<th>Edge peak height (P_h)</th>
<th>PNR (P_h/ N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) Sphere</td>
<td>ζ</td>
<td>0.0037</td>
<td>0.032</td>
<td>8.6</td>
</tr>
<tr>
<td>(b) Sphere</td>
<td>α</td>
<td>0.015</td>
<td>0.26</td>
<td>17.0</td>
</tr>
<tr>
<td>(c) Sphere</td>
<td>sum</td>
<td>0.03</td>
<td>0.22</td>
<td>7.3</td>
</tr>
<tr>
<td>(d) Tube</td>
<td>ζ</td>
<td>0.01</td>
<td>0.032</td>
<td>3.2</td>
</tr>
<tr>
<td>(f) Tube</td>
<td>α</td>
<td>0.018</td>
<td>0.25</td>
<td>15.5</td>
</tr>
</tbody>
</table>

Table 4-5: Peak-to-noise ratio (PNR) comparison for images in Fig. 4-13.
5 Conclusion

One frequently developed phase contrast imaging technique employs multiple fine pitch gratings of the order of 1 μm. In this work, a grid-based phase contrast imaging technique was investigated that requires a single coarse grid of period in the range of 100 μm. The single shot, raw images of the object with a superimposed grid of the period 200-300 μm pitch or wire mesh grid of 75×97 μm pitch were acquired and processed in the Fourier domain. The first order harmonic yields phase information that generates an image with enhanced edges that are highly distinguishable from the surrounding material. The differential phase contrast (DPC) images were retrieved using rectangular, square and Gaussian window functions. Ringing artifacts were observed in the DPC images retrieved by using rectangular and square window functions, which may be responsible for higher noise. The edge enhancement peak height-to-noise (PNR) ratio was observed to be higher for the Gaussian window. Different window widths were tested for harmonic separation and higher PNR values were obtained for a window of FWHM approximately $\frac{sw}{4}$. Increasing the window width creates ringing artifacts whereas decreasing width causes blur in the image. A significant increase in PNR values was observed for the simple mesh due to its smaller period, as compared to the linear grids. Images were constructed by combining DPC images $\psi_{1,0}$ and $\psi_{0,1}$ in multiple ways and higher PNR values were observed for a Pythagorean sum. Because absorption, high contrast DPC, and scattering amplitude images with significant edge enhancement can all be retrieved from a single raw image, the technique is promising for future use in clinical diagnostic imaging.
6 References


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