Fiscal policy, Laffer curves and economic growth

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FISCAL POLICY, LAFFER CURVES
AND ECONOMIC GROWTH

by

Si Gao

A Dissertation
Submitted to the University at Albany, State University of New York
in Partial Fulfillment of
the Requirements for the Degree of
Doctor of Philosophy

College of Arts and Sciences
Department of Economics

2015
FISCAL POLICY, LAFFER CURVES
AND ECONOMIC GROWTH

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Si Gao

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To my parents: Jiaqiang Gao, Xiumei Yue
And
My wife: Xin Li
ABSTRACT

The doctoral dissertation consists of three essays on the topics of economic growth and fiscal policy.

The first essay lies in field of economic growth empirics. It is well-known that an economy will converge to its steady state, determined by a country’s production factors including labor force, physical and human capital. Such ‘conditional convergence’ property stays in line with the neoclassical economic growth theory. I investigate how the physical resources allocated to education affect an economy’s standard of living in steady state, as well as the speed of convergence towards the steady state. I find that human capital investment, either the time or physical input, plays a vital role in explaining cross-country income variations. Moreover, I propose a structural panel model to estimate the unobserved country-effects in a large cross-country sample. Empirical results suggest that the unobserved effect is closely tied to a country’s income and human capital levels. This finding provides evidence in favor of the view that human capital strongly affects a country’s technology adoption.

The next two essays contribute to the literature on Laffer curves and fiscal policy. The Laffer curve limits a government’s ability to raise taxes by increasing tax rates because the tax-rate distortion will eventually dominate the revenues when tax rate is high. Standard literature computes Laffer curves under the assumption that government spending is not productive - tax revenues are either thrown into ocean or redistributed as lump-sum transfer payments. The purpose of the essays is to investigate how the shape of Laffer curves changes when I make the more realistic assumption that tax revenues yield benefits. I allow government spending to be productive and recalculate the Laffer curves for the US.

Two types of government spending are considered, utility-enhancing spending on public goods and investment spending which enhances productivity. In Chapter 3, I model the productive government spending as investment in public capital; in Chapter 4, the productive investment is modeled as subsidies to education, essentially subsidies to private investment in human capital. I demonstrate that productive gov
ernment spending does affect the shape of Laffer curves. Both labor and capital tax
Laffer curves have higher peaks which occur at larger tax rates when the government
chooses to optimally allocate marginal tax revenue between public goods and produc-
tive investment, compared with those ‘traditional’ Laffer curves with tax revenues
allocated only to spending. Therefore, when governments use marginal tax revenues
for productive purposes, they are able to collect more tax revenue than otherwise.
Yet, whether or not the government should raise tax revenues by increasing tax rates
depends on the implications for welfare, not for tax revenue.
ACKNOWLEDGMENT

I would never have been able to finish my dissertation without the guidance of my committee members, help from friends, and support from my family.

First and foremost, I would like to express my deepest gratitude to my advisor Professor Betty Daniel for continuous support of my PhD study and research, for her patience, enthusiasm, encouragement. Her guidance and insight helped me through extremely difficult times over the course of the doctoral studies. I could not have imagined having a better advisor and mentor for my PhD study.

Besides my advisor, I would like to thank the rest of my dissertation committee: Professor John Jones, and Professor Zhongwen Liang for their insightful advice and thoughtful criticism.

I would also like to thank Professor Fang Yang, Professor Michael Jerison, Professor Adrian Masters, and Liu Yang for their helpful comments, time and attention during the busy semesters.

I would additionally like to thank my colleagues and friends for their support and encouragement, including but not limited to: Fang Song, Pu Li, Herbert Zhao, Na Cheng, Yunan Lu, Rui Cheng, Chen Cao, and Jungtaek Lee.

Last but not the least, I would like to extend my sincere appreciation to my parents Jiaqiang Gao and Xiumei Yue and my wife Xin Li, without whose love, support and understanding could I never have completed the doctoral degree.
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CHAPTER 1

Introduction

Fiscal policy is the use of taxation and government expenditures to influence the economy. As an extremely important instrument, the changes on the level and composition of taxation and government spending can affect a number of key macroeconomic variables in an economy. There exists an enormous literature centering on the fiscal policy and its roles in stabilizing business cycle fluctuations, distribution of income, providing public goods, and so on. However, searching through the literature on fiscal policy, the answers to several key questions still remain ambiguous and vague, for instance: what are the roles of public education expenses in explaining cross-country variations in per capita income? How do productive government expenditures impact the tax revenues that the government can collect, i.e. the shape of Laffer curves? What is the welfare implication to long-run taxation when tax revenue is productive to the economy, rather than ‘thrown into ocean’ in the standard literature? Answering these questions requires efforts of further exploration on top of the existing literature, which motivates the three essays in this dissertation.

Chapter 2 discusses economic growth empirics within cross-country context. There has been enormous empirical work on cross-country economic growth in recent years. Among the most influential and widely cited, Mankiw, Romer, and Weil [53] (hereinafter MRW) empirically study cross-country per capita income growth disparity. They find an augmented version of Solow model with human capital can explain most of the income variations, with implied physical and human capital share both close to one third. However, measurement of human capital investment is solely based on schooling enrollment rate, which could wrongly measure a country’s real investment in human capital, since education expenses are not considered. I provide a human capital production function that requires physical resources as well as human capital and time for schooling, and find these coefficients are significant, implying the human capital investment is mismeasured in MRW.
Another potential source of bias in estimation is the ignorance of a country’s TFP. The country’s unobserved total-productivity factor includes a country’s endowment, technology and other characteristics that contribute to production, which is in general correlated with a country’s income and other production factors. To tackle this problem, I set up a dynamic panel data framework to capture the time-invariant country effect. GMM is applied to obtain consistent estimators. The findings include: 1. There is strikingly strong evidence in favor of an important role of human capital in short-term economic growth dynamics, in contrast to most of findings in the literature. 2. Public spending in education takes around a decade in order to become effective to economic growth. 3. The estimated ‘country-effect’ is highly correlated with a country’s per capita income and enrollment rate. When the estimated unobserved factor is accounted for in MRW’s cross-country regression, the coefficient on human capital is reduced from one-third to around one-fifth. This is consistent with the existing literature that the human capital coefficient is less than the advocated one-third by Mankiw, Romer, and Weil [53].

Chapter 3 investigates the welfare implications of productive public investment under neoclassical growth model framework. Government spending is explicitly modeled in the utility function and public capital is productive. The benevolent government funds itself through raising tax revenues and issuing bonds, and endogenously determines the allocation between government consumption, government investment after repaying exogenous transfer payments and debts. For a given level of tax revenues, government spending yields welfare gains directly to the representative household, while government investment boosts output so that the representative household benefits from a raise in wages and dividends. The government is fully informed of the social benefit and cost, and sets the government expenditure to maximize the welfare. The model is calibrated to the quarterly US economy. I assume the government optimally chooses the combination of public investment and public consumption in the long run and propose a set of parameter values so that the optimal steady state government investment to output ratio matches the historical data. I find the calibrated Laffer curves in my model can reach higher peaks than those in the traditional counterparts.
In Chapter 4 I extend the standard model with human capital to study role of valued government expenditures in characterizing Laffer curves and optimal long-run fiscal policies. Governments choose to incur the costs of distortionary taxation because they can use tax revenue to provide benefits. However, a standard assumption in the macroeconomics literature and in graduate textbooks is that tax revenue provides no benefits - it is either thrown into the ocean or is redistributed as lump-sum transfer payments.

In this model, government subsidizes human capital investment and public spending yields utility. The benevolent government funds itself through raising tax revenues and issuing bonds, and endogenously determines the allocation between public spending and government subsidies after repaying exogenous transfer payments and debts. Human capital exhibits externalities in production function, and subsidies to human capital investment is available to reduce the magnitude of distortion. The government is fully informed of the social benefit and cost, and allocate the government expenditures to maximize the welfare in the long-run. We provide a role for government subsidies to education by assuming that human capital production exhibits spillovers. However, in presence of distortional labor taxation, the government would optimally choose to provide subsidies even in the absence of externality, in order to offset some of the distortion created by the labor tax. I find higher peaks that occur at a larger tax rate along the Laffer curve when the government optimally allocate tax revenues compared with no subsidies. Subsidies offset some distortion from distortionary labor taxes and externalities, implying an increase in welfare and final output.

My dissertation contributes to the current literature in two folds: First, along the branch of economic growth empirics, I investigate the role of education expenses and the unobserved country-effect (TFP) in explaining per capita income variations across countries. Education expenditures have a significant impact on either long-run or short-run economic dynamics. Since the education sector is heavily subsidized in most of countries, the result has strong policy implications. Under a dynamic panel framework, the unobserved country-effect, or the total factor productivity (TFP), is estimated explicitly across countries, and is found significantly contributive to inter-
national income difference; Second, the dissertation extends the literature on fiscal policy and Laffer curves. The existing literature, e.g. Trabandt and Uhlig [71, 72], computes Laffer curves under the assumption that government spending is not productive, and finds that some countries are precariously close to their ‘slippery slope’. I allow tax revenue the government collects to be productive, and recalculate Laffer curves. I first consider productive government capital that contribute to final output. Next, I extend the model with human capital sector, in which human capital production exhibits spillovers. Tax revenue can be collected to subsidize the education expenses. In either case, peaks are higher, and slopes at low tax rates are larger, implying that productive government spending places countries further from their ‘slippery slopes’, than a conventional Laffer curve would suggest.

The dissertation unfolds as follows: Chapter 2 discusses the economic growth empirics; Chapter 3 considers productive government capital and its impact in shaping Laffer curves; Chapter 4 extends the model with human capital sector, and discusses the role of education subsidies in welfare implications and Laffer curves. Next, Chapter 5 concludes the dissertation. Technical derivations, data sources, and additional details for Chapter 2 to Chapter 4 are provided in Chapter 6 as appendix.
CHAPTER 2

Economic Growth Empirics: Beyond Mankiw-Romer-Weil

2.1 Introduction

There has been enormous empirical work on cross-country economic growth in recent years. Among the most influential and widely cited, Mankiw, Romer, and Weil [53] (hereinafter MRW) performed the neoclassical growth framework, which was first proposed by Solow [70], Cass [20] and Koopmans [47], to empirically study cross-country per capita income growth disparity. MRW first found estimates of the textbook Solow model predicted a capital share of about 0.6, too high compared to the prevalent value of about one-third. So they considered an augmented version of Solow model with human capital as an input for final goods production. The result turns out to be extraordinarily well: the implied physical and human capital share are both close to one third, and most of the cross-country per capita income variations can be explained by physical and human capital levels and population growth rates. Following MRW, an outburst of research work has been focusing on empirical studies in neoclassical growth and issues of convergence.¹

Albeit the excellent outcome, we are not fully comfortable with MRW’s measurement for human capital investment. Secondary school enrollment rate is only a partial measurement for the time endowment that workers spend on education. Its variations across the countries might not appropriately represent true variations of time spent on human capital production across countries. In addition, the appropriateness of using enrollment as the sole human capital investment remains in question. Production in human capital requires time inputs (the students’ forgone time), as well as physical investment, which includes teachers’ wages, fixed-assets, devices, textbooks and all relevant facilities. Ignoring these factors undermines the reliability of estimation for human capital and physical capital coefficients.

¹A non-exhaustive list includes Islam [39], Cho and Graham [23], Klenow and Rodriguez-Clare [46], Lee et al. [49], Caselli et al. [19], Hamilton and Monteagudo [36], Bernanke and Gurkaynak [14], Young [75], Durlauf and Quah [29], Barossi-Filho et al. [7], and Fischer [32].
We provide a human capital production function in a more realistic specification. Public spending on education is used as a proxy for education expenditures. In most countries government heavily subsidizes education sector, especially primary and secondary education, so we argue public spending is an appropriate proxy. We find that both types of investments in education are significant in the regression. Hence, we conclude that the physical education investment is important in explaining international income disparity.

The second goal of this paper is capture and characterize the underlying ‘country-effect’ in the augmented Solow model. The underlying unobserved effect is assumed to be randomly distributed across countries in MRW. However, it is in general likely that the unobserved factor correlates with a country’s income, since it includes a country’s endowment, technology and other characteristics that contribute to production. One branch of the literature uses the panel data approach to allow country-specific effect. An important contribution along this direction is made by Islam [39]. He reformulates the regression equation into a dynamic panel data model, and uses the Least Squares Dummy Variable (LSDV) approach to estimate it. He then reports that the role of human capital in economic growth process is ambiguous, as opposed to MRW’s conclusion. In fact, the anomalies regarding the role of human capital are often observed in the literature (for example, De Gregorio [28], Benhabib and Spiegel [13], Caselli et al. [19], and Hamilton and Monteagudo [36]), which is often regarded as a puzzle.

We follow this strategy to capture the individual country effect in a dynamic panel data model. However, the with-in estimators adopted by a number of existing empirical cross-country studies (e.g. Islam [39], and Barossi-Filho, Silva, and Diniz [7]) are not consistent estimators when applied in the dynamic panel, which weakens the validity of results from within-group estimations. In order to address the problem, this paper applies a generalize method of moments (GMM) estimator to the dynamic panel of augmented Solow model. Following the proposed method by Holtz-Eakin, Newey, and Rosen [38] and Arellano and Bond [3], sometimes called the difference GMM, the inconsistency is appropriately eliminated through instrumenting the difference of the dynamic panel equation with the lagged explanatory variables.
In contrast to the predominant view in panel economic growth studies, we find strikingly strong evidence in favor of an important role of human capital in explaining economic growth process. Notably, after allowing time-invariant country specific effect, the implied coefficient on human capital reduces to about half of that in single cross-country regression, while the estimated coefficient on physical capital remains in the ballpark of the conventional value, one-third. Although the size of human capital shrinks, the impact of human capital on economic growth is still significant with positive sign. To our knowledge, it is the first to document a significant impact of human capital in a panel augmented Solow model.

Similar to the practice in single cross-country regression, we also estimate the effect of education expenditures. However, this coefficient is insignificant. In fact, this finding is consistent with Hamilton and Monteagudo [36], which concludes that ‘the suggestion that countries can significantly improve their growth by further investments in public education does not seem to be supported by the data’. One possible reason is that education investment takes a long time to make any difference to economic growth. To test this possibility, we estimate the dynamic panel under an alternative assumption that the human capital in the current period depends on physical investment in schooling one decade ago, or 2-period lagged public spending on education. We find the coefficients on education investment become positive and significant. Therefore, our finding suggests the opposite of Hamilton and Monteagudo [36]: investment in public education is necessary and important for an economy’s sustainable growth, although it may take up to a decade to be effective.

Finally, the dynamic panel framework enables us to estimate the unobserved individual effect for each country. We find the unobserved effect is highly correlated with per capita income and education investment, particularly the forgone time. Next, we bring the estimated country-effects back to single-cross country estimation, treating them as ‘observed’. We find that the individual effect is significant in the regression. At the same time, other coefficients barely change except those on human capital investments. The size roughly reduces by half. The yielding result implies that ignoring the differences in individual factors might cause bias in estimation. The implied share of human capital is lower than the estimate in MRW.
The paper is organized as follows: Section 2.2 revisits MRW’s single cross-country regression. Section 2.3 discusses the effects of education expenditures. Section 2.4 introduces the dynamic panel data model framework and discusses about the unobserved country-effect. Section 2.5 concludes. Additional details and data are in the appendix.

2.2 Revisit MRW

In this section we start by discussing about the MRW framework to assess the human capital augmented Solow model, update the estimates using more recent data, and then modify it. In this model the production function is specified as follows:

\[ Y_t = K_t^\alpha H_t^\beta (A_t L_t)^{1-\alpha-\beta} \]  \hspace{1cm} (2.1)

\(K_t, \, H_t\) denote aggregate physical and human capital, respectively. \(L_t\) denotes aggregate labor force. We assume it’s inelastic. Each worker provides 1 unit of labor force. \(Y_t\) is output at time \(t\). Slightly different from MRW, we directly work with the aggregate values, \(X\) rather than dividing them by \(AL\) for normalization. After rearrangement, we can rewrite equation (2.1) to yield

\[ \frac{Y_t}{L_t} = A_t(K_t/Y_t)^{-\frac{\alpha}{1-\alpha-\beta}}(H_t/Y_t)^{-\frac{\beta}{1-\alpha-\beta}} \]  \hspace{1cm} (2.2)

Per capita output is now a function of an unobserved factor, \(A\), and two capital intensities. Different from the usual accounting exercise that directly substitutes human and physical capitals with their inputs, such an arrangement gives a clear picture of per capita output, the object of interest, and its explanatory variables on the right-hand side. \(A_t\) grows exponentially at rate \(\psi\): \(A_t = A_0 e^{\psi t}\). Labor grows at rate \(n\): \(L_t = L_0 e^{nt}\).

Next, capital and human capital accumulate according to:

\[ K_{t+1} = (1 - \delta)K_t + I_{k,t} \]  \hspace{1cm} (2.3)

\[ H_{t+1} = (1 - \delta)H_t + I_{h,t} \]  \hspace{1cm} (2.4)
So MRW assume the two capitals adopt the same production technology, and they depreciate at the same rate. In steady state,

\[ K/Y = \frac{I_k/Y}{\psi + \delta + n} \]  

(2.5)

\[ H/Y = \frac{I_h/Y}{\psi + \delta + n} \]  

(2.6)

Along the balanced growth path, taking the logarithm on both sides of equation (2.2) yields

\[
\log \frac{Y}{L}(t) = \log A_0 + \psi t + \frac{\alpha}{1 - \alpha - \beta} \log \frac{I_k/Y}{\psi + \delta + n} + \frac{\beta}{1 - \alpha - \beta} \log \frac{I_h/Y}{\psi + \delta + n}
\]

(2.7)

This equation is the MRW’s estimation framework. Essentially, MRW construct the steady-state capital intensities through specifying production technologies for both capitals that accumulate in the same technology. Next MRW use the average ratio of secondary enrollment to the working-age population from UNESCO yearbook over 1960-1985 as a proxy for \( I_h/Y \), which MRW denote as ‘SCHOOL’. For \( Y/L \), MRW use real GDP in 1985 divided by the working-age population in that year from Summers-Heston data set, the Penn World Tables (PWT). \( I_k/Y \) is the average Summers-Heston investment rate over 1960-1985. \( n \) is the average working-age population growth rate over 1960-1985 from UNESCO yearbook. MRW assume that \( \psi + \delta = 0.05 \).\(^2\)

MRW argue that \( \log A_0 \) term reflects not only technology but resource endowments, climate, institutions and so on. It differs across countries randomly: \( \log A_0 = a + \epsilon \), where \( a \) is a constant and \( \epsilon \) is a country-specific shock. Thus, this assumption allows MRW to run OLS regression on equation (2.7) to estimate \( \alpha \) and \( \beta \), the shares of physical and human capital, respectively. They provide estimates in three samples: non-oil countries excluding oil producers (98 countries); intermediate countries that have population larger than one million (75 countries); OECD countries (22 countries). They find that the estimates of physical/human capital shares are both close to one-third. Especially in non-oil sample, \( \alpha = 0.31 \) and \( \beta = 0.28 \), with \( R^2 = 78\% \).

\(^2\)None of the results are sensitive when we use 0.04 or 0.06 instead in a sensitivity test.
The first experiment we do is to revisit MRW with updated data. Our sample period is 1970-2010. Since MRW, the data quality has improved so accurate enrollment information is available. ‘SCHOOL’ measures the ratio of secondary enrollment over the working-age population. In MRW, this variable is approximated by using secondary enrollment rate multiplied by fraction of the working-age population that is at school age (aged 15 to 19). In order to obtain a more accurate measure of ‘SCHOOL’, we directly use the secondary school enrollment population divided by the working-age population as a regressor.

Ideally, the variable ‘SCHOOL’ is intended to measure the fraction of population that involves in education. Thus, we also propose an alternative measure that seems more in line with this intention. We calculate the ratio of secondary and tertiary school enrollment over the working-age population, and use this ratio as a proxy for $I_h/Y$ instead of ‘SCHOOL’ by MRW. The data is from UNESCO over 1970-2010. To distinguish it from the original measure in MRW, it is denoted as ‘SCHOOL2’.

The Summers-Heston data has been updated regularly, and the latest version, PWT 7.1 is up to 2010. From PWT 7.1 we use real GDP in 2010 divided by the working-age population in that year for $Y/L$, the average working-age population growth rate for $n$ and average investment share of GDP for $I_h/Y$, over 1970-2010. Some countries are dropped out of the data set, so the sizes are reduced to 89 and 69 for non-oil and intermediate sample, respectively.

For comparison purposes, we run two versions of OLS regressions with updated data, one using MRW’s ‘SCHOOL’ and another one using ‘SCHOOL2’ as described above. We denote the first regression as MRW1 and second one as MRW2. The results are summarized in Table 2.1.

In general we find the results with updated data are similar to those in the MRW. Both MRW1 and MRW2 explain over 70% of the variation in per capita income in the non-oil and intermediate samples. The implied coefficients for physical and human capital are also reasonable. Both $\alpha$ and $\beta$ are around one-third in the non-oil and intermediate samples. They are slightly larger than those estimated with data over

---

3One might question that the real GDP per capita in 2010 is too close to the financial crisis and thus underestimates the per capita output on the balanced growth. We estimated using PWT 6.3 from 1970-2007, and found the results are very close: none of the coefficients change significantly.
Table 2.1:
Estimation Of the Augmented Solow Model

Dependent Variable: Log Real GDP per working-age person in:

<table>
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<th>1985</th>
<th>2010</th>
<th>2010</th>
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<tr>
<td></td>
<td>MRW0 Non-oil</td>
<td>MRW0 Intermediate</td>
<td>OECD</td>
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<td>Sample Observations:</td>
<td>98</td>
<td>75</td>
<td>22</td>
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<tr>
<td>Constant</td>
<td>6.89***</td>
<td>(1.17)</td>
<td>7.81***</td>
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<tr>
<td>log($I_k/Y$)</td>
<td>0.69***</td>
<td>(0.13)</td>
<td>0.70***</td>
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<tr>
<td>log($\psi + \delta + \pi$)</td>
<td>-1.73***</td>
<td>(0.41)</td>
<td>-1.50***</td>
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<tr>
<td>log(SCHOOL)</td>
<td>0.66***</td>
<td>(0.07)</td>
<td>0.73***</td>
</tr>
<tr>
<td>R²</td>
<td>0.78</td>
<td>0.77</td>
<td>0.24</td>
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<td>Restricted Regression:</td>
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<tr>
<td>Constant</td>
<td>7.86***</td>
<td>(0.14)</td>
<td>7.97***</td>
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<td>log($\frac{I_k}{Y}$)</td>
<td>0.73***</td>
<td>(0.12)</td>
<td>0.71***</td>
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<td>log(SCHOOL)</td>
<td>0.67***</td>
<td>(0.07)</td>
<td>0.74***</td>
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<tr>
<td>R²</td>
<td>0.78</td>
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<td>p-value</td>
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<td>0.89</td>
<td>0.97</td>
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<tr>
<td>Implied Coefficients:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Implied $\alpha$</td>
<td>0.31</td>
<td>0.29</td>
<td>0.14</td>
</tr>
<tr>
<td>Implied $\beta$</td>
<td>0.28</td>
<td>0.30</td>
<td>0.37</td>
</tr>
</tbody>
</table>

*a* MRW0: the original observation period 1960-1985; MRW1: use updated data over 1970-2010 from PWT 7.1; MRW2: the same as MRW1, but use the ratio of secondary and tertiary school enrollment to the working-age population rather than just secondary enrollment.

*b* The standard errors are in parentheses immediately below.

*c* *, ***, *** indicate significance at 10%, 5% and 1% level, respectively.

*d* The investment, working-age population growth rates and SCHOOL are averaged over the corresponding observation period; ($\psi + \delta$) is assumed to be 0.05.
1960-1985. Implied $\beta$’s are in particular larger with more recent data, especially in MRW2. In the OECD sample, the results in more recent data have a much higher explanatory power than the original. $R^2$ equals 49% in MRW1, and equals 64% in MRW2, which more than double the result in MRW0 with a 28% $R^2$. The implied $\alpha$ in OECD is essentially the same, but the implied $\beta$ in OECD sample rises with more recent data, from 0.37 in MRW0 to 0.46 and 0.51 in MRW1 and MRW2, respectively. When comparing the two estimation results over 1970-2010, we find that MRW2 with two enrollment explains the cross-country per capita income variation unambiguously better than MRW1 which uses only secondary enrollment: $R^2$ rises by 4 percentage points in the non-oil and intermediate sample, and by 15 percentage points in the OECD sample. So we conclude that inclusion of tertiary enrollment in ‘SCHOOL’ improves estimation accuracy. Thus we will use the two enrollment from now for all the following regressions in place of ‘SCHOOL’, which we denote as ‘SCHOOL2’.

Some issues are also noteworthy. The implied $\alpha$ is still relatively low and insignificant in the OECD sample, but its size is similar to that in the original MRW estimation. Another issue is the rejection of test for over-identifying restriction. Equation (2.7) predicts that coefficients on $\log I_k/Y$, $\log SCHOOL2$, $\log(\psi + \delta + n)$ sum to zero. As shown in the bottom of Table 1, this restriction is rejected at 1% level for non-OECD samples over 1970-2010. The sum of the three coefficients appears to be negative given a large negative coefficient on $\log(\psi + \delta + n)$ in either non-oil or intermediate sample. Actually this issue is not found for the first time in the literature. Bernanke and Gurkaynak [14] report that the restriction is rejected at the 1% level when using PWT 5.6 over 1960-1990 and PWT 6.0 over 1960-1995 in the non-oil sample, and rejected at the 5% level using PWT 6.0 over the period 1960-1985. Such problems with estimation of production relationships are not uncommon. As Bernanke and Gurkaynak [14] commented, ‘... we have found that the over-identifying restriction implied by the Cobb-Douglas structure is often rejected, but problems with estimation of production relationships are not uncommon. Very possibly, these statistical rejections are not of great economic significance’.
2.3 Physical Investment in Human Capital

Through more recent data and more accurate measure of ‘SCHOOL’, we are able to obtain decent values of coefficients with a high $R^2$. But we are still not fully comfortable with the MRW framework. Human capital in the economy should be the workers’ knowledge, skills that contribute to aggregate production. In MRW, the investment in human capital is solely measured by the percentage of population that enroll in (secondary) schooling. Denote population in schooling as $L_h$, then MRW assume $I_h/Y = L_h/L$. This is essentially the fraction of workers’ time spent on human capital production. There is considerable literature in areas of labor economics and macroeconomics that employs similar human capital production technology (for example, see Lucas [51], Heckman et al. [37], Jones [41], and Erosa et al. [30]).

However, as some researchers argue, the contributions of physical investment and some fraction of human capital are important to human capital formation. For example, see Ben-Porath [12], Kendrick [45], Rebelo [65], Keane and Wolpin [44], just to name a few. According to Digest of Education Statistics, expenditures on education averaged about 7% of GDP over 1995-2007 in United States. Kendrick [45] estimates that about 50% of investment in human capital in the United States represents the opportunity cost of student time, and the remaining 50% is composed of expenditures on teachers (human capital) and facilities (physical capital). These components indicate the quality of schooling, which have a large impact in human capital production. The quality of schooling is especially important in international research, since it varies significantly across countries. In richer countries, students enjoy better facilities and better teachers, so the human capital should be higher than other countries, even at the same enrollment rate. MRW’s estimates totally ignore physical investment in human capital as a factor, thus the reliability of estimation for human capital and physical capital coefficients remain in question.

Assume that human capital investment is a composite of physical expenditures and human capital. We now modify the equation (2.4) to:

$$H_{t+1} = (1 - \delta)H_t + E_t^\omega H_{h,t}^{1-\omega},$$  \hspace{1cm} (2.8)
where $E_t$ is the aggregate education expenditures, and $H_h$ is the human capital devoted to human capital production. In the infinitely-lived representative framework, the teacher and student is the same person. So the human capital invested in schooling is the fraction of workers’ time spent on schooling multiplied by the total human capital. Define $N_h = L_h/L$. Equation (2.8) can be rewritten as:

$$H_{t+1} = (1 - \delta)H_t + E_t^\omega (H_t N_{h,t})^{1-\omega} \tag{2.9}$$

Here $N_h$ is essentially the workers’ time spent on human capital production. Based on (2.9), production in human capital can be thought of as a combination of two investments: physical investment multiplied by human capital, and workers’ forgone time: $H_{t+1} = (1 - \delta)H_t + (E_t^\omega H_t^{1-\omega})N_{h,t}^{1-\omega}$. The steady state of equation (2.9) yields:

$$H/Y = \frac{E/Y}{\psi + \delta + n} \left( \frac{N_h}{\psi + \delta + n} \right)^{(1-\omega)/\omega} \tag{2.10}$$

Different from equation (2.6), this equation indicates that both the time component and physical investment will affect the steady state of human capital intensity. Their relative importance is determined by the coefficient $\omega$. When $\omega = 1$, this equation will consolidate back to equation (2.6), the one used in MRW. If $\omega = 0$, $E$ in equation (2.9) is wiped out, leaving only the time component and human capital. This is essentially the discrete-time version of human capital production in Lucas [51]. In that case, steady state of human capital is undetermined. Thus equation (2.9) is a generalized version to describe human capital production process.

The goal is to see whether such a specification is reasonable in explaining cross-country income differences. So I use (2.10) in place of equation (2.6), and plug it into the final output, the equation (2.2). After rearrangement we have:

$$\log \frac{Y}{L}(t) = \log A_0 + \psi t + \frac{\alpha}{1 - \alpha - \beta} \log \frac{I_k}{Y} + \frac{1 - \omega}{\omega} \frac{\beta}{1 - \alpha - \beta} \log \frac{N_h}{\psi + \delta + n}$$

$$+ \frac{\beta}{1 - \alpha - \beta} \log \frac{E/Y}{\psi + \delta + n} \tag{2.11}$$

I continue to use Summers-Heston data (PWT 7.1) for the investment share,
real GDP. The working-age population is from *World Development Indicators*. As discussed in the previous section, the ratio of secondary and tertiary enrollment to the working-age population, from *UNESCO* database, is used as a proxy for \( N_h \). To distinguish it from the variable ‘SCHOOL’ in MRW, we denote it as ‘SCHOOL2’. Note that in MRW, ‘SCHOOL’ is used to approximate total human capital investment, while it should be a proxy for \( N_h \), the time component in our model. \( E/Y \) measures the percentage of output allocated to produce human capital. It includes all physical resources devoted to education, ideally both formal and informal. To the best of our knowledge, there is no such accurate measure in the available world data set. As an alternative, we use the public spending on education as a percentage of GDP as a proxy from the World Bank Database. Public spending takes a substantial portion of education expenditures in most of the countries, especially in OECD countries. Thus it is plausibly a good candidate for \( E/Y \). Yet if public spending on education as a % of GDP is proportional to \( E/Y \), then we can still use it to estimate (2.11); the factor of the proportionality will be in the constant term.

Therefore we can empirically estimate (2.11). The real GDP per working-age population in 2010 is used as the dependent variable. Physical investment share, the working-age population growth rates, SCHOOL2, and public spending as a percentage of GDP are the average over 1970-2010. The results are displayed in Table 2.2.

We find that the estimated result appears to be very encouraging. All the signs of the coefficients are consistent with the prediction. The public spending on education is significant in non-oil and OECD sample at 5% level in the unrestricted regression. In the restricted regression, it is significant in all samples at 5% level. \( R^2 \) is 80%, 78% and 78% in non-oil, intermediate and OECD samples. Notably, \( R^2 \) in OECD is now comparable with that in other samples, a sharp rise to 78% compared with only 24% in the original MRW. Note now the null hypothesis is that the sum of coefficients on \( \log I_k/Y, \log(\psi + \delta + n), \log\text{SCHOOL2} \) and \( \log E/Y \) is zero. The over-identifying restriction is rejected at 5% level in non-oil and intermediate samples, similar to the results in the previous section.

The values of implied \( \alpha \) are 0.36, 0.42 and 0.17 which are close to 0.31, 0.33 and 0.14 in MRW2 for non-oil, intermediate and OECD samples, respectively. However,
### Table 2.2: Estimation Of the Modified-MRW Model

Dependent Variable: Log Real GDP per working-age person in 2010

<table>
<thead>
<tr>
<th>Sample</th>
<th>Non-oil</th>
<th>Intermediate</th>
<th>OECD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observations</td>
<td>89</td>
<td>69</td>
<td>22</td>
</tr>
<tr>
<td>Constant</td>
<td>4.11**</td>
<td>4.34**</td>
<td>11.57***</td>
</tr>
<tr>
<td></td>
<td>(1.66)</td>
<td>(1.78)</td>
<td>(1.49)</td>
</tr>
<tr>
<td>\log I_k/Y</td>
<td>0.80***</td>
<td>0.95***</td>
<td>0.45**</td>
</tr>
<tr>
<td></td>
<td>(0.19)</td>
<td>(0.22)</td>
<td>(0.20)</td>
</tr>
<tr>
<td>\log(\psi + \delta + n)</td>
<td>-3.68***</td>
<td>-3.59***</td>
<td>-0.97*</td>
</tr>
<tr>
<td></td>
<td>(0.52)</td>
<td>(0.53)</td>
<td>(0.49)</td>
</tr>
<tr>
<td>\log SCHOOL2</td>
<td>0.97***</td>
<td>1.05***</td>
<td>0.78**</td>
</tr>
<tr>
<td></td>
<td>(0.11)</td>
<td>(0.15)</td>
<td>(0.32)</td>
</tr>
<tr>
<td>\log E/Y</td>
<td>0.38*</td>
<td>0.24</td>
<td>0.52***</td>
</tr>
<tr>
<td></td>
<td>(0.19)</td>
<td>(0.22)</td>
<td>(0.16)</td>
</tr>
<tr>
<td>(R^2)</td>
<td>0.80</td>
<td>0.78</td>
<td>0.78</td>
</tr>
</tbody>
</table>

#### Restricted Regression:

<table>
<thead>
<tr>
<th>Constant</th>
<th>8.14***</th>
<th>7.90***</th>
<th>9.51***</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(0.25)</td>
<td>(0.33)</td>
<td>(0.46)</td>
</tr>
<tr>
<td>\log(\frac{I_k}{Y})|_{\psi + \delta + n}</td>
<td>0.88***</td>
<td>1.00***</td>
<td>0.29</td>
</tr>
<tr>
<td></td>
<td>(0.19)</td>
<td>(0.22)</td>
<td>(0.17)</td>
</tr>
<tr>
<td>\log(\frac{SCHOOL2}{Y})|_{\psi + \delta + n}</td>
<td>1.06***</td>
<td>1.17***</td>
<td>0.83**</td>
</tr>
<tr>
<td></td>
<td>(0.11)</td>
<td>(0.15)</td>
<td>(0.33)</td>
</tr>
<tr>
<td>\log(\frac{E/Y}{Y})|_{\psi + \delta + n}</td>
<td>0.56***</td>
<td>0.43**</td>
<td>0.42**</td>
</tr>
<tr>
<td></td>
<td>(0.18)</td>
<td>(0.21)</td>
<td>(0.15)</td>
</tr>
<tr>
<td>(R^2)</td>
<td>0.79</td>
<td>0.77</td>
<td>0.75</td>
</tr>
</tbody>
</table>

#### Test of restriction:

| p-value | 0.02 | 0.05 | 0.17 |

#### Implied Coefficients:

<table>
<thead>
<tr>
<th>Implied (\alpha)</th>
<th>0.36</th>
<th>0.42</th>
<th>0.17</th>
</tr>
</thead>
<tbody>
<tr>
<td>Implied (\beta)</td>
<td>0.23</td>
<td>0.18</td>
<td>0.25</td>
</tr>
<tr>
<td>Implied (\omega)</td>
<td>0.35</td>
<td>0.27</td>
<td>0.34</td>
</tr>
</tbody>
</table>

\(^a\) The standard errors are in parentheses immediately below.

\(^b\) The investment, working-age population growth rates, public spending on education and SCHOOL2 are averaged over the observation period 1970-2010; \((\psi + \delta)\) is assumed to be 0.05.

\(^c\) *, **, *** indicate significance at 10%, 5% and 1% level, respectively.
the estimated $\beta$ is lower: 0.23, 0.18 and 0.25, respectively. It appears to be in the range of one-fifth and one-quarter, rather than around one-third in MRW estimation. We believe that public education spending is a better representative of the physical resources allocated to human capital production. Education enrollment, on the other hand, represents workers’ time invested in education. The refinement of variables changes the estimation of $\beta$, while the values of implied $\alpha$ are still consistent with the conventional view. The coefficient on human capital is around 0.18-0.25.

Another finding is that the estimated $\omega$ revolves around one-third in the three samples: 0.35, 0.27 and 0.34 in non-oil, intermediate and OECD, respectively. It indicates that physical investment accounts for the share of around one-third, and the combination of workers’ time and human capital takes the rest of the two-thirds in human capital production:

$$H_{t+1} = (1 - \delta)H_t + E_t^{1/3}H_t^{2/3}N_{h,t}^{2/3}$$

Is this estimation sensible? Kendrick [45] estimated that about 50% of investment in human capital in the United States represents the opportunity cost of student time, and the remaining 50% is composed of expenditures on teachers (human capital) and facilities (physical capital). Klenow and Rodriguez-Clare [46] found that expenditures on teachers represent about 80% of all expenditures, according to 1996 Digest of Education Statistics. Thus they estimated factor shares of 10%, 40% and 50% for physical resources, human capital and raw labor in the production of human capital. In a Cobb-Douglas function form, this implies:

$$H_{t+1} = (1 - \delta)H_t + E_t^{0.1}H_t^{0.4}N_{h,t}^{0.5}$$

Our model imposes the underlying assumption that the coefficients on human capital and students’ time are the same. The framework is different from the factor share estimation. To make it comparable, our model predicts that the factor shares of physical investment, schooling time, and human capital are 0.2, 0.4 and 0.4, respectively, after normalization through dividing their corresponding coefficient by the sum of the three coefficients; the factor share estimation gives shares of 0.1, 0.4
and 0.5, respectively. Note that it is actually a very rough comparison because the teachers’ wages include compensations for hours worked and human capital in factor share estimation. Also, in our model public spending on education includes physical investment and part of the teachers’ compensation. That probably explains why the estimated share of physical investment is larger in our estimation than that in Klenow and Rodriguez-Clare [46]. But in general our model estimates are consistent with evidence based on factor shares. So we conclude that physical investment on education has a significant and positive effect on per capita income in the long-run.

2.4 Capturing the Country-Effect

2.4.1 The Dynamic Panel Data Model

In the previous section, we explored extensions of MRW to include measurement of education expenditures in the single-cross country framework. One important underlying assumption in single-cross country regression is that the unobserved factor is uncorrelated with the explanatory variables and same across countries. This allows the OLS regression. As argued by MRW, this assumption is common and is also made in several other growth models. In models where saving and population growth are endogenous but preferences are isoelastic, the explanatory variables, the saving rate, population rate and human capital variables are independent of the unobserved Solow residual.

But this isoelastic preference imposes an additional restriction. As Klenow and Rodriguez-Clare [46] discussed, a country’s unobserved factor is likely to be correlated with a country’s human and physical capital level in general. The unobserved country effects include basically all the country’s attributes that capital and human capital fail to capture in explaining economic growth, for example, technology, culture, language, and possibly international spill-overs of technology and human capital. A change in one factor may impact the others. For instance, a policy that contributes to human/physical capital production, might also raise the technology adoption which is in the Solow residue. Suppose that the true unobserved factor, A is highly correlated with human capital, and A explains international per capita income variations
significantly, then it is still consistent with the view that human capital explains international income disparity, though not necessary proof.

Thus we attempt to deal with this problem by considering an alternative framework. As shown by Islam [39], the panel data framework makes it possible to allow for differences the unobserved individual country-effects. This approach assumes there exist time-invariant and country-specific technological differences across economies. In order to fit in the panel data framework, we divide the entire 40-year observation sample period into eight 5-year intervals. However, the short time intervals will invalidate the steady-state assumption because of business-cycle fluctuations.\(^4\) We need a framework that is able to accommodate states of economies which possibly deviate from the steady state. Denote \(x = \frac{X}{A}\) and use \(*\) subscript to indicate steady-state values. In the steady state the aggregate production function can be normalized as

\[
y^* = k^* h^{\gamma\beta} \tag{2.12}
\]

As shown in MRW, approximating (2.12) around the steady state yields

\[
\frac{d \log y(t)}{dt} \approx \lambda [\log y^* - \log y(t)] \tag{2.13}
\]

where

\[
\lambda = (\psi + \delta + n)(1 - \alpha - \beta)
\]

Equation (2.13) implies that given \(\tau = t_1 - t_0\),

\[
\log y(t_1) = (1 - e^{-\lambda\tau}) \log y^* + e^{-\lambda\tau} \log y(t_0). \tag{2.14}
\]

Therefore, from (2.14) the logarithm of income at time \(t_1\) is a weighted average of the initial level at \(t_0\) and the steady state. This suggests a dynamic framework that applies for the economy at and around the steady state. Given that \(\log y_t = \log Y/L(t) - \log A_t = \log Y/L(t) - \log A_0 - \psi t\), (2.14) can be written as

\[
\log \frac{Y}{L}(t_1) = e^{-\lambda\tau} \psi t + (1 - e^{-\lambda\tau}) \log \frac{Y^*}{L^*}(t_1) + e^{-\lambda\tau} \log \frac{Y}{L}(t_0) \tag{2.15}
\]

\(^4\)In fact, I tried the panel data regression in four 10-year periods and found the regression performs poorly, which implies the steady-state assumption remains in question at 10-year interval.
(2.15) represents a partial adjustment that the ‘target’ value of the dependent variable is determined by the initial position and the explanatory variables in the steady state. It is the equation that MRW used to study the process of convergence.

Next, we drop * subscript for the steady-state (more accurately, balanced-growth) variables. Substituting $\log Y^*/L^*$ with equation (2.7) yields the dynamic panel MRW model:

$$
\log \frac{Y}{L}(t_1) = e^{-\lambda r} \log \frac{Y}{L}(t_0) + (1 - e^{-\lambda r}) \log A_0 + \psi(t_1 - e^{-\lambda r} t_0) \\
+ (1 - e^{-\lambda r}) \frac{\alpha}{1 - \alpha - \beta} \log \frac{I_k/Y}{\psi + \delta + n} + (1 - e^{-\lambda r}) \frac{\beta}{1 - \alpha - \beta} \log \frac{I_h/Y}{\psi + \delta + n}
$$

(2.16)

In MRW, $t_0$ is 1960 and $t_1$ is 1985, so $r$ is a 25-year interval, and the estimation remains a single cross-country OLS regression. Similar to Islam (1995), we allow $\log A_0$ to be country-specific and time-invariant. We assume $r$ is a 5-year interval over 1970-2010, and $\log(Y/L)(t)$ is the per capita income at the end of each interval. Since $\log(Y/L)(t-1)$ is on the right hand-side, the model has a dynamic panel structure. So we call it dynamic panel MRW model to distinguish it from the single cross-country MRW regression.

Substituting $\log(Y^*/L^*)$ with equation (2.11) yields the modified dynamic panel MRW model with education expenditures:

$$
\log \frac{Y}{L}(t_1) = e^{-\lambda r} \log \frac{Y}{L}(t_0) + (1 - e^{-\lambda r}) \log A_0 + \psi(t_1 - e^{-\lambda r} t_0) \\
+ (1 - e^{-\lambda r}) \frac{\alpha}{1 - \alpha - \beta} \log \frac{I_k/Y}{\psi + \delta + n} + (1 - e^{-\lambda r}) \frac{\beta}{1 - \alpha - \beta} \log \frac{N_h}{\psi + \delta + n} \\
+ (1 - e^{-\lambda r}) \frac{\beta}{1 - \alpha - \beta} \log \frac{E/Y}{\psi + \delta + n}
$$

(2.17)

Therefore, (2.16) and (2.17) each represents a dynamic panel data model framework, where $\log A_0$ is the time-invariant country effect. To see this more clearly we use the following notations to rewrite (2.16) and (2.17):

$$
y_{i,t} = \rho y_{i,t-1} + x_{i,t}^\prime \beta + \mu_t + \eta_t + \epsilon_{i,t}
$$

(2.18)
where

\[ y_{i,t} = \log \frac{Y}{L}(t_1) \]

\[ y_{i,t-1} = \log \frac{Y}{L}(t_0) \]

\[ x_{i,t} = \left[ \log \frac{I_k/Y}{\psi + \delta + n}, \log \frac{I_h/Y}{\psi + \delta + n} \right]' \]

or

\[ x_{i,t} = \left[ \log \frac{I_k/Y}{\psi + \delta + n}, \log \frac{N_h}{\psi + \delta + n}, \log \frac{E/Y}{\psi + \delta + n} \right]' \]

\[ \mu_i = (1 - e^{-\lambda \tau}) \log A_0 \]

\[ \eta_t = \psi(t_1 - e^{-\lambda \tau} t_0) \]

\[ \epsilon_{i,t} \sim i.i.d(0, \sigma_\epsilon) \]

Here the idiosyncratic shock, \( \epsilon_{i,t} \) is Gaussian white noise. \( \mu_i \) captures a country’s time-invariant attributes, including the culture, endowments and other related factors. This model is based on approximation around the steady state, so it is supposed to capture the dynamics towards the steady state. The explanatory variables, \( x_{i,t} \) have two alternative sets, from dynamic panel MRW model as in (2.16) or its modified version in (2.17) with education expenditures included.

Islam (1995) adopts the Least Squares Dummy Variable (LSDV) framework to estimate the dynamic model. However, the LSDV estimator, which is based on the fixed-effect assumption in panel data model, might cause inconsistency arising from the dynamic character of the model. The lagged dependent variable on the right hand-side gives rise to ‘dynamic panel bias’, since it is correlated with \( \mu_i \). But within-groups transformation does not eliminate dynamic panel bias: although each observation is subtracted by the average over \( T \) for each individual to eliminate the fixed effect, the demeaned lagged dependent variable is still correlated with the demeaned error term. Therefore, LSDV estimator is biased especially when \( T \) is small. The coefficient on the lagged dependent variable would be biased downward due to its negative correlation with the ‘within transformed’ error term. As \( T \) rises, the bias dwindles. But according to simulation studies by Judson and Owen [43], a bias equal to 20% of the coefficient is found even when \( T=30 \). So the bias could be large for small \( T \). This issue severely
weakens LSDV estimator as a qualified estimator.

2.4.2 GMM Estimation

We turn to an alternative approach for dynamic panel data estimation. Arellano and Bond [3] proposed using GMM to address this issue. The levels of the lagged dependent variables are used to instrument the first-difference equation. It is a suitable solution for our panel model with dynamic process with lagged dependent variables influencing the current one. In addition, it is robust to possibilities of idiosyncratic disturbances that may have patterns of heteroskedasticity but uncorrelated across individuals. Also, it is able to eliminate the potential inconsistency arising from predetermined explanatory variables, i.e. they may be correlated with disturbances from previous periods and are thus not strictly exogenous. Moreover, the Arellano-Bond estimator is particularly designed for a small number of time periods of available data (‘small T, large N’): our sample has T=8 periods and N ranging from 22 to 89 individual countries. Given reasons above we consider GMM as an ideal methodology for our model.

First Arellano-Bond estimation requires first-difference transformation. So (2.18) is transformed from level equation to first-difference equation:

\[ y_{i,t} - y_{i,t-1} = \rho(y_{i,t-1} - y_{i,t-2}) + (x'_{i,t} - x'_{i,t-1})\beta + (\eta_t - \eta_{t-1}) + (\epsilon_{i,t} - \epsilon_{i,t-1}) \]  

(2.19)

Individual effects are eliminated through first difference transformation. But endogeneity is still present because of the correlation between \( y_{i,t-1} \) and \( \epsilon_{i,t-1} \). Arellano and Bond [3] instrument this transformed equation with \( y_{i,t-2} \) and its longer lags, which are uncorrelated with the error term but is necessarily correlated with \( y_{i,t-1} - y_{i,t-2} \). The instruments are drawn internally within the data set, and are considered as GMM-style instruments. When some of the explanatory variables are predetermined, i.e. they are correlated with shocks from previous periods, endogeneity will also be incurred due to non-zero correlation between \( x_{i,t} \) and \( \epsilon_{i,t-1} \). Similar to the treatment to \( y_{i,t-1} \), these variables should be instrumented by their lagged values (\( x_{i,t-1} \) and its longer lags). The strictly exogenous regressors instrument themselves as the ordinary IV style.
In implementation, an efficient and robust GMM estimator is obtained through a two-step estimation: perform an initial GMM regression based on a minimally arbitrary assumption about errors, like homoskedasticity; obtain the residuals to construct the weighting matrix; use the weighting matrix to return the second-step GMM estimation. Theoretically the two-step GMM estimator is robust to heteroskedasticity, while one-step GMM requires homoskedasticity in the errors. But historically, researchers often find finite-sample downward bias in the computed standard errors in two-step results. Windmeijer [74] proposed a small-sample correction for the two-step standard errors so this problem is greatly reduced. When performing Arellano-Bond regressions on simulated panels, he found that two-step GMM outperforms one-step GMM with lower bias and standard errors. In particular, the estimated standard errors with his correction is quite accurate. So two-step first-difference GMM with corrected errors seems superior to one-step estimation.

However, one disadvantage of the Arellano-Bond estimation is that first-difference transformation magnifies gaps in unbalanced panel. A missing value in $y_t$ results in missing values of both $\Delta y_t$ and $\Delta y_{t+1}$. This motivates Arellano and Bover [4] to propose an alternative transformation instead of first-difference, sometimes called orthogonal deviations transformation. The difference equation is obtained by subtracting the average of all future available observations of a variable, rather than the one-period lagged observation, from the contemporaneous observation of the variable. Numerically, the estimated results are identical when the panel is balanced. But they are different in unbalanced panel since orthogonal deviations transformation reduces information loss due to missing data.

In order to minimize data loss and maximize estimation efficiency, we employ the two-step system GMM estimation with orthogonal-deviation transformation and finite-sample corrected errors. We consider all explanatory variables are potentially predetermined except the population growth rate. To eliminate the possible inconsistency we treat all the lagged explanatory variables in either restricted or unrestricted regression as GMM-style instruments except $\log(\psi + \delta + n)$.

For comparison, LSDV is also implemented to estimate the panel. We choose 5 years as the time span for each of the eight periods over 1970-2010. According
to Islam (1995), the yearly time spans would be too short to be appropriate since short-term disturbances may loom large in such brief time spans. $y_{i,t}$ is the log real GDP per working-age person at the end of the interval (for example, 1980 for the interval 1975-1980, or 1985 for interval 1981-1985). $x_{i,t}$ is the average over the 5-year interval. Thus the errors are for 5-year spans and are thought to be less impacted by business-cycle fluctuations. We first estimate the dynamic panel MRW model (equation 2.16). The results are summarized in Table 2.3.

We find strikingly excellent results. The two estimations produce consistent outcomes. Almost all the estimate coefficients are significant with correct sign. The Wald test is in favor of not rejecting the imposed linear restriction in any sample. The Hansen test of overidentifying restrictions is to test the exogeneity of instruments as a group, which is a crucial assumption for the validity of GMM. The null hypothesis that the instruments are exogenous for all of the samples are not rejected in all samples. Almost all of the per capita income variations are explained in the model with extremely high $R^2$’s. The GMM estimation produces higher implied coefficients but lower convergence rates (except for the non-oil sample) than LSDV does. The discrepancies are not large but are quite consistent across the three samples.

Notably, education investment, or ‘SCHOOL2’ is positive and significant in all samples. To our knowledge, this is the first to identify significance of education investment in a panel data model. The existing literature predominantly report an ambiguous role of human capital investment. Our results tend to reject the prevalent view and are more in line with MRW’s conclusion. GMM coefficients on human capital are 0.16, 0.13, and 0.23, respectively. They are smaller than the conjecture of one-third in single cross-country estimation (in MRW2), only around half of the size. The implied coefficients on capital are 0.39, 0.43, 0.37, respectively, which are larger than those in MRW2 but still appear consistent with the conventional value of one-third. Especially, an $\alpha$ of 0.37 in OECD sample is much more reasonable compared with a mere 0.17 in single cross-country. The estimated yearly convergence rates range from 3% to 4%, which are in line with the consensus of 2%-3% per year in the literature.\(^5\) Therefore introducing individual effect lowers the estimated values of

\(^5\)For example, Barro [9], and Sala-i Martin [67].
### Table 2.3: Estimation of Dynamic Panel MRW Model

Dependent Variable: Log Real GDP per working-age person at the end of each 5-year interval, log Y/L(t)

<table>
<thead>
<tr>
<th>Sample</th>
<th>LSDV</th>
<th>GMM</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Non-oil</td>
<td>Intermediate</td>
</tr>
<tr>
<td>Countries:</td>
<td>89</td>
<td>69</td>
</tr>
<tr>
<td>Observations:</td>
<td>614</td>
<td>482</td>
</tr>
<tr>
<td>log Y/L(t − 1)</td>
<td>0.80***</td>
<td>0.79***</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td>(0.02)</td>
</tr>
<tr>
<td>log I_k/Y</td>
<td>0.16***</td>
<td>0.16***</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td>(0.03)</td>
</tr>
<tr>
<td>log(ψ + δ + n)</td>
<td>-0.13**</td>
<td>-0.23***</td>
</tr>
<tr>
<td></td>
<td>(0.06)</td>
<td>(0.07)</td>
</tr>
<tr>
<td>log SCHOOL2</td>
<td>0.05**</td>
<td>0.03*</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.02)</td>
</tr>
</tbody>
</table>

R^2

Hansen Test of Over-id.

| p-value | 0.30 | 0.85 | 0.99 |

Restricted Regression:

log Y/L(t − 1) | 0.79*** | 0.80*** | 0.84*** | 0.81*** | 0.79*** | 0.84*** |
|              | (0.02)  | (0.02)  | (0.03)  | (0.03)  | (0.04)  | (0.02)  |

log (I_k/Y) | 0.15*** | 0.17*** | 0.13*** | 0.16*** | 0.20*** | 0.15*** |
|            | (0.02)  | (0.02)  | (0.04)  | (0.03)  | (0.05)  | (0.05)  |

log (SCHOOL2) | 0.05** | 0.04** | 0.09** | 0.06** | 0.06** | 0.09*  |
|              | (0.01)  | (0.02)  | (0.04)  | (0.02)  | (0.03)  | (0.05)  |

R^2

Hansen Test of Over-id.

| p-value | 0.41 | 0.86 | 0.99 |

Wald Test for Restriction:

| p-value | 0.19 | 0.65 | 0.47 | 0.42 | 0.99 | 0.91 |

Implied Coefficients:

| Implied λ | 0.0470 | 0.0456 | 0.0351 | 0.0417 | 0.0475 | 0.0341 |
| Implied α | 0.36   | 0.41   | 0.33   | 0.39   | 0.43   | 0.37   |
| Implied β | 0.12   | 0.09   | 0.24   | 0.16   | 0.13   | 0.23   |

*Estimation of MRW model in panel data framework over 1970-2010 (equation 16), with eight 5-year intervals. LSDV: Least Squares Dummy Variable Regression; GMM: Two-step difference GMM with orthogonal deviations transformation and corrected errors for finite sample.

The panel is unbalanced due to missing values. In a balanced panel the observations should be 89×8, 69×8 and 22×8, i.e. 712, 552 and 176 for non-oil, intermediate and OECD samples, respectively. GMM estimation requires forward orthogonal deviations transformation so the last period is wiped out. The balanced panel would have 623, 483 and 154 observations, respectively.

log Y/L(t) and log Y/L(t − 1) are the real log GDP per working-age person at the end of current and previous periods, respectively. The investment, working-age population growth rates and SCHOOL2 are averaged over the corresponding observation period; (ψ + δ) is assumed to be 0.05.

The standard errors are in parentheses immediately below.

* , ** , *** indicate significance at 10%, 5% and 1% level, respectively.
the elasticity of output with respect to human capital, but the coefficients on capital are still in conformity with commonly accepted values.

Next we consider the effect of adding education expenditures into the dynamic panel. The modified dynamic panel MRW model is based on equation (2.17). The average of public spending on education over each 5-year interval is used as a proxy for education expenditures on the right hand-side. One additional variable also brings a larger missing-data issue. The sample size reduces to 84 and 66 for non-oil and intermediate samples in GMM. Size of the OECD sample remains unchanged. The results are shown in Table 2.4 below.

The result in Table 2.4 show that the model still explains most of the per capita income variations. Inclusion of public spending on education, however, produces a somewhat different result from the previous one. The coefficient on education expenditures is negative and insignificant. The negative values deteriorate the calculation for implied values of \( \alpha, \beta \) and \( \omega \). Values of \( \alpha \) are much higher, out of the reasonable range; \( \beta \) and \( \omega \) are even negative. The anomalous results seem to deny the contribution of education expenditure in explaining economic growth dynamics.

One plausible explanation for the ‘anomaly’ is that education expenditures take a very long time to become effective to economic growth. In the regression, the proxy for education investment is public spending on education, which is mostly spent on primary and secondary education system. It might take a decade for the investment in education system to take effect when these students graduate and join the labor force. The contemporaneous elasticity of output with respect to education investment is likely to be negative because it requires resources that could be used to produce final output, and human capital it produces will not be productive until years later. This is probably why a negative coefficient on public spending on education has been observed in the regression.

To test this explanation, we use the public spending on education from one decade ago, or 2-period lagged as a regressor instead in non-oil and intermediate samples. 1-period lagged public spending on education is used in the OECD sample since we believe OECD economies are closer to the steady state. Also, data loss
## Table 2.4:
### Estimation of Dynamic Panel Modified-MRW Model
Dependent Variable: Log Real GDP per working-age person at the end of each 5-year interval, log \( Y/L(t) \)

<table>
<thead>
<tr>
<th>Sample</th>
<th>LSDV</th>
<th>GMM</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Non-oil</td>
<td>Intermediate</td>
</tr>
<tr>
<td>Countries:</td>
<td>87</td>
<td>67</td>
</tr>
<tr>
<td>Observations:</td>
<td>489</td>
<td>401</td>
</tr>
<tr>
<td>log ( Y/L(t-1) )</td>
<td>0.82***</td>
<td>0.81***</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td>(0.02)</td>
</tr>
<tr>
<td>log ( I_k/Y )</td>
<td>0.17***</td>
<td>0.18***</td>
</tr>
<tr>
<td></td>
<td>(0.03)</td>
<td>(0.03)</td>
</tr>
<tr>
<td>log(( \psi + \delta + n ))</td>
<td>-0.16**</td>
<td>-0.23**</td>
</tr>
<tr>
<td></td>
<td>(0.07)</td>
<td>(0.08)</td>
</tr>
<tr>
<td>log ( SCHOOL2 )</td>
<td>0.06***</td>
<td>0.03</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td>(0.02)</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.99</td>
<td>0.99</td>
</tr>
<tr>
<td>Hansen Test of Over-id. Restriction:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>p-value</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

### Restricted Regression:

| log \( Y/L(t-1) \) | 0.81*** | 0.81***     | 0.84***     | 0.83*** | 0.81***     | 0.84***     |
| log(\( k/Y \))     | 0.17*** | 0.18***     | 0.11**      | 0.20*** | 0.25***     | 0.14*       |
|              | (0.02)    | (0.02)      | (0.03)      | (0.03)    | (0.04)      | (0.08)      |
| log(\( SCHOOL2 \)) | 0.06*** | 0.03        | 0.12***     | 0.06**   | 0.04        | 0.11*       |
|              | (0.02)    | (0.02)      | (0.05)      | (0.03)    | (0.04)      | (0.06)      |
| \( R^2 \)     | 0.99      | 0.99        | 0.97        | 0.99      | 0.99        | 0.96        |
| Hansen Test of Over-id. Restriction: |  |
| p-value       | -         | -           | -           | 0.98      | 0.99        | 0.99        |

### Wald Test for restriction:

| p-value       | 0.71      | 0.63        | 0.34        | 0.16      | 0.52        | 0.87        |
| Implied Coefficients: |  |
| Implied \( \lambda \) | 0.0410 | 0.0420      | 0.0347      | 0.0384    | 0.0410      | 0.0243      |
| Implied \( \alpha \)  | 0.53     | 0.53        | 0.49        | 0.57      | 0.58        | 0.55        |
| Implied \( \beta \)   | -0.12    | -0.07       | -0.21       | -0.06     | -0.09       | -0.17       |
| Implied \( \omega \)  | -2.01    | -3.06       | -0.69       | -0.54     | -0.05       | -0.63       |

---

a Estimation of Modified-MRW model in panel data framework over 1970-2010 (Equation 21), with eight 5-year intervals.
LSDV: Least Squares Dummy Variable Regression; GMM: Two-step first-difference GMM with orthogonal deviations transformation and corrected errors for finite sample.

b log \( Y/L(t) \) and log \( Y/L(t-1) \) are the real log GDP per working-age person at the end of current and previous periods, respectively. The investment, working-age population growth rates and \( SCHOOL2 \) are averaged over the corresponding observation period; \( (\psi + \delta) \) is assumed to be 0.05.

c The standard errors are in parentheses immediately below.

d **, *** indicate significance at 10%, 5% and 1% level, respectively.
can be reduced. Other regressors are the same as in modified dynamic panel MRW. Inclusion of 2-period lags wipes out 2 periods of observations, so the GMM regression covers 5 periods (6 for OECD countries). Number of countries in the samples further reduces to 72, 59, and 21 in non-oil, intermediate and OECD samples, respectively. Table 5 reports the estimation results.

Very encouraging results can be found as shown in Table 2.5. The model still fits well with high $R^2$. Estimated coefficients are all significant with correct sign in non-oil and intermediate samples. Estimation in OECD sample, however, seems to suffer from data loss with high standard errors. But the sign of lagged public spending on education turns positive. The implied coefficients of $\alpha$ and $\beta$ are reasonable and consistent with estimation in dynamic panel MRW model.

In particular, lagged public spending on education is very significantly positive in non-oil and intermediate samples. It confirms our conjecture that physical education investment is very slow to become effective and productive to the economy. It has strong policy implications: education expenditures promote economic growth. When government spends more on education, income will ultimately grows sufficiently to recover the investment, although around a decade is needed. The views of empirical papers that study the relationship between public education expenditures and economic growth are mixed. This paper provides strong empirical evidence to support a positive impact of education expenditures on growth.

For summary, we list several of comparable estimated results in relevant literature and this paper in Table 2.6. MRW estimates convergence rates in single cross-country OLS regression over 1960-1985. The education investment proxy is ‘SCHOOL’ which was discussed previously. Islam [39] estimates over 1960-1985 in a fixed-effect panel model using LSDV method. Years of schooling is directly used as a proxy as human capital level rather than as education investment. Caselli et al. [19] (CEL) apply first-difference GMM to the panel model over 1960-1985, and use secondary enrollment rates as education investment proxy. The rest of the columns are results from this paper. ‘OLS’ is a simple pooled regression of dynamic panel MRW model over 1970-2010. ‘LSDV’ and ‘GMM(1)’ are extracted from the left and right panel of Table 2.3; ‘GMM(2)’ is from the right panel of Table 2.4 for modified
Table 2.5:  
GMM Estimation of Dynamic Panel Modified-MRW Model  
With Lagged Public Spending  

Dependent Variable: Log Real GDP per working-age person at the end of each 5-year interval, log \( Y/L(t) \)

<table>
<thead>
<tr>
<th>Sample</th>
<th>Non-oil</th>
<th>Intermediate</th>
<th>OECD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Countries:</td>
<td>72</td>
<td>59</td>
<td>21</td>
</tr>
<tr>
<td>Observations:</td>
<td>257</td>
<td>223</td>
<td>113</td>
</tr>
<tr>
<td>log ( Y/L(t-1) )</td>
<td>0.70***</td>
<td>0.75***</td>
<td>0.79***</td>
</tr>
<tr>
<td>(0.04)</td>
<td>(0.05)</td>
<td>(0.06)</td>
<td></td>
</tr>
<tr>
<td>log ( I_k/Y )</td>
<td>0.14**</td>
<td>0.20***</td>
<td>0.19*</td>
</tr>
<tr>
<td>(0.07)</td>
<td>(0.06)</td>
<td>(0.10)</td>
<td></td>
</tr>
<tr>
<td>log ( \psi + \delta + n )</td>
<td>-0.20**</td>
<td>-0.23**</td>
<td>-0.19</td>
</tr>
<tr>
<td>(0.08)</td>
<td>(0.11)</td>
<td>(0.25)</td>
<td></td>
</tr>
<tr>
<td>log ( SCHOOL2 )</td>
<td>0.15***</td>
<td>0.12***</td>
<td>0.09</td>
</tr>
<tr>
<td>(0.05)</td>
<td>(0.04)</td>
<td>(0.08)</td>
<td></td>
</tr>
<tr>
<td>Lagged log ( E/Y )</td>
<td>0.13***</td>
<td>0.14***</td>
<td>0.05</td>
</tr>
<tr>
<td>(0.04)</td>
<td>(0.03)</td>
<td>(0.05)</td>
<td></td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.99</td>
<td>0.98</td>
<td>0.95</td>
</tr>
</tbody>
</table>

Hansen Test of Over-id. Restriction:

| p-value | 0.88 | 0.99 | 0.99 |

Restricted Regression:

| log \( Y/L(t-1) \) | 0.76*** | 0.73*** | 0.80*** |
| (0.04) | (0.05) | (0.05) |
| log \( \frac{I_k}{Y}(\psi+\delta+n) \) | 0.15**  | 0.23*** | 0.18*  |
| (0.07) | (0.06) | (0.09) |
| log \( \frac{SCHOOL2}{\psi+\delta+n} \) | 0.16*** | 0.14*** | 0.03   |
| (0.04) | (0.04) | (0.06) |
| Lagged log \( \frac{E}{Y}(\psi+\delta+n) \) | 0.11*** | 0.14*** | 0.06   |
| (0.04) | (0.03) | (0.04) |
| \( R^2 \) | 0.99    | 0.98         | 0.95   |

Hansen Test of Over-id. Restriction:

| p-value | 0.90 | 0.99 | 0.99 |

Test of restriction:

| p-value | 0.05 | 0.11 | 0.57 |

Implied Coefficients:

| Implied \( \lambda \) | 0.0548 | 0.0639 | 0.0454 |
| Implied \( \alpha \) | 0.30   | 0.36   | 0.40   |
| Implied \( \beta \) | 0.22   | 0.22   | 0.14   |
| Implied \( \omega \) | 0.42   | 0.50   | 0.66   |

- Two-period lagged public spending as a percentage of GDP is used for regression in non-oil and intermediate samples. In OECD sample, one-period lag is used instead.
- Due to missing data the panel is unbalanced; a balanced panel would have 72×5=360, 59×5=295 and 21×6=126 observations for non-oil, intermediate and OECD samples, respectively.
- The investment, working-age population growth rates, public spending on education and SCHOOL2 are averaged over the observation period 1970-2010; \( (\psi + \delta) \) is assumed to be 0.05.
- The standard errors are in parentheses immediately below.
- *, **, *** indicate significance at 10%, 5% and 1% level, respectively.
dynamic panel MRW model. Finally, ‘GMM(3)’ is from Table 2.5.

We find that the convergence rates in this paper are more in line with estimation in Islam (1995). Convergence rate in ‘CEL’ is the highest. In fact, a high convergence rate (as high as 10%) in that paper is cited as key evidence against exogenous growth theory. However, our results are mostly in the range of 3%-4%, much lower than their estimates. The estimated convergence is much lower in MRW and ‘OLS’, around 1%. This is largely due to bias of OLS regression, arising from the strong correlation between the lagged dependent variable and the observed individual effect. Thus it indicates such a downward bias in λ is huge! Individual effects are strong and will have to be well controlled for in a dynamic panel.

The implied shares of capital are unreasonably high in the previous literature, close to 50%. On the contrary, our model predicts are much closer to the widely accepted value. Moreover, implied β’s from ‘GMM(1)’ and ‘GMM(3)’ predict a share of human capital at around 0.2, which is consistent with our estimates in single cross-country steady state estimates in modified MRW model. In either Islam or CEL, the estimates are negative. Thus our predicted implied shares are desirably very stable across several scenarios, with α around one-third, and β around one-fifth.


\[ \text{Table 2.6: Comparison of Augmented Solow Model Estimation} \]

<table>
<thead>
<tr>
<th>Model</th>
<th>MRW</th>
<th>Islam</th>
<th>CEL</th>
<th>OLS</th>
<th>LSDV</th>
<th>GMM(1)</th>
<th>GMM(2)</th>
<th>GMM(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Countries:</td>
<td>98</td>
<td>79</td>
<td>97</td>
<td>89</td>
<td>89</td>
<td>89</td>
<td>84</td>
<td>72</td>
</tr>
<tr>
<td>Observations:</td>
<td>98</td>
<td>395</td>
<td>377</td>
<td>614</td>
<td>614</td>
<td>525</td>
<td>402</td>
<td>257</td>
</tr>
<tr>
<td>Implied λ</td>
<td>0.0142</td>
<td>0.0375</td>
<td>0.0679</td>
<td>0.0065</td>
<td>0.0470</td>
<td>0.0417</td>
<td>0.0384</td>
<td>0.0548</td>
</tr>
<tr>
<td>Implied α</td>
<td>0.48</td>
<td>0.52</td>
<td>0.49</td>
<td>0.58</td>
<td>0.36</td>
<td>0.39</td>
<td>0.57</td>
<td>0.30</td>
</tr>
<tr>
<td>Implied β</td>
<td>0.23</td>
<td>-0.20</td>
<td>-0.26</td>
<td>0.21</td>
<td>0.12</td>
<td>0.16</td>
<td>-0.06</td>
<td>0.22</td>
</tr>
</tbody>
</table>

*MRW, Islam and CEL report the results in Mankiw, Romer and Weil (1992, Table VI), Islam (1995, Table V) and Caselli, Esquivel and Lefort (1996, Table 3), respectively. The column OLS performs a pooled OLS regression of equation (2.16) at 5-year interval from 1970-2010. LSDV reports estimation of Panel MRW model from equation (2.16) in LSDV approach; GMM(1) is the estimation of (2.16) in two-step difference GMM approach. GMM(2) applies the same GMM method on the modified Panel MRW model from (2.17); GMM(3) is the same as GMM(2) except that it uses the lagged public spending as a regressor.
2.4.3 Discussion Of Estimated Country-Effects

Panel estimation not only allows for individual effects to provide more accurate estimates of parameters, but also enables us to obtain estimates of the unobserved individual effects. In GMM implementation, parameters are estimated from the difference equation where the individual effects have been eliminated. The underlying individual effects can be restored in the following steps. First, define variables that average over individuals as \( \bar{x}_i = \frac{1}{N} \sum x_{i,t} \). Subtracting them from both sides of equation (2.18) yields:

\[
y_{i,t} - \bar{y}_t = \rho(y_{i,t-1} - \bar{y}_{t-1}) + (x_{i,t} - \bar{x}_t)\beta + (\mu_i - \bar{\mu}) + (\epsilon_{i,t} - \bar{\epsilon}_t)
\]

(2.20)

The time effect, \( \eta_t \) is eliminated since \( \eta_t = \frac{1}{N} \sum \eta_t = \bar{\eta}_t \). Define \( \Delta \bar{x}_{i,t} = x_i - \bar{x}_i \). Next, the estimated \( \Delta \bar{\mu}_i \) are obtained as:

\[
\Delta \hat{\mu}_i = \Delta \hat{y}_{i,t} - \hat{\rho} \Delta \hat{y}_{i,t-1} - \Delta \hat{\bar{x}}_i \hat{\beta},
\]

(2.21)

where \( \Delta \hat{y}_{i,t} = \sum_{t=1}^{T_i} \Delta \hat{y}_{i,t}, \Delta \hat{y}_{i,t-1} = \sum_{t=0}^{T_i-1} \Delta \hat{y}_{i,t-1}, \) and \( \Delta \hat{\bar{x}}_i = \sum_{t=1}^{T_i} \Delta \hat{x}_{i,t} \). \( T_i \) is the number of observations for each individual, possibly different across countries because of missing data in some period. \( \Delta \bar{\mu}_i \) indicates the strength of the individual effect relative the international average. Finally, define the average individual unobservable effect as \( \bar{A}_0 = \left( \prod_{i=1}^{N} A_{0,i} \right)^{1/N} \). Then the estimated relative individual effect is:

\[
\log \hat{A}_{0,i} / \bar{A}_0 = \Delta \hat{\mu}_i / (1 - e^{-\hat{\lambda}T})
\]

(2.22)

The estimator in (22) describes a country’s relative strength of unobserved factor that contributes to economic growth, which includes, but not limited to aggregate productivity, technology adoption efficiency, endowment, culture, language, political system, etc. As Islam [39] mentioned, it is very close to the conventional concept of total factor productivity (TFP). A positive value indicates a strong individual effect relative to the average level; on the contrary, a value smaller than 0 implies the underlying individual factor is weak.

We estimate \( \Delta \mu_i \)'s and \( A_{0,i}/\bar{A}_0 \) from dynamic panel MRW model with the non-
oil sample. The estimated values are listed in Appendix A.1. In order to illustrate
the distributions of individual effects, we categorize countries into six groups: 1.
very low \((A_{0,i}/\bar{A}_0 < 0.5)\); 2. below average \((0.5 < A_{0,i}/\bar{A}_0 < 1)\); 3. above average
\((1 < A_{0,i}/\bar{A}_0 < 2)\); 4. high \((2 < A_{0,i}/\bar{A}_0 < 3)\); 5. very high \((3 < A_{0,i}/\bar{A}_0 < 4)\); 6 super
high \((A_{0,i}/\bar{A}_0 > 4)\). Appendix A.2 displays the histogram. We find the individual
effects are slightly skewed toward the bottom end. Around 52% of the countries are
below average; 30% of countries fall into the lowest group, which are all less than
half of the international average. On the other side, Singapore, United States and
Norway rank the top 3 on the list. In particular, Singapore has the highest value, 5.34
times of the cross-country average, which leads with a large margin compared with
the next country on the list (United States). Some countries seem remarkably high
on the list, for example, Hong Kong, Israel, which are higher than some traditional
developed countries, including Germany, Japan, and Spain. Islam [39] estimates \(A_0\)
as the ratio over the internationally lowest \(A_0\) from the textbook Solow model using
LSDV regression. In general, the relative rankings are very similar: the top three
countries are Hong Kong, Canada and USA in his results, with Norway and Singapore
at the fifth and sixth. But Islam’s results seem much more skewed to the bottom
end. 66% in his sample fall into the last two groups. Such difference illustrates the
effects in controlling for human capital investment in economic growth dynamics. The
unobserved effects are essentially the leftover after controlling for a number of factors
that contribute to a country’s per capita income growth. From our model’s point of
view, the estimated country effects in Islam [39] are, in some sense, a combination
of country-specific human capital investment and the true unobserved factor. That
probably explains why our estimated individual effects appear less widespread across
countries than in Islam [39].

\footnote{In fact, two models are available for estimation: the dynamic panel MRW and its modified model. The modified model accounts for the effect of (lagged) public spending on education. However, the accuracy of estimation may be influenced by reduction in number of observations when 2-period lags are used instead as a regressor. The 2-period lags wipe out observations in the first and second periods, and difference GMM with orthogonal deviations transformation truncates the last period. So the number of observations for each individual country reduces to at most 5 in the difference equation. As a result, the gaps arising from missing data are magnified. As in Table 2.5, 72 countries remain in the sample. Therefore, we do not show estimated \(\mu_i\)’s from the modified model in this paper. In fact, we find that the estimated individual effects exhibit comparable patterns: the correlation between the two versions of estimated \(\log A_{0,i}/\bar{A}_0\) is as high as 0.948.}
Figure 2.1: Scatter Plot of $A_{0,i}/\bar{A}_0$ and $\log Y/L$ in 2010

Table 2.7:
Correlations Between $\log(A_{0}/\bar{A}_0)$ and Other Variables:

<table>
<thead>
<tr>
<th></th>
<th>log $Y/L_{2010}$</th>
<th>log SCHOOL2</th>
<th>log $I_h/Y$</th>
<th>log $I_k/Y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\log A_{0}/\bar{A}_0$</td>
<td>0.8287</td>
<td>0.5850</td>
<td>0.3004</td>
<td>0.2006</td>
</tr>
</tbody>
</table>

$^a$ $\log A_{0}/\bar{A}_0$ are the estimated individual effects relative to the cross-country average from GMM estimations of the dynamic panel augmented Solow model (equation 16).

Figure 2.1 illustrates the relationship between $\log A_{0,i}/\bar{A}_0$ and the real log GDP per working-age person in 2010. A strong and positive relationship is found: the simple correlation between $\log A_{0,i}/\bar{A}_0$ and $\log Y/L$ in 2010 is 0.8287. Similarly, the counterpart correlations reported by Islam (1995) are even higher: 0.87 and 0.96 associated with $\log Y/L$ in 1960 and 1985, respectively. The correlations between $\log A_{0,i}/\bar{A}_0$ and other explanatory variables including enrollment, investment and public education spending are shown in Table 2.7. Among these, ‘SCHOOL2’ is particularly associated with individual effect, with a correlation of 0.5850. In fact, the result lends strong support to findings of Klenow and Rodriguez-Clare [46], who estimate $\ln A$ in a different approach and report correlations of 0.93 and 0.57 between
\[ \ln A \text{ and } \ln(Y/L), \ln(H/Y), \text{ respectively, which highly coincide with ours.} \]

Are the estimates reasonable? As Klenow and Rodriguez-Clare [46] explained, one possible reason for correlation of 0.5850 between \( \log A_0 \) and \( \log SCHOOL2 \) is that education investment facilitates technology adoption. High level of human capital improves the efficiency of technology conversion to productivity. That explains why the coefficient on human capital is smaller in panel estimation than in single cross-country regression. Note that the role of human capital is not necessarily weakened, because it is possible to facilitate production in another channel: through raising technology adoption, or TFP.

The analysis above shows that \( \log A_0 \) is not only highly correlated with a country’s per capita income, but is also associated with education enrollment. Thus ignoring the country-specific effects in MRW regression might be a problem. This is the main limit of a single cross-country regression. Therefore, the next question arises: what would happen when a single cross-country incorporates the unobserved effects? To test the effect, we first rewrite equation (2.7) as:

\[
\log(Y/L)(t) = (\psi t + \log \overline{A}_0) + \log(A_0/\overline{A}_0) + \frac{\alpha}{1 - \alpha - \beta} \log \frac{I_k/Y}{\psi + \delta + n} \\
+ \frac{\beta}{1 - \alpha - \beta} \log \frac{I_h/Y}{\psi + \delta + n}
\]  

(2.23)

Now \( \log A_0/\overline{A}_0 \) enters the equation as a regressor, and \( \psi t + \overline{A}_0 \) is the fixed intercept. We then run the unrestricted and unrestricted regressions as in the previous single cross-country estimation with the non-oil sample (89 countries). \( \log(Y/L) \) is the real log GDP per working-age person in 2010, and the regressors are the average over 1970-2010. The only difference is to add one more variable, \( \log A_0/\overline{A}_0 \) into regression. The results are shown below.

Unrestricted:

\[
\log(Y/L)(t) = 5.59 + 0.68 \log(A_0/\overline{A}_0) + 0.93 \log I_k/Y - 2.38 \log(\psi + \delta + n) + 0.55 \log SCHOOL2 \\
(0.91) \quad (0.05) \quad (0.11) \quad (0.33) \quad (0.08)
\]  

(2.24)
Restricted:

$$\log(Y/L)(t) = 7.87 + 0.72 \log(A_0/\bar{A}_0) + 1.00 \log \frac{I_k/Y}{\psi + \delta + n} + 0.59 \log \frac{SCHOOL2}{\psi + \delta + n}$$

$$\begin{array}{llll}
(0.05) & (0.12) & (0.12) & (0.08) \\
\end{array}$$

(2.25)

The standard errors are reported in the parenthesis immediately below. All the coefficients, including $$\log(A_0/\bar{A}_0)$$, are significant at 1% level. The coefficient on country-effect is around 0.7, less than the expected unity but not too far away. The noteworthy finding is that the coefficient on ‘SCHOOL2’ shrinks significantly. The values in unrestricted and restricted regressions are 0.55 and 0.59, compared with 1.00 and 1.16 in the original single cross-country regression. The size reduces by around 50%, which is greatly similar to what we find in panel data model! Now the implied coefficients of $$\alpha$$ and $$\beta$$ are 0.39 and 0.23. Again, $$\beta$$ falls into the range of one-fifth and one-quarter.

The next experiment is to include physical investment in education. We rewrite (2.11) in the same way to allow country-specific effects. The results are shown below:

Unrestricted:

$$\log(Y/L)(t) = 6.18 + 0.67 \log(A_0/\bar{A}_0) + 0.92 \log I_k/Y - 2.34 \log(\psi + \delta + n)$$

$$\begin{array}{llll}
(1.01) & (0.05) & (0.11) & (0.33) \\
\end{array}$$

$$+ 0.55 \log SCHOOL2 + 0.16 \log E/Y$$

$$\begin{array}{llll}
(0.08) & (0.12) \\
\end{array}$$

(2.26)
Restricted:

\[
\log(Y/L)(t) = 8.06 + 0.69 \log(A_0/A_0) + 0.96 \log \frac{I_k/Y}{\psi + \delta + n} + 0.57 \log \frac{SCHOOL2}{\psi + \delta + n} + 0.24 \log \frac{E/Y}{\psi + \delta + n}
\]

\[(0.15) \quad (0.05) \quad (0.11) \quad (0.08) \quad (0.11)\]

Note that \( E \) in this modified model measures the physical investment in education, for which public spending on education is used as a proxy; In MRW model, \( I_h \) indicates a broader sense of education investment, approximated by ‘SCHOOL2’. The implied \( \alpha \) and \( \beta \) are 0.44 and 0.11, respectively. However, the results should be interpreted with caution. \( E \) is narrowed to specifically measure the physical investment in education. In addition, the individual effects, \( \log(A_0/A_0) \) might partially capture measurement in education quality, since it is calculated from the dynamic panel MRW model where schooling quality is not controlled for. As a result, coefficient on \( E/(\psi + \delta + n) \) could be underestimated.

In fact, in another experiment in which the estimated individual effects from the modified dynamic panel MRW model are used instead as a regressor, I find the the implied \( \alpha \) and \( \beta \) are 0.32 and 0.20, respectively in single cross-country regression. Therefore, the coefficients fall into a reasonable range again once the physical investment is properly accounted for.

The discussion above has further confirmed that a country’s unobserved individual effect is important in explaining economic growth. Individual effects are not only correlated with a country per capita income, but are also highly associated with a country’s human capital. Ignoring country-specific effects might lead to overestimation of coefficients on human capital. We find the implied share of human capital appears to be between 0.20-0.25 again. The findings shed light on the role of human capital in economic growth dynamics. Given the empirical evidence above, we believe that focus of future research on constructing a structural model that properly links

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7The regression results are not shown in the paper but are available upon request.
human capital to final output and also to TFP is a necessary direction in order to understand human capital in more depth.

2.5 Conclusion

In this paper, we investigate the role of education investment, schooling quality, and the country-specific unobserved effects in explaining cross-country economic growth under exogenous growth framework, and estimate the implied share of human capital in production function. We have several noteworthy findings. First, education expenditures, approximated by public spending on education, are found to be influential to long-run economic growth. Education is usually heavily subsidized by the government, especially in primary and secondary schooling. Thus public spending on education is a plausible indicator of physical investment on education. Education enrollment, on the other hand, is more appropriate to be interpreted as the time component in human capital production. We find that physical investment explains around one-third in human capital production, and a combination of the forgone time and human capital explains the remaining two-thirds.

Next, we explore the country-specific unobserved effects in a dynamic panel model where the country-effects are assumed to be the time-invariant heterogeneous fixed intercepts. GMM is adopted and implemented to eliminate potential inconsistencies arising from the lagged dependent variable and non-strictly exogenous explanatory variables. We find that schooling enrollment, or the forgone time, continues to impact economic growth dynamics at 5-year interval. The finding is in sharp contrast with the predominant result that the contribution of human capital investment to economic growth is vague, and even negative in a panel model.

However, the estimated coefficient on physical investment in education mostly becomes negative and insignificant. Instead, when we use the one-decade lagged public spending on education as a regressor, this estimated coefficient restores to significantly positive. Based on this fact, we conclude that physical education expenditures need around a decade to become effective to economic growth. Public spending is mostly spent on primary and secondary schooling, so one decade lag seems reasonable. The finding has strong policy implementation. The increase in a
country’s income due to public spending on education will finally be large enough to recover the initial spending. Education expenditures are important to a country’s sustainable growth.

Dynamic panel framework enables us to estimate the relative strength of a country’s unobserved effect. So we list all countries’ the estimated individual effect in our sample. We find that the estimated individual effect is highly correlated with a country’s per capita income. The simple correlation of log $A_0$ and log $Y/L$ in 2010 is 0.8287. In addition, individual effects are highly associated with a country’s human capital investment, especially enrollment. It lends support to the popular view that human capital investment facilitates technology adoption, which is usually considered to be included in the unobserved TPF.

Finally, we find the estimated human capital share in production function mostly falls into the range of 1/5-1/4. Although the MRW model still predicts a share of one-third with more recent data, the estimates might be inflated because of a positive correlation between the unobserved country effect and human capital. When the country effect is controlled for, predicted human capital share shrinks. The estimated physical capital share is reasonable in the sense that it’s consistent with the conventional value of one-third. We find the estimates are consistently stable in either single cross-country or panel data regression.
CHAPTER 3

Distortionary Taxation and Productive Public Capital: A Long-Run Welfare Analysis

3.1 Introduction

This paper extends the current literature to investigate the role of valued government expenditures in characterizing Laffer curves and optimal long-run fiscal policies. Government spending is explicitly modeled in the utility function, and public capital is productive. The benevolent government funds itself through raising tax revenues and issuing bonds, and endogenously determines the allocation between government consumption, government investment after repaying exogenous transfer payments and debts. For a given level of tax revenues, government spending yields welfare gains directly to the representative household, while government investment boosts output so that the representative household benefits from a raise in wages and dividends. The government is fully informed of the social benefit and cost, and sets the government expenditure to maximize the welfare.

The model is calibrated to the quarterly US economy. I assume the government optimally chooses the combination of public investment and public consumption in the long run and propose a set of parameter values so that the optimal steady state government investment to output ratio matches the historical data. I also investigate the effects of tax rate changes in the long run and characterize quantitatively the Laffer curves for labor and capital income taxation. Trabandt and Uhlig [71, 72] quantitatively characterize the Laffer curves for US and EU economies from 1975-2000 and 1995-2007, respectively. In their paper, government investment is absent and government spending is exogenous. Laffer curves are obtained by depicting steady state tax revenues varying tax rates. Government spending does not affect the shape of the Laffer curves.

This paper sheds light on the existing literature of Laffer curves. Laffer curves differ from those in the previous literature in that the government decisions of public
investment and public consumption alter the shape of Laffer curves. Public investment affects the total output as well as price levels and thus shifts labor supply and capital accumulation. So rules of allocation of government expenditure need to be specified to pin down Laffer curves. In our model, the government optimally allocates the two types of government expenditures for any given level of taxes in the steady state instead of 'keeping all others the same' in the previous literature. For comparison purposes, another group of Laffer curves are depicted under the alternative rule that public investment is fixed for all tax rates. It is equivalent to the traditional Laffer curves. I find the calibrated Laffer curves in my model can reach higher peaks than those in the traditional counterparts. This reflects the government’s decisions of expenditures. The calibrated results show that the US is on the left of Laffer curves. The peaks of Laffer curves occur at labor and capital tax rates both around 0.73.

My paper also contributes to the literature of optimal taxation. As noted by Trabandt and Uhlig [71], 'the level of taxation which delivers maximum tax revenues is not in general a policy that maximizes welfare’. However, they do not provide welfare analysis to further discuss the issue. In this paper, I explore explicitly on this direction.

I measure welfare levels obtained for different combinations of labor and capital taxes in the long run. Using the measure of welfare loss proposed by Lucas [50], I quantify the welfare cost of deviating from the optimal fiscal policies. Under my parametrization, I find welfare is maximized when the government levies only labor income taxes and imposes zero capital income taxes. It is consistent with the famous optimal zero capital taxes in the literature. Another important finding is that output as well as private investment exhibits a hump shape for varying only labor income taxes. It means up to some point, cutting labor tax results in output recession in the long run.

The optimal taxation problem in this paper is different from solving a Ramsey problem. I focus on the optimal welfare in the long run instead of looking at short run along the transition path. Also, lump-sum transfers and debts are calibrated to US data, so that they are held fixed. In other words, we are searching for policies of taxes, government spending and government investment that maximize the welfare
while keeping the government explicit and implicit liabilities unchanged. The solution from Ramsey problem indicates that a government should accumulate claims and fund itself through interest earnings. However, it is rarely observed in the real world. The US government has been running a budget deficit for most of the years. This difference between theory and reality remains a controversial issue. Many researchers look at this problem from political economy perspective. Government can appropriate part of tax revenues for unproductive public consumption called 'pork-barrel spending’, i.e. political rents. Pork-barrel spending can be thought of as favors paid to 'friends' of the government or public employees. Voters can replace a government that abuses its power, but in equilibrium they generally cannot push rents all the way to zero. This idea shows that political agency can lead to excessive debt accumulation due to the problem of moral hazard. The binding borrowing constraints problem is well explained under this framework.

A discussion of interactions between voters and political agent is beyond the point of this paper. But the conclusion of this branch of literature provides reasonable explanation for the current US accumulated debts, which cannot be fully justified in Ramsey problem. I assume in the long run, government debts bind at the calibrated share of GDP. Government expenditures have to be funded by imposing distorting labor and capital taxes. Since government spending is utility-enhancing and public investment is productive, the government has an active role to guide the economy. Sensitivity analysis indicates that zero optimal capital tax rate in the long run is not as robust under our model framework as in Ramsey problem.

The paper is organized as follows: Section 3.2 summarizes the existing literature. I specify the model in section 3.3 and discuss its parameterization and calibration in section 3.4. Section 3.5 discusses the results. Section 3.6 provides sensitivity analysis. Section 3.7 concludes. Further details are included in the appendix.

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1See e.g. Alesina et al. [2], Battaglini and Coate [10].
3.2 Related Literature

There is large literature discussing about effects of fiscal policy on aggregate fluctuations and growth. One of the central issues focuses on the macroeconomic effects of labor and capital income tax and government purchases in an exogenous growth context. Baxter and King [11] were one of the first authors who analyzed the effects of fiscal policy in general dynamic equilibrium using one-sector neoclassical growth model with productive government capital. Braun [15], McGrattan [55], Burnside et al. [17], among others, studied the effects of fiscal shocks in Real Business Cycles model. These papers mainly focus on dynamic interactions of capital and labor in presence of exogenous technology shocks and temporary/permanent fiscal shocks. On the other hand, Trabandt and Uhlig [71], Trabandt and Uhlig [72] quantitatively build up a neoclassical growth model and calibrated US and EU-14 economies. They characterize Laffer curves for labor and capital income taxation and investigate the effects of tax rate change in the long run.

This branch of literature usually assumes that the government is subject to exogenous streams of outlays and finance itself through issuing bonds and collecting distortionary taxes from the private sector. The outlays are assumed ‘thrown into ocean’. However, such a model cannot fully characterize the government’s role to guide the economic activity through providing public goods and services and public investment, and thus it has a limited ability to provide policy implication. Another disadvantage is its limitation to provide welfare analysis. Baxter and King [11], and Trabandt and Uhlig [71], Trabandt and Uhlig [72] simply include government spending in the utility function without explicitly specifying it. Typically this term is omitted for simplicity. Of course, when government outlays are assumed to be exogenous processes to both consumers and the government, then it makes little difference whether to include them in social preferences. Such a setup is widely accepted due to its simplicity and success in simulations of historical data. However, this assumption limits the capability of the model to evaluate government’s role to guide the economy, especially from the welfare perspective.

Barro [8] extends one strand of the endogenous-growth model, known as the AK model, to include tax-financed government services that affect production or u-
tility. He shows that the welfare and growth maximization are equivalent in the competitive economy and growth rate in the competitive economy is lower than social optimum under income taxation. Ganelli and Tervala [33] investigated the welfare effect of a trade-off between productivity-enhancing public infrastructures and utility-enhancing public consumption by the government in a two-country New-Open-Economy-Macroeconomics model. The rest of literature that considers effects of government spending mainly focuses on the complement/substitute relationship of private and public consumption (See, for example, Mountford and Uhlig [60] ).

On the other hand, productive government capital is often modeled to investigate the effect of government investment on economic growth (For example, see Aschner [5], Aschauer [6], Barro [8], Corsetti and Roubini [26], Bruce and Turnovsky [16] and Turnovsky [73]). Baxter and King [11] extend the neoclassical growth model to include productive government capital to investigate the long-run/short-run macroeconomic consequences of changes in public investment. The public investment and public consumption are calibrated to post-war US economy. However, government spending is not explicitly modeled in preferences. There is no trade-off between the two types of government expenditures.

Another central issue of fiscal policy literature is the optimal taxation. Chamley [21] builds up a basic neoclassical model with a government that finances an exogenous stream of government purchases. The production factors are raw labor and physical capital on which the government levies distorting flat-rate taxes. He finds the optimal policy is eventually to set the tax rate on capital to zero. This conclusion is robust whether the government can issue debt or must run a balanced budget in each period. Whether the government can issue debt or must run a balanced budget in each period. [42] show that under somewhat stringent assumptions, the return to human capital should not be taxed in the limit and therefore all taxes should be zero in the steady state. The optimal policy is to amass claims on the private economy under a transition period and finally use only interest earnings to finance government expenditures in the long run. Although results for non-capital taxes require somewhat rigid assumptions, a zero capital tax in a non-stochastic steady state seems very robust. Lucas and Stokey [52] analyze a model without physical capital in stochastic environment. They
propose the Primal approach to solve the Ramsey problem, which shows Ramsey
problem is equivalent to optimizing the welfare subject to resource constraint and an
implementability condition. They find labor income tax is nonzero and smooth over
states. Chari et al. [22] perform numerical simulations in a stochastic model with
complete markets and find that labor tax rate should be very smooth and expected
capital tax rate should be roughly zero every period. Schmitt-Grohe and Uribe [68]
Schmitt-Grohe and Uribe [69] consider stochastic models with imperfect competition
and still find the optimality of smooth labor income taxes. The conclusion of zero
capital taxes and smooth labor income taxes is commonly drawn in the existing
literature. I find the result consistent with the previous in this paper, although it is
sensitive to the parameter values. The model is specified in the following section.

3.3 The Model

I construct a neoclassical growth model that permits a variety of fiscal inter-
ventions. The government collects distortionary labor and capital taxes, and issues
government bonds to finance the lump-sum transfer payments, the government con-
sumption expenditure and government investment. I model the US economy as an
infinite-horizon closed economy.

3.3.1 The Private Sector

The representative household’s problem is to maximize the discounted sum of
life-time utility subject to the flow budget constraint and a capital flow equation:

$$
\max_{c_t,n_t,i_t,k_{t+1},b_{t+1}} E_0 \sum_{t=0}^{\infty} \beta^t \{ u(c_t, n_t) + f(g_t) \} \\
\text{s.t. } c_t + i_t + b_{t+1} = (1 - \tau_{n,t})w_t n_t + (1 - \tau_{k,t})(d_t - \delta)k_t + \delta k_t + s_t + R_{bt}b_t + \Pi_t \tag{3.1} \\
k_{t+1} = (1 - \delta)k_t + i_t \tag{3.2}
$$

where $c_t$, $n_t$, $k_{t+1}$, $b_{t+1}$, $i_t$, $s_t$, $\delta$ denote consumption, hours worked, private capital,
government bonds, private investment, lump-sum transfer payments and depreciation
rate, respectively. The household takes the government consumption, $g_t$ as given. As
in Baxter and King [11], $g_t$ enhances utility, and is assumed to be separable with the private counterpart. It rationalizes government consumption such as military purchases and fees maintaining the law system as well as other goods and services provided by the public sector, and they do not directly affect private consumption decisions. $u(\cdot, \cdot)$ and $f(\cdot)$ are both concave and twice continuously differentiable. The subjective discount factor is represented by $\beta \in (0, 1)$. The household has to pay labor income taxes $\tau_{n,t}$ and capital income taxes $\tau_{k,t}$, and receives interest earnings $R_{b,t}$, wages $w_t$, dividends $d_t$, and profits $\Pi_t$. Note that capital taxes are levied on dividends net-of-depreciation as in Trabandt and Uhlig [71, 72], Prescott [62, 63] and in line with Mendoza et al. [57]. Finally, the maximization problem above presumes another condition, namely a constraint that prevents running Ponzi schemes.

I consider the following preferences for our analysis:

$$u(c_t, n_t) = \log c_t + \alpha_n \log (1 - n_t),$$

$$f(g_t) = \alpha_g \log g_t,$$

which is a commonly employed utility function form in the macroeconomic literature. It can be derived a more general utility function: $u(c_t, n_t) + f(g_t) = \frac{c_t^{1-(1-n_t)^{1-\eta}}}{1-\eta} + \frac{g_t^{1-\eta}}{1-\eta}$, with $\eta = 1$. The coefficient of risk aversion is 1 throughout this paper.\(^2\)

The output at date $t$ is the result of private capital $k_t$, labor supply $n_t$ and public capital $k_{g,t}$ in a Cobb-Douglas production function form:

$$y_t = z_t k_t^{\theta} n_t^{1-\theta} (k_{g,t})^{\gamma},$$

(3.3)

where $\theta, \gamma \in (0, 1)$. Total factor productivity is denoted by $z_t$, and $z_t = \xi^t$. I assume there are constant returns to scale over private inputs of labor and capital, and the government provided capital is not subject to congestion, which is in line with Baxter and King [11]. The public capital evolves according to:

$$k_{g,t+1} = (1 - \delta)k_{g,t} + i_{g,t},$$

(3.4)

\(^2\)It would be interesting to use a general utility function and let the coefficient of risk aversion vary. However it is beyond the point of this paper. I leave it as one direction of further exploration for future work.
where I assume public capital depreciates at the same rate as private capital.

The representative firm’s maximizes its profit in a perfectly competitive market:

$$\max_{k_t, n_t} y_t - d_t k_t - w_t n_t.$$ 

Therefore profits at date $t$ are:

$$\Pi_t = \frac{\partial y_t}{\partial k_t} k_t + \frac{\partial y_t}{\partial n_t} n_t = 0, \quad \forall t. \quad (3.5)$$

### 3.3.2 The Government

The benevolent government allocates public consumption and productive public investment, and faces exogenous streams of lump-sum transfer payments. These expenditures are financed by levying distortionary labor and capital taxes, and by issuing government bonds. The government’s flow budget constraint is:

$$i_{g,t} + g_t + R_{b,t} b_t + s_t = b_{t+1} + T_t, \quad (3.6)$$

where government tax revenue $T_t$ is

$$T_t = \tau_{n,t} w_t n_t + \tau_{k,t} (d_t - \delta) k_t. \quad (3.7)$$

### 3.3.3 Equilibrium

In the long run equilibrium all the variables except for hours worked $n$, interest rates $R_b$ and tax rates $\tau_n, \tau_k$ grow at a constant rate

$$\psi = \xi^{\frac{1}{1-\theta-\gamma}}.$$ 

Assume that $\xi \geq 1$ and after-tax return on capital $R_t > 1$. I detrend all variables that are non-stationary by the balanced growth path $\psi^t$ so that a stationary solution can be achieved. All the detrended variables are denoted by a tilde sign, e.g. $\tilde{y}_t = \frac{y_t}{\psi^t},$
\( \tilde{c}_t = \frac{c_t}{\psi t} \), etc, except for \( n_t, R_t, R_{b,t}, \tau_{n,t}, \) and \( \tau_{k,t} \).

The household chooses plans to maximize the utility and the firm solves its profit maximization problem. The detrended first order conditions are as follows: Households Euler equation for capital:

\[
\beta E_t \left[ \frac{u_c(t+1)}{u_c(t)} R_{t+1} \right] = 1
\]  (3.8)

Real after-tax return on capital:

\[
R_t = (1 - \tau_{k,t})(\theta \frac{\tilde{y}_t}{k_t} - \delta) + 1.
\]  (3.9)

Households labor supply decision:

\[- \frac{u_n(t)}{u_c(t) \psi t} = (1 - \tau_{n,t})(1 - \theta) \frac{\tilde{y}_t}{n_t} \]  (3.10)

Households Euler equation for government bonds:

\[
\beta E_t \left[ \frac{u_c(t+1)}{u_c(t)} R_{b,t+1} \right] = 1
\]  (3.11)

A competitive equilibrium is a set of plans \( \{ \tilde{c}_t, \tilde{n}_t, \tilde{b}_{t+1}, \tilde{k}_{t+1}, \tilde{i}_t \} \) satisfying equations (3.8) to (3.11), two detrended capital accumulation equations:

\[
\psi \tilde{k}_{t+1} = (1 - \delta) \tilde{k}_t + \tilde{i}_t,
\]  (3.12)

\[
\psi \tilde{k}_{g,t+1} = (1 - \delta) \tilde{k}_{g,t} + \tilde{i}_{g,t},
\]  (3.13)

detrended government budget constraint equation:

\[
\tilde{i}_{g,t} + \tilde{g}_t + R_{b,t} \tilde{b}_t + \tilde{s}_t = \psi \tilde{b}_{t+1} + \tilde{T}_t,
\]  (3.14)

detrended tax revenues equation

\[
\tilde{T}_t = \tau_{n,t}(1 - \theta) \tilde{y}_t + \tau_{k,t}(\theta \frac{\tilde{y}_t}{k_t} - \delta) \tilde{k}_t,
\]  (3.15)
detrended production function

\[ \tilde{y}_t = \tilde{k}_t^{\theta} n_t^{1-\theta} (\tilde{k}_{g,t})^{\gamma}, \]  

(3.16)

and detrended resource constraint:

\[ \tilde{c}_t + \tilde{g}_t + \tilde{i}_{g,t} + \tilde{i}_t = \tilde{y}_t, \]  

(3.17)

given exogenous processes \{τ_{n,t}, \tau_{k,t}, \tilde{g}_t, \tilde{i}_{g,t}, \tilde{k}_{g,t+1}, \tilde{s}_t\} and initial condition \{k_0, k_{g,0}, b_0\}.

The government’s optimal allocation problem in the steady state is defined as follows. Assume steady state transfer payments and debts are exogenous. For any given tax rates, the benevolent government will allocate tax revenues between utility-enhancing government spending and public investment optimally by solving the following problem:

\[
\max V(\tilde{i}_g, \tilde{g}) = \max \{ \log c(\tilde{i}_g, \tilde{g}) + \alpha_n \log (1 - \bar{\pi}(\tilde{i}_g, \tilde{g})) + \alpha_g \log \tilde{g} \}
\]

s.t.

\[
\bar{g} + \bar{i}_g + \bar{R}_b b + \bar{s} = \psi \tilde{b} + \tau_n (1 - \theta) \tilde{g} + \tau_k \tilde{g} (\tilde{i}_g) \tilde{g} - \tau_k \delta \tilde{k} (\tilde{i}_g, \tilde{g}),
\]

\[
\psi \tilde{k}_g = (1 - \delta) \tilde{k}_g + \tilde{i}_g,
\]

\[
\psi \tilde{k}(\tilde{i}_g, \tilde{g}) = (1 - \delta) \tilde{k}(\tilde{i}_g, \tilde{g}) + \tilde{i}(\tilde{i}_g, \tilde{g}),
\]

where \( V(i_g, g) \) is the indirect utility function that solves the consumer’s problem as a function of \( i_g \) and \( g \) given \( \tau_n, \tau_k, s \) and \( b \). Given the complexity of this problem, we will solve it numerically. The calibration will be discussed in the next section.

### 3.4 Calibration and Parameterization

I carefully calibrate the model to the quarterly data of the US economy. The data starts from 1995 to 2007. All the data we use for calibration come from Federal

Table 3.1: Average tax rates in the literature

<table>
<thead>
<tr>
<th>$\tau_k$</th>
<th>$\tau_n$</th>
<th>Time Span</th>
<th>Authors</th>
<th>Methodology</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.37</td>
<td>0.26</td>
<td>1975-2000</td>
<td>Carey and Rabesona [18]</td>
<td>Mendoza et al. [57]</td>
</tr>
<tr>
<td>0.36</td>
<td>0.28</td>
<td>1995-2007</td>
<td>Trabandt and Uhlig [72]</td>
<td>Mendoza et al. [57]</td>
</tr>
<tr>
<td>0.37</td>
<td>0.22</td>
<td>1970Q1-2010Q2</td>
<td>Fernandez-Villaverde et al. [31]</td>
<td>Jones [41]</td>
</tr>
<tr>
<td>0.184</td>
<td>0.223</td>
<td>1960Q1-2008Q1</td>
<td>Davig, Leeper, and Walker [27]</td>
<td>Jones [41]</td>
</tr>
</tbody>
</table>

Tax rates: I use flat taxes instead of marginal tax rates in the model because the former are relatively easy to be constructed on a quarterly basis, and this is an usual caveat in the literature. I use the results of Trabandt and Uhlig [72] who calculate average tax rates following the prevalent methodology developed by Mendoza, Razín, and Tesar [57] for 1995-2007. See Appendix B.3 for a detailed description of this methodology. They find average capital income taxes and labor income taxes: $\tau_k = 0.36$, $\tau_n = 0.28$. Table 3.1 lists calculated tax rates for different periods in the literature using the methods of Mendoza et al. [57] and Jones [41]. We can see the results calculated using these two methods do not substantially differ.

Growth rate: I set the exogenous balanced growth factor $\psi$ to 1.0075 which corresponds to the annual average growth rate of real US GDP of roughly 3% from data.

The interest rate: I calculate the average annual real interest rate from 1995 to 2007. Divided by 4, it results in the quarterly real gross interest rate, $\bar{R}_b = 1.012$. In the steady state, equation (3.11) becomes:

$$
\bar{R}_b = \frac{\psi}{\beta},
$$

Another prevalent methodology is developed by Jones [41], which is closely related to Mendoza et al. [57]. These two methodologies are similar to the procedures used in Joines [40] and McGrattan [55]. The main difference between Mendoza et al. [57], Jones [41] and Joines [40], McGrattan [55] is that the Joines and McGrattan estimate the personal income tax rate as a marginal tax rate from tax records, rather than as an average rate from the national accounts. See Jones [41] for a detailed comparison of his tax rate measures and those of Joines [40] and McGrattan [55].
which is used to calibrate the subjective discount factor, $\beta$.

Depreciation rate $\delta$: It is calculated from the steady state of equation (3.12) and (3.13):

$$\bar{t} = (\psi - 1 + \delta)\bar{k}, \quad \text{(3.19)}$$

$$\bar{i}_g = (\psi - 1 + \delta)\bar{k}_g. \quad \text{(3.20)}$$

If I define total investment as $I \equiv i + i_g$, total capital stock as $K \equiv k + k_g$. Then we have the steady state of total capital accumulation equation:

$$\bar{I} = (\psi - 1 + \delta)\bar{K}. \quad \text{(3.21)}$$

Limited by data availability, we only have access to the total net capital stock on annual basis. In accordance with capital stock, I calculate the average annual investment for the total economy. The ratio is used to calibrate $\bar{I}/\bar{K}$. Note that the ratio needs to be divided by 4 to obtain the quarterly counterpart because the annual investment is a flow while the annual capital is a stock. Plug it into equation (3.21) and I can solve for $\delta$.

The government debt to output ratio: I use the average of the federal debt held by the public for $\bar{b}$. In accordance with government debt, the average of nominal GDP is used to calibrate $\bar{y}$. The ratio of two nominal data cancels out the effect of price levels and is equal to the real counterpart, which is in line with Trabandt and Uhlig [71, 72]. To match the quarterly calibrated model, $\bar{b}/\bar{y} = 0.38 \times 4$. Trabandt and Uhlig [72] consider the an alternative measure of US public indebtedness by using the total government public debt. It amounts to 63% of nominal GDP in their sample. They find none of the quantitative results change noticeably by using this larger US debt to GDP ratio.

The government investment to output ratio: From the data, the real average public investment to the real average GDP is roughly 3.2%, which is used as $\bar{i}_g/\bar{y}$. Note that I do not distinguish between defense and non-defense government expenditure and investment in the calibration. Baxter and King [11] assume that utility enhancing government consumption expenditures include military purchases and they do not directly affect private consumption and production decisions. My model is consistent
with this assumption. Government consumption includes defense and non-defense public consumption expenditure.

Public investment doesn’t exclude defense investment as well. The reason is that this type of investment can contribute to the production of military goods which may also increase utility in the form of government consumption. Besides, this type of investment has a relatively small share of GDP (roughly 0.6%). So I do not exclude military purchase and investment from the government sector in the model.

Government consumption, \(\overline{y}\) is calculated by subtracting public investment from total government expenditure. See Appendix B.2 for details of data calculation.

Parameters in production function: I solve the steady state of equation (3.9) for \(\theta\):

\[
\frac{\bar{k}}{\overline{y}} = \theta (\frac{\bar{R}}{1 - \bar{r}_k} + \delta)^{-1}. \tag{3.22}
\]

Since the private capital to output ratio is not observable, I first divide both sides of equation (3.19) by \(\overline{y}\):

\[
\frac{\bar{i}}{\overline{y}} = (\psi - 1 + \delta)\frac{\bar{k}}{\overline{y}}.
\]

Then plug in values of \(\bar{i}/\overline{y}, \psi\) and calculated \(\delta\) to solve for \(\bar{k}/\overline{y}\). As a result, \(\bar{k}/\overline{y} = 8.03\). \(\theta\) is calibrated to 0.25. This parameter value is lower than the conventional value of one-third, because the previous literature assumes away government capital and only includes private capital and labor supply as inputs. However, the data for total capital stock in the economy is usually used to calibrate the total private capital, \(\bar{k}\) (e.g. Trabandt and Uhlig [72]), due to data availability. Under our model framework, I am able to correct this underestimation, and to recalibrate the parameter value for \(\theta\), using this proposed method. This discrepancy for calibration justifies a smaller value of \(\theta\).

Baxter and King [11] propose a rule of thumb to calibrate the parameter of government capital, \(\gamma = s_g \equiv i_g/y\). The argument is: government capital works like a productivity shift from the standpoint of the private determination of capital and labor input. If we treat private capital and labor as unresponsive to government, then steady state net output, \(y - i_g\), is maximized when \(i_g/y = \gamma\), if we take the first order condition with respect to \(i_g\). Of course, the private capital and labor will respond
to marginal change of \( i_g \) because it raises prices, but as a rough way of estimate, this rule of thumb provides us some insight into calibration of \( \gamma \). So I set \( \gamma = 0.03 \) following this method.

The government transfer payments to output ratio: I rewrite the steady state government budget as:

\[
\frac{s}{y} = \frac{T}{y} - (\bar{R}_b - \psi)\bar{b}/y - \tilde{i}_g/y - \bar{g}/y. \tag{3.23}
\]

where \( \frac{T}{y} = \bar{\pi}(1 - \theta) + \bar{\pi}_k \theta - \bar{\pi}_k \delta \bar{k}/y \). I calibrate \( \frac{s}{y} \) so that ratios on the right hand-side of the equation above match the observed average. Based on the calculation in my sample, I set this ratio \( \frac{s}{y} = 0.07 \).

The parameters in the utility function: I calibrate hours worked \( \bar{n} = 0.25 \), which is consistent with Prescott [62, 63], McGrattan and Rogerson [56] and Trabandt and Uhlig [71, 72]. It means workers supply on average roughly 40 hours per calendar-week. Steady state consumption to output ratio, \( c/y \) can be solved from the steady state of equation (3.1):

\[
c/y = (1 - \tau_a)(1 - \theta) + [(1 - \tau_k)(\theta \bar{y}/\bar{k} - \delta) - (\psi - 1)]\bar{k}/y + s/y + (\bar{R}_b - \psi)\bar{b}/y, \tag{3.24}
\]

by plugging in all the calibrated ratios on the right-hand side. Given \( \bar{n} = 0.25 \) and \( c/y \), \( \alpha_n \) can be solved in the steady state of equation (3.10).\(^4\)

The government preference parameter \( \alpha_g \): This parameter measures how the representative household values the government consumption expenditure. To calibrate it, I assume the government is behaving optimally to allocate public investment and public consumption in the long run, subject to the steady state government budget constraint:

\[
\tilde{i}_g + \bar{g} + \bar{R}_b \bar{b} + \bar{s} = \psi \bar{b} + \bar{T}, \tag{3.25}
\]

where the tax revenues, government debt and transfer payments calibrated and fixed to quarterly data. Essentially, the government faces the trade-off between the two types of government expenditures. Government consumption directly yields the

\(^4\)See Appendix B.1.
household’s utility and government investment raises total output. \( \alpha_g \) determines how the household values government spending and thus directly affects the government decision for the allocation. I set \( \alpha_g = 0.25 \) so that at the optimal allocation, government investment share on GDP is roughly 3.2\% of GDP.

Figure 3.1 depicts the hump-shaped welfare arising from increase in \( \bar{\bar{i}}_g/\bar{y} \), which indicates the trade-off the government faces between the two types of public expenditures. With our calibrated parameters, \( \bar{\bar{i}}_g/\bar{y} \) is roughly 3.2\% at the optimal. By doing this we have implicitly assumed that the two public expenditures are in fact optimally allocated by US government in the long run. Calibrated parameters are summarized in Table 3.2.

![Graph](image.png)

Figure 3.1: Welfare against \( i_g/y (\tau_n = 0.28, \tau_k = 0.36) \)

### 3.5 Results

#### 3.5.1 Steady States and Optimal Government Investment

Detailed description of the steady state of the model is provided in Appendix B.1. Table 3.3 compares the government, consumption and private investment share on GDP of the data and the model. Since US economy is an open economy, so the sum of shares of government expenditures, investment, private consumption on GDP
Table 3.2: Baseline Calibration

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
<th>Description</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_b$</td>
<td>1.012</td>
<td>Interest Rate</td>
<td>Data</td>
</tr>
<tr>
<td>$\psi$</td>
<td>1.0075</td>
<td>Exogenous growth rate</td>
<td>Data</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.9956</td>
<td>Subjective time discount rate</td>
<td>$\psi/R_b$</td>
</tr>
<tr>
<td>$\alpha_n$</td>
<td>2.45</td>
<td>Parameter on $n_t$ in utility</td>
<td>$\pi = 0.25$</td>
</tr>
<tr>
<td>$\alpha_g$</td>
<td>0.25</td>
<td>Parameter on $g_t$ in utility</td>
<td>$\tilde{g}/\tilde{y} = 3.2%$</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.25</td>
<td>Parameter on $k_t$ in production</td>
<td>Equation (3.22)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.03</td>
<td>Parameter on $k_{g,t}$ in production</td>
<td>Set $\gamma = \tilde{g}/\tilde{y}$</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.013</td>
<td>Depreciation rate</td>
<td>Equation (3.19)</td>
</tr>
<tr>
<td>$\tilde{b}/\tilde{y}$</td>
<td>0.38×4</td>
<td>Government Debt to GDP</td>
<td>Data</td>
</tr>
<tr>
<td>$I/K$</td>
<td>0.02</td>
<td>Total Investment to Capital stock</td>
<td>Data</td>
</tr>
<tr>
<td>$\tilde{\pi}/\tilde{y}$</td>
<td>0.07</td>
<td>Transfer payments to GDP</td>
<td>Equation (3.23)</td>
</tr>
</tbody>
</table>

differs from 100% due to net import. To reconcile such discrepancy, I recalculate these shares by excluding net import/export so that they will sum up to 1, consistent with our model. In the data the government consumption to GDP ratio is 15.4% and our model predicts 15.4%. Consumption share on GDP is 65.8% in the data and 65.3% in our model. Private and public investment shares on GDP are both calibrated to the data. So we argue the model is roughly able to match these shares on GDP.

Table 3.3: Shares on GDP(%)  

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Government consumption</td>
<td>15.4</td>
<td>15.4</td>
</tr>
<tr>
<td>Government Investment</td>
<td>3.1</td>
<td>3.2</td>
</tr>
<tr>
<td>Consumption</td>
<td>65.8</td>
<td>65.3</td>
</tr>
<tr>
<td>Private Investment</td>
<td>15.7</td>
<td>16.1</td>
</tr>
</tbody>
</table>

The government has two types of expenditures: government consumption and government investment. Government consumption directly boosts the welfare since it appears in the utility function. This assumption can be justified by the goods and services that are provided by general government, like law enforcement service and public goods, which raise the overall welfare. On the other hand, government investment consists of purchase of new structures, equipment and software by the government which facilitate the firm’s production. Figure 3.1 shows the trade-off between the two choices. The output function and the utility are strictly concave in
government investment and government consumption, respectively. So taking the tax
rates as fixed for now, increasing government investment from a low level increase
welfare significantly because the marginal product of government investment is large
and the marginal utility of government consumption is relatively low. As government
investment increases, the welfare increases and reaches the peak, then starts to fall
at an accelerating pace.

To further illustrate the magnitude of welfare costs, I use Lucas measure of
welfare loss in much of the welfare analysis. Lucas [50] measures the welfare cost of
consumption fluctuations as the percentage increase in consumption, across all dates
and states, required to leave a representative agent indifferent between consumption
fluctuations and a smooth consumption path. According to this definition, he obtains
an estimate of 0.042% of consumption in the US.\(^5\) In his measurement Lucas employs
a model based on homogenous agent, perfect capital markets, trend-stationary con-
sumption process and a CRRA utility function. A large body of research focuses on
altering the model framework and remeasure the welfare cost.\(^6\) In this literature, the
welfare cost of consumption fluctuations ranges from 0.1% to 4.31%, depending on
the utility function form and specification of the relative risk aversion.

In much of the welfare analysis I employ this method to quantify the welfare
cost. First I choose the benchmark welfare level (usually the optimal welfare level
or the welfare with baseline parameters is used). Then for all the rest of the welfare
levels, I calculate how much additional percentage of consumption is required to leave
the representative household indifferent between the welfare level and the benchmark
welfare level. This percentage of consumption measures the welfare cost of deviating
from the benchmark welfare level. The cost could also be negative, indicating a
welfare improvement, as can be seen in some cases later.

In Figure 3.2 I depict the welfare obtained for different \(\bar{i}_g/\bar{y}\)'s in Lucas measure of
welfare loss. As discussed above, I obtain the welfare loss by calculating the additional
percentage of consumption that is required to make the representative indifferent
\(^5\)This result is obtained with a coefficient of relative risk aversion equal to 5. For risk aversion
levels of 10 and 20, Lucas estimates the welfare cost of fluctuations equal to 0.084% and 1.7% ,
respectively.
\(^6\)See e.g., Obstfeld [61], Krusell et al. [48]
between each welfare level against $\tilde{\tau}_g/y$ and the optimal welfare (at $\tilde{\tau}_g/y=0.03$). I find that the magnitude of welfare loss is significant. The welfare cost is almost 1% of consumption if the government investment is reduced to less than 2% or raised to more than 5% of GDP. Compared to 0.042%, the welfare cost of eliminating all economic fluctuations in Lucas model, this welfare loss of deviating from optimal public investment is quite large. If we also notice the coefficient of risk aversion is just 1 in our model, compared to a level of 5 in Lucas’ model, the degree of welfare loss might be even larger. The public investment-consumption choice has significant effects on the overall welfare.

![Trade-off between ig and g](image)

Figure 3.2: Welfare loss against ig/y($\tau_n = 0.28, \tau_k = 0.36$)

### 3.5.2 Steady State Laffer Curves

In this section I characterize the Laffer curves for labor taxes and capital taxes. Laffer curves are obtained by varying the steady state labor/capital tax rate, while holding the capital/labor tax rate and parameters fixed. However, Laffer curves in my model differ from those in the previous literature (e.g. Trabandt and Uhlig [71, 72]) in that the government optimally allocate public consumption and investment for any labor and capital income tax rates in steady states. It means for different tax revenues with varying tax rates, the government spending and investment will both change. In the Trabandt and Uhlig [71], productive capital is absent and government spending adjusts corresponding to various tax revenues on the Laf-
fer curve. Implicitly, it is equivalent to adopting a fiscal rule which is always to fix government investment level and to change only government spending in our model. Recall that government capital works like a productivity shift from the private sector: 

\[ y_t = z_t(k_{g,t})^\gamma k_i^\theta n_i^{1-\theta} = A_t k_i^\theta n_i^{1-\theta}, \]

where \( A_t \equiv z_t(k_{g,t})^\gamma \). If public capital is non-productive, we can also interpret it as public capital always stays unchanged at benchmark. Therefore, Laffer curves constructed at fixed government investment can be thought of as the ‘traditional’ Laffer curves.

![Laffer Curve For Labor Taxes](image1)

(a)

![Laffer Curve For Capital Taxes](image2)

(b)

Figure 3.3: Laffer Curves For Labor and Capital Taxes

Figure 3.3a and 3.3b depict the steady state Laffer curves for labor and capital income taxes, respectively. The dotted lines in both figures depict the Laffer curves
under fiscal rules of fixing government investment, corresponding to the traditional literature. The solid lines are the Laffer curves under optimal allocation rules. The tax revenues at the calibrated tax rates ($\tau_n = 0.28$, $\tau_k = 0.36$) are taken as 100, which are marked red in both figures. First, we can see both current US labor and capital tax rates are on the left of all the Laffer curves. This finding is consistent with Trabandt and Uhlig [71] and Trabandt and Uhlig [72]. Second, I find that our model predicts that US can increase tax revenues by 94.2% by raising labor taxes and 12.5% by raising capital income taxes, while the traditional Laffer curves indicate that US tax revenues can be increased by around 87.5% and 11.5% through raising labor taxes and capital taxes, respectively. The peaks of Laffer curves occur at labor and capital tax rates both around 0.73.

Our model predicts a slightly larger percentage of tax revenues that can be collected by either labor or capital taxes. The increase in the peak reflects the role of productive public capital. It works as a productivity shift to the private sector and rises real wages and dividends. Suppose the economy is at some point on the left of Laffer curves. Given a marginal increase in the tax rate results in a rise in tax revenues. In our model, the government will optimally allocate the additional revenues between the two expenditures and this leads to increase in both of them. Compared to a traditional model in which government only increases government spending, the tax revenues increase at a faster pace because the government investment raises real wages and real dividends. This explains the gap between the two Laffer curves.

Next, I conduct experiments to investigate the long run macroeconomic effects of the taxes. I allow labor(capital) taxes to vary, while keeping the capital(labor) tax rate fixed. Still, the government adopts optimal government expenditure allocation rule in the steady state for any given level of tax revenues.

Figure 3.4 shows the effects of labor taxes on steady state output, private investment, consumption and hours worked. Except for hours worked, all of them are presented in percentage by taking the level at $\tau_n = 0.28$ as 100. What is worth noted is that hump-shape output and private investment curves are observed. Thus up to some point, increasing labor tax rates will boost the output and private investment in the steady state. It might be somewhat surprising at the first glance,
because straightforwardly, levying labor taxes reduces the real wage and thus suppresses the output by reducing labor supply. A reduction in labor supply also reduces marginal product of capital and thus tends to suppress private investment. But at the same time, government raises the government capital stock through public investment, which has a force to drive up the marginal product of capital. When labor tax is small, the government has a relatively low public capital stock and thus the marginal product of public capital is large. It offsets the reduction of hours worked so that the total output actually increases at the beginning. As labor tax rate keeps increasing, government capital stock is also growing so the marginal product of public capital keeps falling, then at some point it can no longer offset the reduction in hours worked. This explains a hump shaped output. Consumption level also exhibits a small hump shape when labor taxes are quite small. Consumption decreases at a faster rate than output because government spending is increasing in this process, which absorbs resources and suppresses consumption.

![Figure 3.5: Optimal Labor Tax](image)

I find that if we keep capital tax fixed at 0.36, the optimal steady state labor tax rate is equal to 0.27 which is very close to the average of observations in US data.

---

7The private investment level is also hump shaped because in the steady state with capital income taxes fixed, capital stock to output ratio always remains a constant, while output exhibit hump-shape. See Appendix B.1 for more details
It implies the US long run labor tax rate is roughly at the optimal, holding capital tax rate at 0.36. The optimal output, however, is not the maximized output level, although they are very close. The steady state output peaks at labor tax rate equal to 0.26. The reason for the little discrepancy between production peak point and welfare maximizing point is due to the utility enhancing government spending, which offsets the welfare loss of the non-maximized output. Figure 3.5 depicts the welfare loss in terms of percentage of consumption.

Figure 3.6 illustrates the effects of capital taxes on steady state output, private investment, consumption and hours worked, while fixing labor income taxes. Except for hours worked, all of them are presented in percentage by taking the level at $\bar{\eta}_k = 0.36$ as 100. The effects of capital taxes exhibit different patterns from those of labor taxes. Hump shapes are absent here. Increasing capital tax rates reduces real dividends on capital and suppresses private investment. But at the same time, government investment increases because of more tax revenues which tends to increase the real wage and the real dividends on capital. However, this tendency cannot fully offset the tendency to reduce private capital. Thus private investment is decreasing against capital taxes. Output is also dominated by the reduction of private capital.
Effects on labor supply are a little complicated. Increase in government investment raises the real wage, while decrease in private investment reduces the real wage. Note income effect also affects labor supply when capital taxes change. Numerically we find that labor supply is increasing against capital taxes. The household is poorer as capital taxes rise and tends to work more at the same real wage. Since the effects of the private investment decrease are large relative to those of government investment increase on real wages (output is monotonously decreasing), it must be that the income effect dominates so that labor supply is increasing.

![Graphs showing the effects of capital tax on output, consumption, private investment, and hours worked.](image)

**Figure 3.6: Effects of Capital Tax**

When we hold labor tax rate fixed at 0.28, the optimal capital tax rate is 0.20. I measure the welfare loss in Figure 3.7. Again, the optimal output is not the maximized level. It decreases at an accelerating pace against capital taxes. An additional 0.87% of consumption is needed to make the representative indifferent between the current welfare level and the optimal. It means the distortion of imposing capital taxes that deviate from the optimal level is large. Also, welfare cost of deviating from the optimal
capital tax does not increase as fast as the cost of deviating from the optimal labor tax.

![Figure 3.7: Optimal Capital Tax](image)

### 3.5.3 Optimal Taxation

In this section, I examine the joint optimality of the two tax rates. I relax both tax rates and find the best combination of \( \tau_n, \tau_k, \bar{y}, \) and \( \bar{i}_g \) that maximizes the welfare in the steady state. The results are summarized in Table 3.4.

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Baseline</th>
<th>Optimal Tax</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tau_k )</td>
<td>0.36</td>
<td>0.36</td>
<td>0</td>
</tr>
<tr>
<td>( \tau_n )</td>
<td>0.28</td>
<td>0.28</td>
<td>0.33</td>
</tr>
<tr>
<td>( \bar{i}_g/\bar{y} )</td>
<td>3.1%</td>
<td>3.2%</td>
<td>2.9%</td>
</tr>
<tr>
<td>( \bar{c}/\bar{y} )</td>
<td>15.7%</td>
<td>16.1%</td>
<td>20.4%</td>
</tr>
<tr>
<td>( \bar{c}/\bar{y} )</td>
<td>65.8%</td>
<td>65.3%</td>
<td>62.5%</td>
</tr>
<tr>
<td>( \bar{g}/\bar{y} )</td>
<td>15.4%</td>
<td>15.4%</td>
<td>14.2%</td>
</tr>
</tbody>
</table>

Now the government not only faces the trade-off between public consumption and public investment, and also chooses the optimal labor and capital taxes in the
steady state. As mentioned in the previous section, this is not a Ramsey problem. In this problem, we are trying to identify the long run optimal public consumption, public investment and taxes, while keeping government transfers and debts on the balanced growth path. Ramsey problem searches for the competitive equilibrium that maximizes the overall welfare out of all competitive equilibria, which is a fully characterized dynamic problem and all fiscal policy variables are endogenous to the government, including government debts. Solution to Ramsey problem suggests the government should accumulate financial claims so that government expenditures can be repaid with interest earnings. However, it is rarely observed in reality. Political economists find the government has excessive debt accumulations with the presence of moral hazard, which provides a good explanation for the observed facts in US historical data. Thus, I assume the government debt is exogenously fixed at calibrated level, so that the optimization problem provides more practical policy implications than Ramsey problem.

I find it particularly interesting to see the optimal capital tax is zero, while the labor tax is 0.33. This result coincides with the seminal paper by Chamley [21], in which he illustrates that the optimal long-run capital tax should be zero by solving a Ramsey problem. The optimal policy in Ramsey problem is to amass claims on the private economy initially under a transition period, accumulate assets and finally use only interest earnings to finance the government expenditures and levies no capital tax. This result is proved robust in the literature. Still, it is somewhat surprising to reach the same conclusion in our model. Government transfers and government debt are are usually considered implicit and explicit liabilities. They are exogenous in the model. The government has to resort to distortionary taxation to meet these obligations and to make government expenditures. Zero capital taxes are still found optimal so that all the tax revenues are collected by labor income taxes. Since the degree of distortion from levying labor taxes is much less than that from imposing capital income taxes, it implies that the US government should relax the current capital taxes and compensate the revenues through a correspondent increase in labor taxes. The labor supply is not very elastic to prices, as illustrated in labor supply curve in Figure 3.4. Also, in Table 3.4, labor taxes increase from 0.28 to 0.33 in the last column, a 18% increase, but only result in 1 percentage point decrease in labor
supply. This explains the large potential tax revenues and a steep Laffer curve for labor taxes.

3.6 Conclusion

It still remains somewhat controversial to evaluate the contribution of government expenditures to the economy in the current literature. This paper provides an attempt to quantitatively measure the macroeconomic effects of government expenditures as well as other fiscal variables. Utility-enhancing public spending and productive government investment make the expenditures measurable from welfare perspective. The model is calibrated to the US economy with the implied assumption that the US is actually optimally allocating government spending and investment in the long run. Under this assumption I characterize Laffer curves and find they can reach higher peaks than the traditional counterparts. I also find when fixing capital tax rate at the observed average, 0.36, output exhibits a hump shape for varying labor taxes. It means up to some point, raising labor taxes will boost output, resulting from productive government capital.

A zero capital tax rate is found optimal when I search for long run fiscal policies that maximize the welfare. It is consistent with the long existing literature of optimal taxation theory. However, since I assume government debt and transfer payments are fixed and exogenous to the government, I am not trying to solve the Ramsey problem and our conclusion is sensitive to calibrated parameter values. It implies that the degree of distortion from labor taxes is relatively smaller than capital taxes so that the government should rely more on labor taxes to finance itself in the long run. When the government is highly indebted in the model, it will choose to mildly cut government spending but drastically raise taxes in the long run. Cutting government expenditures would incur such a large welfare loss that the government would choose to impose higher labor taxes.

This paper has several possible directions for future studies. First, this paper only addresses the long run effects of fiscal policies. Transitional paths to the long run equilibrium remain further explored. Second, an extended model with human capital may better characterize the effect of labor taxes. The presence of human capital may
provide more insight into the effects of labor taxes on economic growth and welfare level. Discussion along this direction is provided in the next chapter.
CHAPTER 4
Laffer Curve Revisited: the Role of Productive Government Spending

4.1 Introduction

How do tax revenues change, as labor or capital taxes are adjusted? To answer this question, existing literature often illustrates by depicting a Laffer curve, i.e. a representation of the relationship between possible rates of taxation and the resulting levels of tax revenue. For example, Trabandt and Uhlig [72] establish a standard neoclassical growth model and explicitly characterize the Laffer curves for labor and capital income taxation for US and EU economy. In most of the literature, the role of government is usually maintained at minimal level: collecting distortionary taxes and making exogenous public purchases. However, tax receipts are mostly spent by the government on providing public goods, subsidizing individual’s education and social securities, and transfer payments, which are all productive to the economy. What’s the role of productive government expenditures in shaping Laffer curves? To answer this question, I extend the current literature to investigate the role of valued government expenditures in characterizing Laffer curves and optimal long-run fiscal policies.

In this chapter, I consider two types of government spending that yields benefits, the utility enhancing government spending and subsidies to education. I set up a model extended with the human capital sector in which human capital exhibits technological spillovers. The government subsidizes human capital investment and makes valued public spending. The benevolent government funds itself through raising tax revenues and issuing bonds, and endogenously determines the allocation between public spending and government subsidies after repaying exogenous transfer payments and debts. Human capital exhibits externalities in production function, and subsidies to human capital investment is available to reduce the magnitude of distortion. The government is fully informed of the social benefit and cost, and allocate
the government expenditures to maximize the welfare in the long-run.

Trabandt and Uhlig [71, 72] quantitatively characterize the Laffer curves for US and EU economies from 1975-2000 and 1995-2007, respectively. They assume that government investment is absent and government spending is exogenous; in their models with human capital, human capital subsidies are also absent. These assumptions neglect the role of valued government expenditures, which could have a significant impact in shaping Laffer curves. In this paper, Laffer curves differ from those in previous literature in that fiscal policies on allocation of valued government expenditures alter the shape of Laffer curves. Subsidies affect human capital as well as total income and final output. Hence fiscal policies on public investment and government spending impact Laffer curves, as well as overall welfare. The models are calibrated to US quarterly economy. The government optimally allocates the two types of government expenditures for any given level of taxes in the steady state. I find Peaks and slopes of Laffer curves are sensitive to subsidies, and are raised much higher with larger subsidies. The large impact on the shape of Laffer curves implies that the government expenditures may have persistent effects on consumers’ income and tax revenues.

A number of papers have empirically assessed the strength of human capital externalities (e.g. Rauch [64]; Rudd [66]; Acemoglu and Angrist [1]; Moretti [58]; Ciccone and Peri [25]). This branch of literature investigates the effects of a change in average level of human capital on individual wages. In general, the current direct empirical evidence on the magnitude of human capital spillovers is mixed, depending on econometric methodology adopted and data sample. But strong evidence confirms the human capital spillovers in city-wide, especially measured using number of college graduates or years of schooling (Rauch [64]; Moretti [58]). The estimated magnitude of external returns is often comparable to that of private returns. Aggregate human capital externalities can also help explain cross-country differences in economic development, the effects of agglomeration on economic growth and other macroeconomic phenomena. Lucas [51] proposes a model (usually referred to as ‘Uzawa-Lucas model’) with average human capital in the aggregate production function, which suggests the human capital externalities play the ‘engine of growth’ and account for the
cross-country income differences. The externality property of human capital leads to underproduction of human capital, and determine the extent government should subsidize human capital accumulation. Gomez [35] proves that the first-best solution can be obtained through human capital subsidies in Uzawa-Lucas model with exogenous leisure. In this paper I show that human capital subsidies have significant impacts on Laffer curves and the overall welfare. Distortion of externalities can be reduced through subsidizing human capital investment.

This paper also investigate the optimal fiscal policies in the long-run. As noted by Trabandt and Uhlig [72], ‘the level of taxation which delivers maximum tax revenues is not in general a policy that maximizes welfare’. However, they do not provide welfare analysis to further discuss the issue. I explore further along this direction by numerically identifying combinations of fiscal policy variables that maximize the overall welfare. I find that raising capital taxes incur larger welfare loss than raising labor taxes, which is consistent with optimal taxation literature. Note that it is different from solving a Ramsey problem since the long-term debt and transfer payments are fixed and calibrated to US data. They are part of endogenous variables in Ramsey problem. Although the solution to Ramsey problem indicates that US should accumulate assets to fund itself instead of imposing distorting taxation, US has been running a budge deficit for most of years. This discrepancy still remains a controversial issue. Political economists have well established models with moral hazard to explain over-spending and over-borrowing.\(^1\) A discussion of interactions between voters and political agent is beyond the point of this paper. But the conclusion of this branch of literature provides reasonable explanation for the current US accumulated debts, which cannot be fully justified in Ramsey problem.

The paper is organized as follows: I introduce the model in section 4.2. Section 4.3 discusses the calibration and parameterizations. Section 4.4 presents the results. Section 4.5 concludes. Relevant details are included in the appendix.

\(^1\)See e.g. Alesina et al. [2], Battaglini and Coate [10].
4.2 The Model

In this section I consider an exogenous growth model with human capital as discussed in Mankiw, Romer, and Weil [53]. Output by the representative firm is produced using a constant-returns-to-scale production function with inputs of physical capital \((K_t)\), human capital \((H_t)\), and labor \((n_{w,t} \times L_t)\), where \(n_{w,t}\) is hours per worker and \(L_t\) is the number of workers. Technology, denoted by \(z_t\), grows at exogenous rate \(\xi\), such that \(z_t = \xi^t\). We augment the constant-returns-to-scale Mankiw-Romer-Weil production function with an externality in the aggregate level of per-worker human capital \((h_{a,t})\). Human capital production requires both physical investment, and time for schooling. This is in line with the seminal work by Ben-Porath [12], and several of the following, for example, Manuelli and Seshadri [54], Rebele [65] and Glomm and Ravikumar [34].

4.2.1 Household

A representative infinitely-lived household maximizes the discounted stream of utility arising from consumption and leisure, by solving the following problem:

\[
\max E_0 \sum_{t=0}^{\infty} \beta^t \{ U(c_t, n_{w,t}, n_{h,t}) + F(g_{c,t}) \}
\]

s.t. \( c_t + (1 - s_t)e_t + i_t + b_{t+1} = (1 - \tau_{n,t})(w_t n_{w,t} + w_{h,t} h_t) + (1 - \tau_{k,t})(d_t - \delta)k_t + \delta k_t + TR_t + R_{b,t} b_t + m_t + \Pi_t \)

\( k_{t+1} = (1 - \delta)k_t + i_t \) \hspace{1cm} (4.1)

\( h_{t+1} = (1 - \delta_h)h_t + e_t^{\omega_h} h_t^{\omega_h} (z_t^{1/1-\theta_k-\phi}) n_{h,t})^{1-\omega_k-\omega_h} \).

The representative household optimally chooses paths of consumption \(c_t\), education expense \(e_t\), labor supply \(n_{w,t}\), time for schooling \(n_{h,t}\), investment \(i_t\), capital \(k_{t+1}\), bond \(b_{t+1}\), human capital \(h_{t+1}\), subject to the flow budget constraint, physical and human capital accumulation equations. The household takes wage \(w_t\), human capital wage
$w_{h,t}$, rental rate $d_t$, labor and capital taxes $\tau_{a,t}$, $\tau_{k,t}$, education subsidies $s_t$, government consumption $g_{c,t}$ and gross interest rate $R_{h,t}$ as exogenously given. Capital taxes are imposed on dividends net-of-depreciation as in Prescott [62, 63] and Trabandt and Uhlig [72]. Physical capital and human capital depreciate at $\delta$ and $\delta_h$, respectively. $TR_t$ and $\Pi_t$ denote government transfer payments and gross profit. The household receives $g_{h,t} = s_t e_t$ units of government subsidies on education expenditures.

$m_t$ is earnings from net foreign assets, which could be either positive or negative. It is used to capture trade balance so that $m_t$ is equated to an economy’s net import/export. Along the balanced growth path, $m_t$ grows at the same rate as the home economy. As discussed by Trabandt and Uhlig [72], it is consistent with an open-economy interpretation with source-based capital income taxation, where the rest of the world grows at the same rate and features households with the same time preferences. The additional variable is necessary in calibrating the model, since in an open economy, net imports are the difference between expenditure by domestic residents and production.

Hours worked, $n_{w,t}$, time for schooling, $n_{h,t}$, and leisure, $1 - n_{w,t} - n_{h,t}$ are between 0 and 1, inclusive. Here the labor taxes are levied jointly on labor supply and time for schooling. $g_{c,t}$ represents direct government spending, which is assumed to be separable from the private sector in a utility function with unity relative risk-aversion coefficient:

$$U(c_t, n_{w,t}, n_{h,t}) + F(g_{c,t}) = \log c_t + \alpha_n \log (1 - n_{w,t} - n_{h,t}) + \alpha_g \log g_{c,t}$$

The human capital accumulation, equation (4.3), is constructed to acknowledge that expenditures on education ($e_t$), human capital ($h_t$), and hours spent on education per worker ($n_{h,t}$) are all inputs into the production function for human capital. Lucas [51] introduces a human-capital-based endogenous growth model in which human capital itself, augmented by hours devoted to human capital accumulation, is the argument in the human capital production. Mankiw, Romer, and Weil [53] do the opposite and assume that human capital is fully produced by expenditures, which they measure as education enrollment. Human capital production in our model includes
both human capital and expenditures as well as hours devoted to education. This three-factor production function is similar to those in Klenow and Rodriguez-Clare [46], Ben-Porath [12], Rebelo [65] and Glomm and Ravikumar [34]. We assume that the production function for human capital is jointly constant returns to scale in these three factors. We also assume that a technology growth factor \( z_t^{1/(\theta_k - \theta_h - \phi)} \), based on the same technology growth factor as output, augments hours, allowing a balanced growth path with constant hours, and with expenditures on education and human capital growing at the rate of growth of the economy.\(^2\) Human capital depreciates at rate \( \delta_h \).

Jones [41] argues that time for schooling, or years of schooling, should be treated as an input to produce human capital, rather than a measurement of human capital stock. In fact, years of schooling is only a partial measure for the time component in human capital production. Human capital stock is considered as stocks of skills and knowledge so as to produce economic value. From standpoint of a nation, the human capital grows over time. So measurement of human capital by years of schooling is inappropriate since such average years will ultimately converge at some level rather than grow continuously. Additionally, the physical cost, \( e_t \), and human capital itself are necessary inputs in human capital sector.\(^3\)

### 4.2.2 Firm

The representative firm maximizes the profit by solving the following problem:

\[
\max_{k_t, n_{w,t}, h_t} y_t - d_t k_t - w_t n_{w,t} - w_{h,t} h_t
\]

The final product requires inputs of physical capital, human capital and labor supply. The total output, \( y_t \) is constant returns to scale with respect the three inputs as in the following fashion:

\[
y_t = z_t h_t^{\theta_k} h_t^{\theta_h, n_{w,t}^{1-\theta_k - \theta_h} h_t^{\phi}}
\]  

\(^2\)I would like to thank John B. Jones for suggesting this specification.
\(^3\)For example, see Kendrick [45], and Klenow and Rodriguez-Clare [46] for discussion of physical investment in human capital production.
where \( \theta_k + \theta_h + \varphi < 1 \). Here, \( h_{a,t} \) is average human capital, which the household treats as exogenous when making work-schooling-leisure decision. The human capital exhibits externality, hence the household and firm fail to fully perceive the contribution of human capital in production function. As a result, human capital is underproduced and it incurs welfare loss. \( z_t \) is total factor productivity with the exogenous growth factor: \( z_t = \xi^t \). Solved from the firm’s problem, wages and rental rates are expressed as:

\[
d_t = \theta_k \frac{y_t}{k_t};
\]

\[
w_t = (1 - \theta_k - \theta_h) \frac{y_t}{n_{w,t}};
\]

\[
w_{h,t} = \theta_h \frac{y_t}{h_t}.
\]

These prices from the perfectly competitive market produce zero profit:

\[
\Pi_t = y_t - d_t k_t - w_t n_{w,t} - w_{h,t} h_t = 0.
\] (4.5)

### 4.2.3 Government

We assume that government purchases three types of goods and services: utility-enhancing government consumption \( (g_{c,t}) \), education subsidies \( (g_{h,t}) \), and something exogenous \( (g_t) \). The government budget constraint requires that total expenditures on goods and services plus transfer payments and interest on government debt equal tax revenue plus new government debt. The budget constraint is given by

\[
g_{c,t} + g_{h,t} + g_t + TR_t + R_{b,t} b_t = T_t + b_{t+1},
\] (4.6)

where government spending on education \( (g_{h,t}) \) is given by

\[
g_{h,t} = s_t e_t,
\]

and tax revenues \( (T_t) \) are

\[
T_t = \tau_{n,t}(w_t n_{w,t} + w_{h,t} h_t) + \tau_{k,t}(d_t - \delta)k_t.
\] (4.7)
We assume that the exogenous government spending, transfer payments \((T R_t)\), and government debt \((b_t)\) all grow exogenously. Government subsidies to education \((s_t)\) and utility-enhancing government consumption \((g_{c,t})\) are chosen jointly optimally with tax rates \((\tau_{k,t}, \tau_{n,t})\) to balance the government budget. This assumption differs from Trabandt and Uhlig [72], who assume that all government spending and debt grow exogenously, requiring transfer payments to adjust endogenously.

Our assumptions on endogeneity are based on the following. We assume that the only reason government raises taxes is to provide benefits to agents who are being taxed. However, a representative-agent model does not capture some of these benefits very well. We set these benefits as exogenous. The primary purpose of government transfers is redistribution, implying that a representative-agent model can yield no information on their optimal level, justifying our assumption that they are exogenous. Additionally, government defense spending is chosen for reasons other than steady-state utility maximization. We set government debt as exogenous because government debt patterns are far from those predicted by the optimal tax literature.

### 4.2.4 Solution and Equilibrium

The first order conditions with respect to \(c_t\), \(i_t\) and \(e_t\), respectively, yield definitions of the multipliers according to

\[
\lambda_t = U_c(t), \quad (4.8)
\]

\[
\lambda_t = \zeta_t, \quad (4.9)
\]

\[
(1 - s_t) \lambda_t = \mu_t \omega_e e_t^{\omega_e} - 1 h_t^{\omega_h} \left( z_t^{1/(1-\theta_k-\theta_h)} n_{h,t} \right)^1 - \omega_e - \omega_h. \quad (4.10)
\]

Equation (4.8) states that the multiplier on the budget constraint is the marginal utility of consumption, as is standard. Equation (4.9) equates the marginal utility of capital with the marginal utility of consumption. The first order conditions with respect to expenditures on physical and human capital, equations (4.9) and (4.10), differ due to both the subsidy on education expenditures and the difference in production functions for human and physical capital. If \(s_t = 0\), \(\omega_e = 1\) and \(\omega_h = 0\), the two first order conditions would be identical, and the marginal utility of human
capital would equal the marginal utility of physical capital.

The first order conditions with respect to \( b_{t+1}, k_{t+1} \) and \( h_{t+1} \) yield Euler equations in each of the three assets according to

\[
\lambda_t = \beta E_t \{ \lambda_{t+1} R_{b,t+1} \},
\]

\[
\zeta_t = \beta E_t \{ \lambda_{t+1}[(1 - \tau_{k,t+1})\left(\theta_k \frac{y_{t+1}}{k_{t+1}} - \delta\right) + \delta] + \zeta_{t+1}(1 - \delta)\};
\]

\[
\mu_t = \beta E_t \{ \lambda_{t+1}\left[\theta_h (1 - \tau_{n,t+1}) \frac{y_{t+1}}{h_{t+1}}\right] + \mu_{t+1}[1 - \delta_h + \\
\omega_h e^{\omega_h} h_t^{\omega_h - 1} (z_t^{1/(1 - \theta_h - \phi)} n_{h,t+1})^{1 - \omega_h - \omega_h} \} \}
\]

where the agent takes \( h_{a,t} \) as exogenous in choosing \( h_t \). In equilibrium the two will be equal.

The first order condition on bonds is the standard Euler equation. Defining \( R_t \) as

\[
R_t = (1 - \tau_{k,t})(d_t - \delta) + 1,
\]

we can substitute into equation (4.12), using equation (4.9), to write the first order condition on capital as

\[
\lambda_t = \beta E_t \{ \lambda_{t+1} R_{t+1} \},
\]

It is useful to compare equations (4.12) and (4.13). The marginal utilities for physical capital and human capital have similar recursive expressions. Each equals the sum of the expected present value of marginal utility of income from the asset plus the marginal utility of human/physical in the next period. The marginal utility for human capital investment contains an additional positive term \( [\mu_{t+1}\omega_h e^{\omega_h} h_t^{\omega_h - 1} (z_t^{1/(1 - \theta_h - \phi)} n_{h,t+1})^{1 - \omega_h - \omega_h}] \). Since human capital is a factor of production in human capital, an additional unit adds to the productive capacity of human capital, raising the marginal product of human capital.

The first order conditions with respect to the allocation of hours, \( n_{w,t} \) and \( n_{h,t} \), can be expressed as

\[
\frac{\alpha_n}{1 - n_{w,t} - n_{h,t}} = \lambda_t w_t (1 - \tau_{n,t}),
\]

(4.15)
\[
\frac{\alpha_n}{1 - n_{w,t} - n_{h,t}} = \mu_t (1 - \omega_c - \omega_h) c_t^\omega c_t^\omega h_t^\omega \left( z_t^{1/(1 - \theta_k - \theta_h - \phi)} n_{h,t} \right)^{-\omega_c - \omega_h}. \tag{4.16}
\]

The marginal utility cost of hours, which is the marginal cost of giving up leisure for both of the two uses of hours, must equal the marginal utility benefit. For labor hours, the later is determined by the after-tax wage multiplied by the marginal utility of consumption. For school hours, the marginal utility benefit is the addition to human capital created by the additional study hours multiplied by the marginal utility of human capital.

Along the balanced growth path, \( n_{w,t}, n_{h,t}, R_t, R_{b,t}, \tau_{n,t}, \tau_{k,t} \) and \( s_t \) are stationary, and all other variables grow at the constant rate \( \psi = \xi^{1/(1 - \theta_k - \theta_h - \phi)} \). We require that government bonds, the exogenous component of government spending, transfer payments, and income from net foreign assets all grow exogenously at the rate of growth of the economy. In order to obtain stationary solutions, we detrend all the growing variables before solving for the system. We denote detrended growing variables with tildes. Non-growing variables are not detrended and retain their original notation.

All variables, which grow at the rate of growth of the economy, are detrended by dividing by \( \psi^t \) to yield \( \tilde{x}_t = x_t / \psi^t \), where the tilde denotes detrended values. Lagrange multipliers grow at a different rate. Multiplying both sides of equation (4.8) by \( \psi^t \) implies that \( \tilde{\lambda}_t = 1 / \tilde{c}_t = \psi^t \lambda_t \). Similarly, equations (4.9) and (4.10) imply that detrended multipliers are expressed as \( \tilde{\zeta}_t = \psi^t \zeta_t \) and \( \tilde{\mu}_t = \psi^t \mu_t \). Therefore all the Lagrange multipliers grow at rate \( \psi^{-1} \) along their balanced growth paths.

A competitive equilibrium is a set of plans \( \{\tilde{c}_t, n_{w,t}, n_{h,t}, \tilde{c}_t, \tilde{b}_{t+1}, \tilde{b}_{t+1}, \tilde{t}_t, \tilde{h}_{t+1}\} \) satisfying the detrended version of equations (1)-(15), given exogenous processes \( \{\tau_{n,t}, \tau_{k,t}, g_{c,t}, g_{h,t}, \tilde{s}_t, \tilde{m}_t, T\tilde{R}_t\} \) and the initial condition \( \{k_0, b_0, h_0, m_0, TR_0\} \). Detailed description of stationary equilibrium is provided in Appendix C.1.1.

4.2.5 Steady State

In the steady state, \( \tilde{x}_{t+1} = \tilde{x}_t \). We denote steady state values of the variables by dropping time subscripts to yield \( \bar{x} = \tilde{x}_{t+1} = \tilde{x}_t \). The full system of steady state equations are described in Appendix C.1.2. Here we present steady-state values of
interest for analysis and calibration. First, the steady state production function is

\[ \bar{y} = \tilde{k}^{\theta_{k}} \tilde{h}^{\theta_{h} + \phi} n_{w}^{1 - \theta_{k} - \theta_{h}}. \]  

(4.17)

Note that in this representative agent model: \( \tilde{h}_{t} = \tilde{h}_{a,t} \). In steady state, the equation for the accumulation of physical capital becomes

\[ (\psi - 1 + \delta) \frac{\bar{k}}{\bar{y}} = \frac{\bar{i}}{\bar{y}}. \]  

(4.18)

The steady state Euler equation with capital as the asset becomes

\[ \frac{\psi}{\beta} - 1 = (1 - \tau_{k}) (\theta_{k} \frac{\bar{y}}{\bar{k}} - \delta). \]  

(4.19)

These two equations link the net-of-tax return to the capital stock and physical investment. Combining the two equations yields the steady state share of investment as a percentage of output

\[ \frac{\bar{i}}{\bar{y}} = \frac{(\psi - 1 + \delta)(1 - \tau_{k}) \theta_{k}}{\psi/\beta - 1 + \delta (1 - \tau_{k})}. \]  

(4.20)

In the steady state, the investment-to-output ratio is increasing in capital’s share \( \theta_{k} \), and decreasing in the capital tax rate, \( \tau_{k} \).

The human capital Euler equation (4.13) in the steady state can be simplified by eliminating \( \lambda \) and \( \mu \), using equations (4.10) and (4.3), to yield

\[ \frac{\bar{e}}{\bar{y}} = \frac{(1 - \tau_{n})}{(1 - s)} \left[ \frac{(\psi - 1 + \delta_{h}) \omega_{h} \theta_{h}}{\psi/\beta - 1 + \delta_{h} - \omega_{h} (\psi - 1 + \delta_{h})} \right]. \]  

(4.21)

The long-run human capital expenditure-to-output ratio is increasing in human capital’s share and in \( \frac{1 - \tau_{n}}{1 - s} \).

Combining equations (4.15) and (4.16) yields the steady state ratio of hours allocated to education and to work as

\[ \frac{n_{h}}{n_{w}} = \frac{(1 - \tau_{n})}{(1 - s)} \left[ \frac{(1 - \omega_{e} - \omega_{h})}{\omega_{e} (1 - \theta_{k} - \theta_{h})} \right] \frac{\bar{e}}{\bar{y}}. \]  

(4.22)
This equation can be further simplified by eliminating \( \frac{\tilde{r}}{y} \), using equation (4.21), to yield
\[
\frac{n_h}{n_w} = \frac{\theta_h}{1 - \theta_k - \theta_h} \left[ \frac{(\psi - 1 + \delta_h)(1 - \omega_h - \omega_h)}{[\psi / \beta - 1 + \delta_h - \omega_h(\psi - 1 + \delta_h)]} \right].
\] (4.23)

This equation implies that the relative time allocation between work and school does not depend on labor taxes or subsidies, and is proportionate to their relative shares \( \frac{\theta_h}{1 - \theta_k - \theta_h} \).

The ratio of capital to output, capital intensity, is derived by solving equation (4.19) to yield
\[
\frac{\tilde{k}}{\tilde{y}} = \frac{(1 - \tau_k)\theta_k}{\psi / \beta - 1 + \delta(1 - \tau_k)}. \] (4.24)

The steady state value of human capital relative to output can be written as
\[
(\psi - 1 + \delta_h) \frac{\tilde{h}}{\tilde{y}} = (\frac{\tilde{e}}{\tilde{y}})^{\omega_h} (\frac{\tilde{h}}{\tilde{y}})^{\omega_h} (\frac{n_h}{\tilde{y}})^{1 - \omega_h - \omega_h}. \] (4.25)

Given solutions for the great ratios and for labor hours, detrended output in the steady state can be expressed as
\[
\tilde{y} = (\frac{\tilde{k}}{\tilde{y}})^{\theta_h/(1 - \theta_h - \theta_h - \phi)} (\frac{\tilde{h}}{\tilde{y}})^{(\theta_h + \phi)/(1 - \theta_h - \theta_h - \phi)} n_w^{(1 - \theta_h - \theta_h)/(1 - \theta_h - \theta_h - \phi)}. \] (4.26)

The consumption-to-GDP ratio can be solved from the equation for the consumer’s budget constraint in steady state, equation (4.1), yielding
\[
\frac{\tilde{c}}{\tilde{y}} = (1 - \tau_n)(\theta_n + \theta_h) + (1 - \tau_k)\theta_k + \tau_k \delta \tilde{k}/\tilde{y} + \tilde{T}/\tilde{y} + (\psi / \beta - \psi)\tilde{h}/\tilde{y} - \tilde{i}/\tilde{y} - (1 - s)\tilde{e}/\tilde{y} + \tilde{m}/\tilde{y}. \] (4.27)

Hours allocated to work and schooling, respectively, in the steady state can be solved by combining the steady state versions of equations (4.15) and (4.16)
\[
n_w = \frac{(1 - \theta_k - \theta_h)}{(1 - \theta_k - \theta_h) + \alpha_n \frac{\tilde{c}}{\tilde{y}} + \frac{(1 - s)\tilde{e}}{(1 - \tau_n)\tilde{y}} \frac{1 - \omega_h - \omega_h}{\omega_h}}. \] (4.28)
\[ n_h = \frac{(1-s) \frac{\bar{e} 1-\omega_c-\omega_h}{(1-\tau_n) \bar{y}}}{(1 - \theta_k - \theta_h) + \alpha_n \bar{y} + \frac{(1-s) \frac{\bar{e} 1-\omega_c-\omega_h}{(1-\tau_n) \bar{y}}}{\omega_c}}. \] (4.29)

Note that taxes and subsidies distort the decision on hours only due to the distortion on consumption since \( \frac{(1-s) \bar{e}}{(1-\tau_n) \bar{y}} \) is independent of both subsidies and taxes from equation (4.21).

Substituting into equation (4.7), detrended long-run tax revenues can be expressed as

\[ \tilde{T} = \tau_n (1 - \theta_k) \bar{y} + \tau_k \theta_k \bar{y} - \tau_k \delta \bar{k}. \] (4.30)

Assume the government is benevolent. In the steady state, the government’s optimal allocation problem can be defined as follows. Assume steady state transfer payments, debts, and foreign financial assets are exogenous. For any given tax rates, the benevolent government will allocate tax revenues between utility-enhancing government spending and education subsidies optimally by solving the following problem:

\[ \max V(s, \bar{g}_c) = \max \{ \log \bar{c}(s, \bar{g}_c) + \alpha_n \log (1 - n_w(s, \bar{g}_c) - n_h(s, \bar{g}_c)) + \alpha_g \log \bar{g}_c \} \]

s.t.

\[ \bar{g}_c + \bar{g}_h + \bar{y} + R_0 \bar{b} + TR = \psi \bar{b} + \tau_n (1 - \theta_k) \bar{y}(s, \bar{g}_c) + \tau_k \theta_k \bar{y}(s, \bar{g}_c) - \tau_k \delta \bar{k}(s, \bar{g}_c) \]

\[ \psi \delta \bar{k}(s, \bar{g}_c) = (1 - \delta) \bar{k}(s, \bar{g}_c) + \bar{i}(s, \bar{g}_c) \]

\[ \psi \bar{h}(s, \bar{g}_c) = (1 - \delta_h) \bar{h}(s, \bar{g}_c) + \bar{c}(s, \bar{g}_c) \omega_c \bar{h}(s, \bar{g}_c) \omega_h n_h(s, \bar{g}_c) (1 - \omega_c - \omega_h) \]

\[ \bar{y}(s, \bar{g}_c) + \bar{m} = \bar{c}(s, \bar{g}_c) + \bar{i}(s, \bar{g}_c) + \bar{c}(s, \bar{g}_c) + \bar{y} + \bar{g}_c \]

\[ \bar{g}_h = \bar{c}(s, \bar{g}_c) s \]

\( V(s, g_c) \) is the indirect utility function that solves the consumer’s problem as a function of \( s \) and \( g_c \) given \( \tau_n, \tau_k, TR, m \) and \( b \).
4.2.6 Externalities and Subsidies

Since the private sector fails to perceive the externality, the price of raising human capital deviates from the social return, resulting in underproduction. The distortion of human capital externalities is illustrated by introducing a central planner’s problem in Appendix C.2. We denote steady-state values for the planner’s detrended optimal values with hats. The planner’s optimal choice for \( k/y, e/y, n_h \) and \( n_w \) can be expressed as

\[
\frac{\dot{k}}{\dot{y}} = \frac{\theta_k}{\psi/\beta - 1 + \delta} \tag{4.31}
\]

\[
\frac{\dot{e}}{\dot{y}} = \frac{\omega_e (\theta_h + \phi)(\psi - 1 + \delta_h)}{[\psi/\beta - 1 + \delta_h - \omega_h(\psi - 1 + \delta_h)]} \tag{4.32}
\]

\[
n_w = \frac{(1 - \theta_k - \theta_h)}{(1 - \theta_k - \theta_h) + \alpha_n \frac{\dot{e}}{\dot{y}} + \frac{1-\omega_e-\omega_h}{\omega_e} \frac{\dot{e}}{\dot{y}}} \tag{4.33}
\]

\[
n_h = \frac{1-\omega_e-\omega_h}{(1 - \theta_k - \theta_h) + \alpha_n \frac{\dot{e}}{\dot{y}} + \frac{1-\omega_e-\omega_h}{\omega_e} \frac{\dot{e}}{\dot{y}}} \tag{4.34}
\]

where hours are understood to be the planner’s choices and are not given hat notation since they are not detrended.

We illustrate the distortions due to taxation and externalities by comparing the planner’s optimal choices with solutions in decentralized economies. Equations (4.24) and (4.31) can be used to demonstrate that the capital-to-output ratio in the decentralized economy is too low when capital taxes are positive. From (4.21) and (4.32), the share of expenditures on education is proportional to its optimal value according to

\[
\frac{\bar{e}}{\bar{y}} = \frac{(1 - \tau_n)}{(1 - s)} \left[ \frac{\theta_h}{\theta_h + \phi} \right] \frac{\dot{e}}{\dot{y}}.
\]

When subsidies and taxes are zero, the externality implies that agents choose too little expenditure on education. Taxes and subsidies can be chosen to offset this externality exactly. However, this leaves the consumption decision distorted, since from equation (4.27), consumption relative to income depends on the levels of taxes and subsidies. The distortion in consumption relative to income implies a distortion in hours worked and in hours allocated to human capital development from equations...
(4.33) and (4.34).

4.3 Calibration and Parameterization

The model is calibrated to the yearly US economy. The data starts from 1995 to 2007. All the data for calibration come from Federal Reserve Economic Database, the World Bank Database, National Income and Product Accounts (NIPA) Data, and the Digest of Education Statistics as described in Appendix C.3.

Tax rates: as a usual caveat in the literature (e.g. Trabandt and Uhlig [72]), we use flat taxes rather than marginal tax rates. We use the calculations by Trabandt and Uhlig [72] that calculate average tax rates for 1995-2007 using the prevalent calculation methodology proposed by Mendoza, Razin, and Tesar [57]. They find average capital income taxes and labor income taxes as: $\tau_k = 0.36$, $\tau_n = 0.28$.

The exogenous balanced growth factor, $\psi$ is set to 1.03, corresponding to an annual real GDP growth rate of 3% from data. For interest rate, I average annual real interest rates over 1995-2007 from the world bank database to yield $R_b = 1.048$. $\beta$ is calibrated from the steady state of equation (4.11):

$$\psi/\beta = R_b.$$  

Capital depreciation rate is calibrated from the steady state of capital production function (4.18). Capital stock and investment data is from NIPA data. All capital and investment, both public and private in the economy are included to calibrate $k$ and $i$, respectively. Hence $\delta$ can be solved in equation (4.18): $\delta = 0.04$

The government transfer payments, $\bar{T}/\bar{y}$ is calibrated using data of government social benefits to persons from NIPA. Based on the calculation in our sample over 1995-2007, I calibrate this ratio $\bar{T}/\bar{y} = 0.11$. $\bar{m}/\bar{y}$ is calibrated as 3.8%, as the average ratio of the net import to GDP for US in our sample. $\bar{b}$ is calibrated as 38% of GDP, according to historical data of federal debt held by the public.$^4$

$^4$An alternative is the gross debt to GDP ratio for US, i.e. 63% in our sample. Quantitative analysis shows no noticeable changes to the results.
The parameter on capital in production function, $\theta_k$, is calibrated from equation (4.19), which can be rewritten as:

$$
\theta_k = \frac{\psi/\beta - 1 + \delta(1 - \tau_k) \bar{k}}{1 - \tau_k} \frac{\bar{k}}{\bar{y}}
$$

(4.35)

where $\bar{k}/\bar{y}$ is calibrated from NIPA data, averaged over the period. Substituting for values of parameters on the right hand-side yields $\theta_k = 0.33$.

To the best of our knowledge, there is yet no literature that provides directly relevant estimate for the parameters, $\theta_h$ and $\phi$, in the human-capital production function. Related literature includes Mankiw, Romer, and Weil [53], which estimates a constant returns-to-scale production function with human capital in a cross-country regression. They find that the coefficient on human capital is 0.28. However, their estimate has been criticized for failing to account for country-specific effects. OLS estimates are biased when unobserved effects are correlated with the right-hand-side variables, in this case, the factors of production. Islam [39] has argued that the extent of the bias could be large. Lucas [51] proposed a different production technology and estimated the coefficient on labor (and also the coefficient on human capital) as 0.75; the externality term has a coefficient of 0.417. However, since the model framework is different, it has limited relevancy to our calibration.

In Chapter 2, I proposed a dynamic panel framework to capture the unobserved country-effects. After accounting for the unobserved country effects, the estimated coefficient on human capital is reduced from 0.28 in Mankiw, Romer, and Weil [53] to 0.23. This finding is in line with other empirical results which find that the coefficient on human capital is much lower than one-third.\(^5\) Additionally, since there is an externality in this model, the total coefficient on human capital is $\theta_h + \phi$ as in equation (4.17), which corresponds to the empirical estimate of human capital coefficient in Chapter 2. Therefore, we calibrate $\theta_h + \phi = 0.23$ and use this estimate in the benchmark analysis.\(^6\)

\(^5\)For example, see Islam [39] and Caselli, Esquivel, and Lefort [19].
\(^6\)The estimation is based on a sample of 89 countries excluding major oil-producers over 1970-
To estimate the coefficient on human capital expenditures ($\omega_e$) and on human capital ($\omega_h$), we adopt the methodology used in Chapter 2 that estimates the coefficients of the human capital production function in a single cross-country regression over 1970-2010. Following his framework, we run the cross-country regression using our human capital accumulation function, over the sample period 1995-2007. Details are contained in Appendix C.4. The regression implies calibrations of $\omega_e = 0.11$, and $\omega_h = 0.61$.

We assume $\delta_h = \delta$, consistent with Mankiw, Romer, and Weil [53] and Trabandt and Uhlig [72].

To separate the externality ($\phi$) from human capital’s share ($\theta_h$), we use equation (4.23) and calibrate $n_h/n_w$, the ratio of the two time inputs in the steady state. The estimates of the parameters $\omega_e$ and $\omega_h$ for the working-age population (age 15-64). Thus, the sample population excludes agents less than 15 years old, most of whom are in school and benefiting from the education subsidies. The presumption is that an agent must acquire education prior to high school to be able to benefit from high-school and higher education. Therefore, he must incur expenditures for all education, but the hours that count toward accumulating human capital are only hours beginning with age 15. According to Barro and Lee’s education attainment data, the average number of years of secondary and tertiary schooling for the US over the sample 1995-2005 is 6.8 years. If the worker devotes the remaining years to work, the years of work will be $64 - 15 - 6.8$, or 42.2 years. This results in school-to-work ratio of 6.8/42.2, or 0.16. Therefore we calibrate $n_h/n_w$ to 0.16, and substitute into equation (4.23), yielding $\theta_h = 0.18$. Hence $\phi$ is calibrated at $0.23 - 0.18 = 0.05$.

$\alpha_n$: I calibrate $\alpha_n$ from the steady state of equation (4.15):

$$\frac{\alpha_n}{1 - \pi_w - \pi_h} = (1 - \theta_k - \theta_h)(1 - \bar{T}_n)\frac{1}{\bar{n}_w \bar{c}/\bar{y}}$$ (4.36)

Hours worked $\bar{n}_w$ is calibrated to 0.25, consistent with Prescott [62, 63], McGrattan

---

2010. As a robustness check, we adopted the same methodology and estimated the coefficients over 1995-2007. The coefficients are almost identical. In particular, the estimated coefficient on human capital is 0.24.

7The education attainment data set is provided at www.barrolee.com, ‘Education Attainment for Population Aged 15 and Over’.
and Rogerson [56] and Trabandt and Uhlig [72]. Following Trabandt and Uhlig [72],
Given that \( n_h/n_w = 0.16 \), hours for schooling is calibrated at 0.04. The consumption-
to-GDP ratio \( c/y \) is calibrated to 66.9% using NIPA data averaged over 1995-2007.\(^8\)
Substituting these values into equation (4.36) yields \( \alpha_n = 1.50 \).

\( \alpha_g \): Given tax rates fixed at calibrated values \((\tau_n = 0.28, \tau_k = 0.36)\), debts and
transfer payments calibrated to data, the government needs to allocate remaining
tax receipts between government consumption expenditures and subsidies. We split
government consumption into an exogenous component \((g)\) and a component which
yields utility \((g_c)\). We let defense spending be the exogenous component and calibrate
\( g \) to be the average of defense spending over the sample yielding 4.3% of GDP. Assume
the government is benevolent, i.e. the allocation aims to maximize the overall welfare.
The values for \( \alpha_g \) and \( s \) are chosen to match the ratio of government spending on
government consumption net of defense and education expenditures, both as a fraction
of GDP. We choose a value for \( \alpha_g \), and calculate the optimal value of subsidies with
taxes fixed at their historical levels. Iterating over values for \( \alpha_g \), we find pairs of \( \alpha_g \)
and \( s \) and choose the one which generates values for \( g_c/y, g_h/y, \) and \( e/y \) closest to
the data. We choose \( \alpha_g = 0.09 \) and \( s = 0.69 \).

Table 4.1 summarizes parameters in the benchmark model calibration. Table
4.2 compares the baseline model predictions with data over 1995-2007. We find the
model prediction fits the data well.

4.4 Results

4.4.1 Steady State Laffer Curves

I characterize the Laffer curves for labor taxes and capital taxes. Laffer curves
are obtained by varying the steady state labor (capital) tax rates, while holding the
capital (labor) tax rate and rest of the parameters fixed. In Trabandt and Uhlig [72],
government expenditures are held exogenous, and transfer payments are endogenous
according to the government budget constraint. In our model, the benevolent govern-
\(^8\)Private spending on education has been deducted from private consumption expenditures; it is
calculated as the difference between total education expenditures and public spending on education.
Table 4.1: Baseline Calibration

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
<th>Description</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\psi$</td>
<td>1.03</td>
<td>Exogenous growth rate</td>
<td>Data</td>
</tr>
<tr>
<td>$R_b$</td>
<td>1.048</td>
<td>Real interest rate</td>
<td>Data</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.9828</td>
<td>Subjective discount rate</td>
<td>$\beta = \psi / R_b$</td>
</tr>
<tr>
<td>$\alpha_n$</td>
<td>1.50</td>
<td>Parameter on leisure in utility</td>
<td>Equation (4.15)</td>
</tr>
<tr>
<td>$\alpha_g$</td>
<td>0.09</td>
<td>Parameter on $g_{c,t}$ in utility</td>
<td>Benchmark</td>
</tr>
<tr>
<td>$\theta_k$</td>
<td>0.33</td>
<td>Parameter on $k_t$ in production</td>
<td>Equation (4.35)</td>
</tr>
<tr>
<td>$\theta_h$</td>
<td>0.18</td>
<td>Parameter on $h_t$ in production</td>
<td>Equation (4.23)</td>
</tr>
<tr>
<td>$\phi$</td>
<td>0.05</td>
<td>Parameter on $h_{a,t}$ in production</td>
<td>Chapter 2</td>
</tr>
<tr>
<td>$\omega_e$</td>
<td>0.11</td>
<td>Parameter on $e_t$</td>
<td>Appendix C.4</td>
</tr>
<tr>
<td>$\omega_h$</td>
<td>0.61</td>
<td>Parameter on $h_t$</td>
<td>Appendix C.4</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.04</td>
<td>Depreciation rate for capital</td>
<td>Data</td>
</tr>
<tr>
<td>$\delta_h$</td>
<td>0.04</td>
<td>Depreciation rate for human capital</td>
<td>$\delta_h = \delta$</td>
</tr>
</tbody>
</table>

Table 4.2: Comparison of Model Prediction and Data (% of GDP)

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Baseline</th>
</tr>
</thead>
<tbody>
<tr>
<td>Government Non-defense Consumption ($g_{c}/y$)</td>
<td>5.7</td>
<td>5.6</td>
</tr>
<tr>
<td>Public Spending on Education ($g_{n}/y$)</td>
<td>5.2</td>
<td>5.0</td>
</tr>
<tr>
<td>Defense Expenditure ($g/y$)</td>
<td>4.3</td>
<td>4.3</td>
</tr>
<tr>
<td>Education Expenditure ($e/y$)</td>
<td>7.2</td>
<td>7.2</td>
</tr>
<tr>
<td>Investment ($i/y$)</td>
<td>19.7</td>
<td>20.1</td>
</tr>
<tr>
<td>Consumption ($c/y$)</td>
<td>66.9</td>
<td>66.7</td>
</tr>
<tr>
<td>Net Import ($m/y$)</td>
<td>3.8</td>
<td>3.8</td>
</tr>
</tbody>
</table>

ment optimally allocates public consumption and subsides for any labor and capital income tax rates in steady states, while keeps transfer payments at a fixed percentage of GDP. That means at different level of tax rates, government spending and subsidies will both change so as to maximize overall welfare. Shapes of Laffer curves are thus sensitive to fiscal policies, i.e. government’s allocation decision of expenditures.

First, I graph the Laffer curve for labor income taxes. At every point along the curve, government spending and subsidies are optimally allocated. I also consider cases of fixed subsidies at the benchmark subsidy rate, i.e. the subsidy rate at current labor and capital tax rates under optimal allocation scheme ($s = 0.69$), and government consumption varies along the curve as taxes change. Tax revenues at $\tau_n = 0.28$ are set as the benchmark and equal to 100. As shown in Figure 4.1, Laffer curve
with optimal subsidies can raise additional 74% tax revenues, while the government has only 43% more tax receipts when government fixes subsidy rate at 0.69. Under optimal allocation, the peak occurs at the labor tax rate equal to 0.69, implying that substantial increases in the tax rate would continue to yield increases in tax revenues; when subsidy rate is fixed at benchmark, the peak occurs at a labor tax rate of 0.63.

![Laffer Curves for Labor Taxes](image)

**Figure 4.1: Laffer Curves for Labor Taxes**

The allocation decision is crucial in that subsidies greatly impact the household’s income and welfare, hence significantly raise the slope and peak of the Laffer curve. As in Figure 4.2, under optimal allocation the optimal subsidy rate is rising rapidly to approach 100% as long-run labor tax rate increases. This implies that expenditures on human capital are increasing rapidly in the tax rate, peaking at an additional 164%. Human capital subsidies also argly impact production through encouraging human capital production. As depicted in Figure 4.3, production increases as the labor tax rate rises from a low rate, eventually peaking and falling. Under full allocation of marginal revenue to government spending, production falls immediately as the tax rate increases, illustrating the importance of the allocation of marginal tax revenue.
Figure 4.2: Optimal Subsidy Rate

Figure 4.3: Production For Labor Taxes

Figure 4.4 summarizes the changes of human capital stock, physical capital, time for schooling and hours worked against labor taxes. Human capital stock could rise up by about 15% at the peak. Given that physical and human capital are complementary, the physical capital is also hump-shaped largely due to a sharp rise in human capital, peaking at only about 1% higher. The two time inputs decrease as labor taxes rise. Human capital subsidies do not encourage time for schooling, but only raise the physical input to human capital. The higher labor tax rate discourages hours allocated to both work and school because the ultimate return on both depends on the after-tax wage.
Figure 4.4: Effects of Labor Taxes

Figure 4.5 displays the two types of government expenditures. As the labor tax rate rises, both the value of subsidies to human capital and expenditures on utility-enhancing government spending exhibit a hump shape under optimal allocation. Government subsidies rise by a larger percentage than government consumption does. From welfare perspective, it indicates that human capital subsidies are highly needed to the economy to offset the distortion from labor taxes and externalities.

Figure 4.5: Government Expenditures For Labor Taxes
The Laffer curves for capital income taxes are graphed in Figure 4.6. Laffer curves for capital taxes exhibit a very similar pattern to those for labor taxes, with a less pronounced increase in slope at lower tax rates and a smaller increase in peak. Government subsidies help raise the slope and peaks further and higher. Under the optimal allocation, there is an addition of 14% tax revenues through raising capital taxes, while the peak yields only 8% more tax receipts with marginal revenue allocated toward government consumption. The peak occurs at capital tax rate equal to 0.67, compared with capital rate 0.63 at peak when subsidy rate is fixed at benchmark.

![Figure 4.6: Laffer Curves for Capital Taxes](image-url)

As capital tax rate increases, more tax revenues are available for the government to make productive public expenditures. Figure 4.7 illustrates the rise in subsidies as a result of capital tax incrementation. Total production is slightly hump-shaped as capital taxes change, which is shown in Figure 4.8. Figure 4.9 reveals that human and physical capital initially move in opposite directions in response to an increase in the capital tax rate. Human capital increases initially, responding to the higher subsidy rate, while physical capital responds negatively to the reduced return implied by the higher capital tax rate. The response of hours is independent of the allocation of marginal tax revenue, as for the case with labor taxes. Figure 4.10 illustrates additional government expenditure on education and on government consumption allowed by the increase in the tax rate. Both are hump-shaped with optimal allocation, while government expenditures on education fall continuously with allocation only to
government consumption, similar to that for labor taxes.

Figure 4.7: Optimal Subsidy Rate For Capital Taxes

Figure 4.8: Production For Capital Taxes
Therefore, subsidizing human capital significantly alter the shape of Laffer curves. In the long-run, government can raise more tax receipts at higher levels of human capital. To our knowledge, there is very little literature that pays attention to the impact of valued government expenditures on shaping Laffer curves and government solvency quantitatively. It has straightforward implication: education subsidies mitigate the government solvency to a greater extent in the long-run with
steeper Laffer curves. The temporary shock of a tax cut and its impact on education subsidies and government solvency are beyond the point of interest in this chapter.

4.4.2 Welfare

In previous sections I focus on the trade-off between the two types of government expenditures while keeping other fiscal variables fixed. However, tax rates are very important fiscal tools so it is interesting to investigate the trade-off between providing productive government expenditures and collecting distortionary taxes in the long run. In the steady state, government debt and transfers are fixed and calibrated to US data, so changes in government expenditures will have to be balanced through raising taxes. It motivates us to investigate the optimal choice of taxes and expenditures in this section.

First, we present welfare curves, which we define as welfare as a function of the tax rate, under the two alternative assumptions about the allocation of marginal tax revenue. Figure 4.11 graphs the welfare curve for the labor tax rate when the capital tax rate is fixed at 0.36. Welfare is maximized at labor tax rate of 0.27, slightly smaller than the current rate of 0.28. Along the capital welfare curve in Figure 4.12 shows that welfare is maximized at capital tax rate of 0.21, given the labor tax of 0.28. Note that a small change in the capital tax rate does not create a very large change in welfare. A reduction in the tax rate from 0.36 to its constrained optimal value of 0.21, a 42% decrease in the capital tax rate, increases welfare by only 1.7% in terms of consumption.⁹

⁹Proposed by Lucas [50], the magnitude of welfare costs is measured as the percentage increase (decrease) in consumption.
Figure 4.11: Welfare for Labor Taxes

Figure 4.12: Welfare for Capital Taxes

To facilitate a better understanding about welfare implications of labor and capital taxes, we conduct the experiment to search for the joint optimality of the taxes and government expenditures. I relax both tax rates and find the best combination of \( \tau_n, \tau_k \) that maximizes the welfare in the steady state. The results are summarized in Table 4.3. We find that at optimal, labor taxes are raised from 0.28 to 0.37, while capital taxes are reduced to 0. Productive government spending does not change these basic results from the optimal tax literature. However, the increase in welfare, moving from the current set of taxes to the optimum is 3% in terms of consumption. The output at the current tax rates is only 92.8% of that at optimal with a decrease of 7.2%.

The last column in Table 4.3 displays the optimal tax rates and aggregate ratios when marginal tax revenue is allocated entirely to government consumption, leaving
no education subsidies. As a result, the optimal labor tax is reduced to 0.31 from 0.37. Education expenditures substantially lower to 2.1%, while consumption-to-GDP rises to 66.7%. Other ratios remain essentially unchanged. However, deviating from the optimal to this scenario implies a welfare loss of 4.6% in terms of consumption, with the output reduced by 8.1% compared with that at optimal. It implies that the benefit of subsidizing education exceeds the cost of distortionary taxes so funding productive government spending through distortionary taxes improves overall welfare.

Table 4.3: Optimal Taxation

<table>
<thead>
<tr>
<th></th>
<th>Baseline</th>
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Figure 4.13 depicts the Laffer curve ‘surface’ along both the capital and labor tax rate dimensions. The surface clearly illustrates the discrepancy in maximizing tax revenues and welfare. The curve is steeper along the labor tax direction; after the peak has been reached, the curve starts to fall sharply along almost any direction. The peak of Laffer curve surface occurs at $\tau_n = 0.68$ and $\tau_k = 0.47$. An addition of 76% of tax revenues can be achieved at the peak.
Note that optimal taxation in this section is distinct from the solving a Ramsey problem in optimal taxation literature in two folds:

First, I focus on the optimal welfare in the long run instead of looking at short run along the transition path. Also, lump-sum transfers and debts are calibrated to US data, so that they are held fixed in steady states. In other words, we are searching for portfolios of taxes, government expenditures that maximize the welfare while keeping the government explicit and implicit liabilities unchanged. This assumption is different from a Ramsey problem, where all fiscal tools, including debts and financing, are available to the government.

The solution from Ramsey problem indicates that a government should accumulate claims and fund itself through interest earnings. However, it is rarely observed in the real world. The US government has been running a budget deficit for most of the years. This difference between theory and reality remains a controversial issue. Many researchers look at this problem from political economy perspective. Government can appropriate part of tax revenues for unproductive public consumption called pork-
barrel spending, i.e. political rents. Pork-barrel spending can be thought of as favors paid to ‘friends’ of the government or public employees. Voters can replace a government that abuses its power, but in equilibrium they generally cannot push rents all the way to zero. This idea shows that political agency can lead to excessive debt accumulation due to the problem of moral hazard. The binding borrowing constraints problem is well explained under this framework. However, investigation of persistent budget deficits is beyond the scope of this paper.

Second, in the previous literature on optimal taxation, government spending is usually assumed exogenous or stochastic, while in our model it is endogenously manipulated to maximize welfare. Most of the literature treats government spending as non-productive or just a source of government insolvency. Though it greatly provides simplicity to model solution, this hides the potential to realize roles of government expenditures as a productive component in the economy and important implications are then missing. In our model, government subsidies have an important role in subsidizing human capital production as an effective fiscal policy tool.

4.5 Sensitivity Analysis: Externalities

There is no consensus among economist about either the existence or magnitude of human capital externalities (Moretti [59]). Therefore, we conduct sensitivity analysis allowing for different magnitudes for the externality.

Arguing for a larger elasticity, Rauch [64] estimates a log wage equation including average schooling in cities and own schooling, in addition to other terms. Define the total labor income as \( W_t \equiv w_t n_{w,t} + w_{h,t} h_t \) Our wage equation in logarithms can be expressed as

\[
\log W_t = \log (1 - \theta_k) + \log z_t + \theta_k \log k_t + \theta_h \log h_t + (1 - \theta_k - \theta_h) \log n_{w,t} + \phi \log h_{a,t} \quad (4.37)
\]

Rauch [64] finds that coefficient on average schooling in cities is 0.033 while the coefficient on own education is 0.048.

We cannot directly use his estimates since there are different conditioning variables in his equation and ours. We use his estimates to reflect the relative magnitudes
of the effects of average and own education on wages, subject to the constraint that the sum of the coefficients equals 0.23. Rauch’s estimates imply that the relative strength of externalities is 0.033/0.048, or roughly 0.69, which corresponds to $\theta_h/\phi$ in (4.37). This ratio implies that $\theta_h = 0.14$ and $\phi = 0.09$. In Lucas [51], the relative strength of externalities is 0.417/0.7, approximately 0.60. Consistent with our assumption, Ciccone and Peri [24] and Moretti [58], also find significant positive spillovers with similar magnitudes. Therefore, we use the relative strength of the elasticities from Rauch [64] as an alternative by considering Laffer curves with $\theta_h = 0.14$, $\phi = 0.09$.

In contrast, Acemoglu and Angrist [1] find no significant positive human capital spillovers. Therefore, we also consider Laffer curves with $\theta_h = 0.23$, $\phi = 0$. For both cases, we hold all other parameters at benchmark values. As a comparison we also depict Laffer curves under the assumption that subsidies fixed at their benchmark rate, which is the subsidy rate at current tax rates under optimal allocation.

Figures 4.14 and 4.15 depict labor-tax and capital-tax Laffer curves with alternative values for the externality parameter $\phi$. The patterns are very similar to those in the benchmark case. Laffer curves with optimal allocation exhibit higher peaks that occur at larger tax rates, even in the absence of externalities. Subsidies to human capital expenditures allow higher tax revenue even in the absence of externalities because they offset some of the distortions created by the labor tax. The benchmark subsidy rate is higher with stronger externalities, implying that subsidies offset more distortions from externalities.
4.6 Concluding Remarks

In this chapter, I recalculate Laffer curves under the implied assumption that the US will optimally allocate between utility-enhancing government consumption and productive subsidies to human capital investment for any given taxes in the long-run. I find the extension of human capital and subsidies significantly affect the shapes of Laffer curves. Laffer curves have higher peaks, which occur at larger tax rates when the government optimally allocates tax revenues. Subsidies offset insufficiency of human capital so that externalities can be effectively mitigated. At the optimum, the government should reduce the capital tax rate to zero, as is consistent with the optimal
tax literature, and raise the labor tax substantially to provide subsidies to human capital. The optimum labor tax rate is 20% higher with productive government spending than without in our calibration, while the optimal capital tax remains at zero.
CHAPTER 5

Conclusion

This dissertation aims to comprehensively investigate economic growth empirics, fiscal policy and Laffer curves.

Chapter 2 empirically examines the role of education investment, schooling quality, and the country-specific unobserved effects in explaining cross-country economic growth under exogenous growth framework. Several findings are noteworthy. First, education expenditures, approximated by public spending on education, contribute to long-run economic growth. Education enrollment, on the other hand, is more appropriate to be interpreted as the time component in human capital production. We find that physical investment explains around one-third in human capital production, and a combination of the forgone time and human capital explains the remaining two-thirds.

Second, I find that schooling enrollment, or the forgone time, continues to impact economic growth dynamics at 5-year interval, in sharp contrast with the predominant view that the contribution of human capital investment to economic growth is ambiguous, and even negative in a panel model. However, physical education expenditures need around a decade to become effective to economic growth.

Finally, the estimated human capital share in production function mostly falls into the range of 1/5-1/4 when unobserved country-effect is accounted for, while the share of physical capital remains in the conventional ballpark of one-third.

Chapter 3 provides an attempt to quantitatively measure response of distortionary taxation to government expenditures when public capital is productive. The model is calibrated to the US economy with the implied assumption that the US is actually optimally allocating government spending and investment in the long run. I characterize Laffer curves for the US. Under this assumption higher peaks can be reached than the traditional counterparts.

In addition, a zero capital tax rate is found optimal when I search for the long
run fiscal policies that maximize the welfare. It is consistent with the long existing literature of optimal taxation theory. It implies that the degree of distortion from labor taxes is relatively smaller than capital taxes so that the government should rely more on labor taxes to finance itself in the long run. When the government is highly indebted in the model, it will choose to mildly cut government spending but drastically raise taxes in the long run. Cutting government expenditures would incur such a large welfare loss that the government would choose to impose higher labor taxes. Output and welfare are a hump-shape for varying labor taxes, meaning that the benefit of providing public investment exceed the cost of raising distortionary labor taxes at low tax rates.

In Chapter 4, I recalculate steady-state Laffer curves for the US under the assumption that marginal tax revenue is optimally allocated between utility-enhancing government consumption and productive subsidies to human capital investment. I find that Laffer curves for both labor and capital taxes have steeper slopes at low tax rates and higher peaks, compared with Laffer curves along which all marginal tax revenue is allocated to government consumption. At the optimum, the government should reduce the capital tax rate to zero, as is consistent with the optimal tax literature, and raise the labor tax substantially to provide subsidies to human capital. The optimum labor tax rate is 20% higher with productive government spending than without in our calibration, while the optimal capital tax remains at zero. Human capital subsidies are shown to be very contributable to overall welfare, which motivates high need for distortionary taxation.

Sensitivity analysis with varying benefits of productive investment confirms the general results about the changes in the shape of the Laffer curve with productive government spending. The stronger the externality in human capital, the more education subsidies are needed to offset the distortion.
CHAPTER 6
Appendices

Appendix A to Chapter 2

Appendix A.1: Data and Estimated Country-Effects

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<td>6.6%</td>
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</tbody>
</table>

Note: a. Above is a list of countries in Non-Oil sample. Columns ‘Intermediate’ and ‘OECD’ indicate the sample group.
b. Y/L: Real GDP per working-age population in 2010; calculated by dividing real GDP by working-age population (15-64), from Summers-Heston PWT 7.1 and World Development Indicators, respectively.
d. n: Average working-age population growth rate over 1970-2010, World Development Indicators.
f. SCHOOL2: Average secondary and tertiary school enrollment as a percentage of working-age population over 1970-2010, UNESCO.
g. A0/\overline{\alpha_0}: estimated unobserved individual effect divided by the international average; ‘Rank’ indicates the rank of the country effect in the sample.
### Appendix A.2: Histogram of Individual Effects

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<th>Country Code</th>
<th>Effect Size</th>
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<td>SEN</td>
<td>(0.48)</td>
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<tr>
<td>3</td>
<td>UGA</td>
<td>(0.48)</td>
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<tr>
<td>27</td>
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<td>(0.12)</td>
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</table>

Note: The figures in parentheses are the corresponding values of $A_i/\bar{A}_0$. 
Appendix B to Chapter 3

Appendix B.1: Steady State and Tax Revenues

In the steady state, all the detrended variables are unchanging over time, i.e., \( \tilde{x}_t = \tilde{x}_{t+1} \). Denote \( \tilde{x} \) for all steady state variables i.e., \( \tilde{x} = \tilde{x}_t = \tilde{x}_{t+1} \). Stationary equilibrium equations become:

\[
\bar{y} = k^\theta \pi^{1-\theta} (\bar{k}^g) \gamma
\]

(6.1)

\[
\bar{r} = (\psi + \delta - 1) \bar{k}
\]

(6.2)

\[
\bar{r}^g = (\psi + \delta - 1) \bar{k}^g
\]

(6.3)

\[
\bar{c} + \bar{y} + \bar{r}^g + \bar{r} = \bar{y}
\]

(6.4)

\[
\bar{R} = (1 - \bar{\tau}_k)(\theta \frac{\bar{y}}{\bar{k}} - \delta) + 1
\]

(6.5)

\[
\bar{R}_b = \bar{R} = \frac{\psi}{\beta}
\]

(6.6)

\[
\bar{T} = \bar{\tau}_n(1 - \theta)\bar{y} + \bar{\tau}_k(\theta \frac{\bar{y}}{\bar{k}} - \delta) \bar{k}
\]

(6.7)

\[
-\frac{\bar{u}_n}{\bar{u}_c} = (1 - \bar{\tau}_n)(1 - \theta)\frac{\bar{y}}{\bar{n}}
\]

(6.8)

\[
\bar{g} + \bar{r}^g + (\bar{R}_b - \psi) \bar{b} + \bar{s} = \bar{T}
\]

(6.9)

The steady state capital-output ratio is obtained from the real return on capital equation:

\[
\frac{\bar{k}}{\bar{y}} = \theta \delta + \frac{\bar{R} - 1}{1 - \bar{\tau}_k} \bar{k}^{-1}.
\]

In the steady state, the consumer’s budget constraint becomes:

\[
\bar{c} + \bar{r} = (1 - \bar{\tau}_n)(1 - \theta)\bar{y} + (1 - \bar{\tau}_k)(\theta \frac{\bar{y}}{\bar{k}} - \delta) \bar{k} + \delta \bar{k} + (\bar{R}_b - \psi) \bar{b}.
\]
By substituting \( \tilde{t} \) with equation (13), and dividing both sides by \( \bar{y} \), I calculate steady state consumption-output ratio

\[
\frac{\tilde{c}}{\bar{y}} = (1 - \tau_n)(1 - \theta) + (1 - \tau_k)(\theta \frac{\bar{y}}{\bar{K}} - \delta) \frac{\bar{K}}{\bar{y}} - (\psi - 1) \frac{\bar{K}}{\bar{y}} + (\bar{R}_b - \psi) \frac{\bar{b}}{\bar{y}}
\]

Since all the terms on the right-hand side are exogenous to the consumer or calibrated values, the consumption to output ratio can be pinned down.

Next, rearrange the labor supply equation. Together with production function, we have a system of two equations. Given the log utility function form in this paper, the equations become:

\[
\frac{1}{1 - \bar{n}} = \frac{1}{\bar{c}}(1 - \tau_n)(1 - \theta) \frac{\bar{y}}{\bar{c}}
\]

\[
\bar{y} = \bar{K}^\theta \bar{\pi}^{1-\theta} (\bar{K}_g)^\gamma
\]

\( \bar{y} \) and \( \bar{n} \) can be pinned down in this system given \( \frac{\bar{r}}{\bar{y}} \) and \( \frac{\bar{K}}{\bar{y}} \):

\[
\bar{n} = \frac{1}{\alpha_n} \frac{1}{(1 - \tau_n)(1 - \theta)} \frac{1}{\bar{y}} + \frac{1}{\alpha_n} (1 - \tau_n)(1 - \theta) \frac{1}{\bar{y}}
\]

\[
\bar{y} = (\bar{\pi})^{1-\gamma} \left( \frac{\bar{K}}{\bar{y}} \right)^{1-\gamma} \left( \frac{\bar{K}_g}{\bar{y}} \right)^{1-\gamma}
\]

Thus, after simplification the steady state tax revenue becomes:

\[
T = \tau_n (1 - \theta) \bar{y} + \tau_k \frac{\theta}{1 + \delta} \frac{1 - \bar{n}}{\bar{R} - 1} \bar{y}
\]

Then the steady state tax revenue can be solved numerically against different levels of tax rates.

**Appendix B.2: Data Details**

The data sets are from Federal Reserve Economic Data (FRED), U.S. Bureau of Economic Analysis data (BEA), Database of the European Commission (AMECO)
and the World Bank Data (WBD). All the data below except for real interest rate are in US dollar. The base year is 2005 for all the quarterly data except for two nominal series: government debt held by the public and GDP. The annual data include real capital stock, GDP, real private investment and nominal total government transfer payments.

Appendix B.2.1: Raw Data

Real Federal Nondefense Investment: Real Federal Nondefense Gross Investment (FRED, NDGIC96)

Real State and Local Public Investment: Real State & Local Government: Gross Investment (FRED, SLINVC96)

Real National Defense Investment: Real National Defense Gross Investment (FRED, DGIC96)

Real Consumption: Real Personal Consumption Expenditures (FRED, PCECC96)

Real Government Consumption and Gross Investment: Real Government Consumption Expenditures & Gross Investment, 3 Decimal (FRED, GCEC96)

Real Total Private Investment: Real Gross Private Domestic Investment, 3 Decimal (FRED, GPDIC96)

Nominal Government Debt Held By the Public: Federal Debt Held by the Public (FRED, FYGFDPUN)

Nominal Government Transfers: Government Transfer Payments to Individuals, Total (BEA, Regional Accounts Data, Annual State Personal Income)

Nominal GDP: Gross Domestic Product, 1 Decimal (FRED, GDP)
**Real GDP**: Real Gross Domestic Product, 1 Decimal(FRED, GDPC1)

**Net Capital Stock**: Net Capital Stock at 2000 prices, Total Economy, Annual(AMECO, OKND)

**Real GDP at 2000 prices**: Real US GDP at 2000 prices, Annual(AMECO, OVGD)

**Real Private Investment at 2000 prices**: Gross Fixed Capital Formation at 2000 prices: Total Economy, Annual(AMECO, OIGT)

**Real Interest Rate**: From WBD, available at:

http://data.worldbank.org/indicator/FR.INR.RINR

**Appendix B.2.2: Data Calculations**

*Real Government Investment*: it is calculated by adding up real federal non-defense investment, real state and local public investment and real national defense investment. There are two reasons that we add national defense investment into the total public investment: 1. it is still considered to be public capital and it affects the overall economy more or less, like a military factory might produce goods that can be used for military services, which is included in the government purchase that increases utility; 2. the defense investment, unlike defense consumption expenditure, is very little(around 0.6% of GDP).

*Real Government Consumption Expenditure*: it is obtained by subtracting real government total investment from real government consumption and gross Investment.

*Ratios of Variables to GDP*: based on the above data, I calculate the observed averages of the variables including consumption, government consumption expendi-
ture, government investment, private investment, government debt held by the public, government transfer payments and net capital stock. Then I divide them by corresponding averages of GDP, i.e., real terms by real GDP, quarterly terms by quarterly GDP, nominal terms by nominal GDP, etc.

Appendix B.3: Tax Rate Calculations: Effective Tax Rates

Mendoza, Razin, and Tesar [57] proposed the following methodology to calculate effective tax rates (data can be found in OECD database):

Personal income tax: \( \tau_p = \frac{1100}{OSPUE + PEI + W} \),

1100: Taxes on income, profits, and capital gains of individuals.

\( OSPUE \): Operating surplus of private unincorporated enterprises.

\( PEI \): Household’s property and entrepreneurial income.

\( GW \): Compensation of employees paid by producers of government services.

\( W \): Wages and salaries.

Labor income tax: \( \tau_n = \frac{\tau^h W + 2000 + 3000}{W + 2200} \),

2000: Total social security contributions.

3000: Taxes on payroll and workforce.

2200: Employer’s contribution to social security.

Capital income tax: \( \tau_k = \frac{\tau^h (OSPUE + PEI) + 1200 + 4100 + 4400}{OS} \)

1200: Taxes on income, profits, and capital gains of corporations.

4100: Recurrent taxes on immovable property.

4400: Taxes on financial and capital transactions.

\( OS \): Total operating surplus of the economy.
Appendix C to Chapter 4

Appendix C.1.1: Stationary Equilibrium

Along a balanced growth path, \( n_{w,t}, n_{h,t}, R_t, R_{b,t}, s_t, \tau_{n,t}, \tau_{k,t} \) and \( s_t \) are constant. All other variables grow at rate

\[
\psi = \xi^{1/(1-\theta_{k}-\theta_{h}-\phi)}.
\]

In order to obtain stationary solutions, all the growing variables are detrended. Define the detrended variables along the balanced growth path as \( \tilde{x} = x_t/\psi^t \). Since variables are constant along the balanced growth path, we drop the time subscripts.

In the economy with the representative agent, \( h_{a,t} = h_t \), implying that the aggregate per worker production function is given by

\[
y_t = z_t k_t^{\theta_k} h_t^{\theta_h} n_{w,t}^{1-\theta_k-\theta_h} h_t^{\theta_h+\phi}.
\]

Dividing both sides of the production function by \( z_t = \psi^t \) yields

\[
\tilde{y}_t = \tilde{k}_t^{\theta_k} h_t^{\theta_h} n_{w,t}^{1-\theta_k-\theta_h} h_t^{\theta_h+\phi}.
\]

The detrended consumer budget contraint is obtained by dividing equation (4.1) by \( \psi^t \) to yield

\[
\tilde{c}_t + (1-s_t)\tilde{c}_t + \tilde{t}_t + \psi \tilde{b}_{t+1} = (1-\tau_{n,t})(\tilde{w}_t n_{w,t} + w_{h,t} \tilde{h}_t) + [(1-\tau_{k,t})(d_t - \delta) + \delta] \tilde{k}_t + \tilde{T} R_t + R_{b,t} \tilde{b}_t + \tilde{m}_t.
\]

Dividing equations (4.2), (4.3), and (4.1) by \( \psi^t \) yields detrended versions of the physical capital and human capital accumulation equations and the government budget constraint

\[
\psi \tilde{k}_{t+1} = (1-\delta) \tilde{k}_t + \tilde{t}_t,
\]

\[
\psi \tilde{h}_{t+1} = (1-\delta_{h}) \tilde{h}_t + \tilde{\epsilon}_t \tilde{h}_t^{\omega_h} n_{h,t}^{1-\omega_{e-h}},
\]

\[
\tilde{g}_t + \tilde{c}_{e,t} + \tilde{T} R_t + s_t \tilde{c}_t + R_{b,t} \tilde{b}_t = \tilde{T}_t + \psi \tilde{b}_{t+1},
\]

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with detrended tax revenues from equation (4.7) given by

\[ \hat{T}_t = \tau_{n,t}(1 - \theta_k)\hat{y}_t + \tau_{k,t}\theta_k\bar{y}_t - \tau_{k,t}\delta\hat{k}_t. \]  

(6.14)

Adding the consumer and government budget constraints and imposing equation (4.5) yields the detrended resource constraint:

\[ \bar{y}_t + \bar{m}_t = \bar{c}_t + \bar{i}_t + \bar{\epsilon}_t + \bar{g}_{c,t} + \bar{g}_t \]  

(6.15)

Along the balanced growth path, the Lagrange multipliers shrink at the rate of growth. Therefore, define \( \tilde{\lambda}_t, \tilde{\mu}_t \) as the detrended Lagrange multiplier along the balanced growth path, where

\[ \tilde{\lambda}_t = \lambda_t\psi^t, \quad \tilde{\mu}_t = \mu_t\psi^t \]

Since the detrended multipliers must be constant along the balanced growth path, \( \lambda_{t+1}/\lambda_t = \mu_{t+1}/\mu_t = \psi^{-1} \).

The detrended first order conditions with respect to \( \bar{c}_t, \bar{i}_t, \bar{\epsilon}_t, \bar{h}_{t+1}, \bar{k}_{t+1} \) are derived by multiplying equations (4.8), (4.9), (4.10), (4.11), (4.12), and (4.13) by \( \psi^t \) to yield

\[ \tilde{\lambda}_t = \frac{1}{\bar{c}_t}, \]  

(6.16)

\[ \tilde{\lambda}_t = \tilde{\zeta}_t, \]  

(6.17)

\[ (1 - \delta_t)\tilde{\lambda}_t = \tilde{\mu}_t\omega \bar{e}_t^{\omega} - 1 \tilde{h}_t^{\omega_0} n_{h,t}^{1-\omega_h-\omega_0}, \]  

(6.18)

\[ \tilde{\lambda}_t = \beta E_t\{\psi^{-1}\tilde{\lambda}_{t+1}R_{b,t+1}\}, \]  

(6.19)

\[ \tilde{\zeta}_t = \beta\psi^{-1}E_t\{\tilde{\lambda}_{t+1}(1 - \tau_{k,t+1})\theta_k y_{\bar{h}_{t+1}} - \delta_{t+1}(1 - \delta_{t+1})\}, \]  

(6.20)

\[ \tilde{\mu}_t = \beta\psi^{-1}E_t\{\tilde{\lambda}_{t+1}\theta_{h}(1 - \tau_{n,t+1})\frac{\bar{y}_{t+1}}{\bar{h}_{t+1}} + \tilde{\mu}_{t+1}[1 - \delta_h + \omega_h \bar{e}_t^{\omega} \tilde{h}_t^{\omega_0} h_{h,t}^{1-\omega_h-\omega_0}]\}. \]  

(6.21)

Next, detrending the first order conditions with respect to \( n_{w,t} \) and \( n_{h,t} \), equa-
tions (4.15) and 4.16), yields

\[
\frac{\alpha_n}{1 - n_{w,t} - n_{h,t}} = \bar{\lambda}_t (1 - \theta_k - \theta_h)(1 - \tau_{n,t})\bar{y}_t / n_{w,t},
\]

\[
\frac{\alpha_n}{1 - n_{w,t} - n_{h,t}} = \bar{\mu}_t (1 - \omega_c - \omega_h)\bar{e}_t \bar{h}_t \omega_h n_{h,t}^{1-\omega_c-\omega_h}
\]

A competitive equilibrium is a set of plans \(\{\bar{c}_t, \bar{i}_t, \bar{e}_t, \bar{b}_{t+1}, \bar{h}_{t+1}, \bar{h}_{t}, n_{w,t}, n_{h,t}\}\) satisfying the equations (6.10) to (4.34), given exogenous processes \(\{\tau_{n,t}, \tau_{h,t}, s_t, TR_t, \bar{b}_t, \bar{m}_t, \bar{g}_{h,t}, \bar{g}_{c,t}, \bar{g}_t\}\) and the initial condition \(\{k_0, b_0, h_0, m_0, g_0, TR_0\}\).

**Appendix C.1.2: Steady State**

We can solve for the steady-state equilibrium by recognizing that in the steady state all detrended variables must be constant. Therefore, we drop time subscripts and denote steady state values as unsubscripted variables. Using equations (6.11) and (6.12), the physical and human capital accumulation equations in steady state are given by

\[
(\psi - 1 + \delta)\bar{k} = \bar{i}
\]

\[
(\psi - 1 + \delta_h)\bar{h} = e^{\omega_c} \bar{h} \omega_h n_h^{1-\omega_c-\omega_h}
\]

From equation (6.10), the steady state production function is given by

\[
\bar{y} = \bar{y}^{\theta_k} n_w^{1-\theta_k} \bar{h}^{\theta_h} \bar{h}^{\theta_\phi}.
\]

Equation (6.15) can be used to derive the steady state resource constraint as

\[
\bar{y} + \bar{m} = \bar{c} + \bar{i} + \bar{e} + \bar{g}_c + \bar{g}.
\]

Using equation (6.13), the steady state budget constraint in steady state becomes

\[
\bar{g}_c + \bar{g}_h + \bar{g} + \bar{T}R + R_b \bar{b} = \bar{T} + \psi \bar{b},
\]

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where, from equation (6.14), taxes are given by

$$\tilde{T} = \tau_n(1 - \theta_k)\tilde{y} + \tau_k\theta_k\tilde{y} - \tau_k\delta\tilde{k},$$  \hspace{1cm} (6.29)

$$\tilde{g}_h = s\tilde{c}.$$ \hspace{1cm} (6.30)

The steady state values for equations (6.17)-(6.21) are given by

$$\tilde{\lambda} = \frac{1}{c}$$ \hspace{1cm} (6.31)

$$\psi = \beta R_b = \beta R$$ \hspace{1cm} (6.32)

$$\tilde{\lambda} = \tilde{\zeta}$$ \hspace{1cm} (6.33)

$$(1 - s)\tilde{\lambda} = \tilde{\mu}\omega c\tilde{\theta}_c\omega_c^{-1}h\omega_h\tilde{n}_h^{1-\omega_c-\omega_h}$$ \hspace{1cm} (6.34)

The steady state equation for $R$ is obtained by dropping time subscripts from equation (4.14)

$$R = (1 - \tau_k)(\theta_k\frac{\tilde{y}}{k} - \delta) + 1$$ \hspace{1cm} (6.35)

Simplification of equation (6.21) using equations (6.25) and (6.34) yields:

$$\frac{\tilde{e}}{\tilde{y}} = \frac{(1 - \tau_n)\omega_c\theta_h(\psi - 1 + \delta_h)}{(1 - s)[\psi/\beta - 1 + \delta_h - \omega_h(\psi - 1 + \delta_h)]}$$ \hspace{1cm} (6.36)

The steady state versions of equations (6.22) and (6.23), after substituting from equation (6.34) for $\tilde{\mu}_t$, become

$$\frac{\alpha_n}{1 - n_w - n_h} = (1 - \theta_k - \theta_h)(1 - \tau_n)\frac{\tilde{y}}{n_w}\tilde{\lambda}$$ \hspace{1cm} (6.37)

$$\frac{\alpha_n}{1 - n_w - n_h} = (1 - s)\frac{1 - \omega_c - \omega_h}{\omega_c}\frac{\tilde{e}}{n_h}\tilde{\lambda}$$ \hspace{1cm} (6.38)

**Appendix C.2: Central Planner’s Problem**

The private sector fails to perceive the average human capital in the production function due to externalities. Prices do not incorporate the externalities, resulting in
underproduction of human capital. The planner is able to internalize this externality. The planner chooses optimal paths of \( c_t, n_{w,t}, e_t, n_{h,t}, i_t, k_{t+1}, g_{c,t}, h_{t+1} \) to maximize the expected discounted sum of utility, subject to the resource constraint and physical and human capital accumulation equations. We assume that \( m_t \) and \( g_t \) grow exogenously along the balanced growth path.

\[
\max_{c_t, n_{w,t}, i_t, n_{h,t}, i_t, k_{t+1}, g_{c,t}, h_{t+1}} \sum_{t=0}^{\infty} \beta^t \{ \log c_t + \alpha_n \log(1 - n_{h,t} - n_{w,t}) + \alpha_g \log g_{c,t} \}
\]

subject to

\[
c_t + e_t + g_{c,t} + n_{h,t} + k_{t+1} - (1 - \delta)k_t = y_t + m_t,
\]

\[
y_t = k_t^{\theta_h} n_{w,t}^{1 - \theta_k} h_t^{\theta_k + \phi},
\]

\[
k_{t+1} = (1 - \delta)k_t + i_t,
\]

\[
h_{t+1} = (1 - \delta_h)h_t + e_t^{\omega_e} h_t^{\omega_h} n_{h,t}^{1 - \omega_e - \omega_h}.
\]

Letting \( \mu_t, \lambda_t \) be Lagrange multipliers for human capital equation and resource constraint, respectively. Then the first order conditions with respect to \( c_t, g_{c,t}, e_t, k_{t+1}, h_{t+1}, n_{w,t}, n_{h,t} \) yield:

\[
1/c_t = \lambda_t;
\]

\[
\alpha_g / g_{c,t} = \lambda_t;
\]

\[
\lambda_t = \mu_t \omega_e e_t^{\omega_e - 1} h_t^{\omega_h} n_{h,t}^{1 - \omega_e - \omega_h};
\]

\[
\lambda_t = \beta E_t[\lambda_{t+1} (1 - \delta + \theta_k y_{t+1}/k_{t+1})];
\]

\[
\mu_t = \beta E_t \{ \mu_{t+1} [1 - \delta_h + \omega_h e_{t+1}^{\omega_e} h_{t+1}^{\omega_h} - 1] n_{h,t+1}^{1 - \omega_e - \omega_h} + \lambda_{t+1} (\theta_h + \phi) y_{t+1}/h_{t+1} \};
\]

\[
\frac{\alpha_n}{1 - n_{h,t} - n_{w,t}} = \lambda_t (1 - \theta_k - \theta_h) y_t / n_{w,t};
\]

\[
\frac{\alpha_n}{1 - n_{w,t} - n_{h,t}} = \mu_t (1 - \omega_e - \omega_h) e_t^{\omega_e} h_t^{\omega_h} n_{h,t}^{1 - \omega_e - \omega_h}.
\]

Along the balanced growth path, \( \lambda_t \) and \( \mu_t \) grow at rate \( \psi^{-1} \); all other variables except \( n_{w,t} \) and \( n_{h,t} \) grow at \( \psi \). Denote the central planner’s detrended variables as \( \hat{x}_t \), and their steady states as \( \hat{x} = \hat{x}_{t+1} = \hat{x}_t \). In the steady state, the human capital
accumulation equation becomes:

\[(\psi - 1 + \delta_h)\hat{h}^{1-\omega_h} = \hat{\epsilon}^{\omega_e} n_h^{1-\omega_e-\omega_h}.\]

The resource constraint in steady state is

\[\hat{c} + \hat{i} + \hat{c} + \hat{g}_c + \hat{g} = \hat{y} + \hat{m},\]

where

\[\hat{y} = \hat{k}^{\theta_k} n_w^{1-\theta_k-\theta_h} \hat{h}^{\theta_h+\phi}.\]

The steady state of the first order conditions can be written as

\[1/\hat{c} = \hat{\lambda};\]

\[\alpha_y/\hat{g}_c = \hat{\lambda};\]

\[\hat{\lambda} = \hat{\mu} \omega_e \hat{\epsilon}^{\omega_e} \hat{h}^{\omega_h} n_h^{1-\omega_e-\omega_h};\]

\[\psi/\beta = 1 - \delta + \theta_k \frac{\hat{y}}{\hat{k}};\]

\[\hat{\mu} \psi/\beta = \hat{\mu} [1 - \delta_h + \omega_h \hat{\epsilon}^{\omega_e} \hat{h}^{\omega_h} n_h^{1-\omega_e-\omega_h}] + \hat{\lambda} (\theta_h + \phi) \frac{\hat{y}}{\hat{k}};\]

\[\frac{\alpha_y}{1 - n_w - n_h} = \hat{\lambda} (1 - \theta_k - \theta_h) \frac{\hat{y}}{n_w};\]

\[\frac{\alpha_n}{1 - n_w - n_h} = \hat{\mu} (1 - \omega_e - \omega_h) \hat{\epsilon}^{\omega_e} \hat{h}^{\omega_h} n_h^{1-\omega_e-\omega_h}.\]

We can obtain the steady state values of \(k/y\), \(e/y\), \(n_w\) and \(n_h\) using the equations above to yield

\[\frac{\dot{k}}{\hat{y}} = \frac{\theta_k}{\psi/\beta - 1 + \delta};\]

\[\frac{\dot{\epsilon}}{\hat{y}} = \frac{\omega_e (\theta_h + \phi) (\psi - 1 + \delta_h)}{[\psi/\beta - 1 + \delta_h - \omega_h (\psi - 1 + \delta_h)]};\]

\[\frac{n_h}{n_w} = \frac{1}{1 - \theta_k - \theta_h} \frac{1 - \omega_e - \omega_h \dot{\epsilon}}{\omega_e \hat{y}}.\]
\[
\begin{align*}
  n_w &= \frac{(1 - \theta_k - \theta_h)}{(1 - \theta_k - \theta_h) + \alpha_n \frac{\xi}{\gamma} + \frac{1 - \omega_k - \omega_h \xi}{\omega_c \gamma}} \\
  n_h &= \frac{1 - \omega_k - \omega_h \xi}{\omega_c \gamma} \left(1 - \theta_k - \theta_h\right) + \alpha_n \frac{\xi}{\gamma} + \frac{1 - \omega_k - \omega_h \xi}{\omega_c \gamma}
\end{align*}
\]

The steady-state equations for the planner differ from those of the agent due to both the distortionary taxes and the externality.

Appendix C.3: Data Details

The data sets for calibration are from National Income and Product Accounts Tables (NIPA) by the Bureau of Economic Analysis, Federal Reserve Economic Data (FRED), the World Bank Data (WBD), and the Digest of Education Statistics (DES). All the data below except for real interest rates are denominated in US dollars.

**GDP**: Gross domestic product (Table 1.1.5, NIPA)

**Investment**: Gross domestic investment (Table 5.1, NIPA)

**Capital Stock**: Fixed assets (private and public) (Table 1.1, Fixed Assets Accounts Tables, NIPA)

**Consumption**: Personal consumption expenditures (Table 1.1.5, NIPA)

**Government Debt**: Federal debt held by the public (FYGFDPUN, FRED)

**Total Government Consumption Expenditures**: Government consumption expenditures (Table 3.1, NIPA)

**Government Defense Expenditures**: Defense expenditures (Table 3.15.5, NIPA)

**Government Subsidies to Education**: Government education expenditures (Table 3.15.5, NIPA)

**Net Import**: Net exports of goods and services (Table 1.1.5, NIPA)

**Transfer Payments**: Government current transfer payments, to persons (Table 3.3, NIPA)

**Education Expenditures (\% of GDP)**: Expenditures of educational institutions as a percent of GDP - all institutions (Table 28, 2011 Tables and Figures, DES)

**Real GDP Growth Rate**: Percent change from preceding period in real gross domestic product (Table 1.1.1, NIPA)

**Real Interest Rate**: Real interest rate, World Bank Data, available at:
Appendix C.4: Estimation of Human Capital Production

We adopt the methodology in Chapter 2 to estimate the coefficient on human capital, $a$ and $b$. Following Mankiw, Romer, and Weil [53], I perform a cross-country regression to estimate coefficients of the production function in Chapter 2. The methodology starts from the aggregate production function:

$$Y_t = z_t K_t^{\theta_k} H_t^{\theta_h} L_t^{1-\theta_k-\theta_h} h_{a,t}^\phi.$$  \hspace{1cm} (6.39)

$K_t$, $H_t$ denote aggregate physical and human capital, respectively. $L_t$ denotes aggregate labor force. Each worker provides 1 unit of labor force. In order to obtain estimated coefficients consistent with our model, we assume there exits externality in the human capital sector. $h_{a,t}$ is per capita human capital with a positive exponent $\phi$. $Y_t$ is output at time $t$. First, dividing (6.39) by $L_t$ yields the per capita production function:

$$y_t = z_t K_t^{\theta_k} H_t^{\theta_h} h_{a,t}^\phi.$$  \hspace{1cm} (6.40)

Under the representative agent model, the per capita human capital is equal to the agent’s own human capital, $h_t = h_{a,t}$. Therefore,

$$y_t = z_t K_t^{\theta_k} h_t^{\theta_h+\phi}.$$  \hspace{1cm} (6.40)

After rearrangement, (6.40) can be rewritten as

$$y_t = z_t^{1/\theta_k-\theta_h-\phi} (k_t/y_t)^{\theta_k/\theta_k-\theta_h-\phi} (h_t/y_t)^{\theta_h+\phi/\theta_k-\theta_h-\phi}$$  \hspace{1cm} (6.41)

Per capita output is now a function of an unobserved factor, $z_t^{1/\theta_k-\theta_h-\phi}$, and two capital intensities. Different from the usual accounting exercise that directly substitutes human and physical capitals with their inputs, such an arrangement gives a clear picture of per capita output, the object of interest, and its explanatory variables on the right-hand side.

Next, Mankiw, Romer, and Weil [53] specifies capital and human capital ac-
cumulation technologies so as to substitute $k_t/y_t$ and $h_t/y_t$ in equation (6.41) with their production factors. To obtain coefficients consistent with our model, we assume capital and human capital accumulate according to equations (4.2) and (4.3). Mankiw, Romer, and Weil [53] assume that the two capitals depreciate at the same rate: $\delta = \delta_h$. Define $A_t = \frac{1}{\lambda_t - \theta_k - \theta_h - \phi}$. Assume that $A_t$ grows exponentially at rate $\psi$: $A_t = A_0 e^{\psi t}$; labor grows at rate $n$: $L_t = L_0 e^{nt}$. In steady state,

$$k/y = \frac{i/y}{\psi + \delta + n} \quad (6.42)$$

$$h/y = \frac{e/y}{\omega_e/(1-\omega_e)} \left( \frac{n_h}{y_t/A_t} \right)^{(1-\omega_e)/(1-\omega_h)} \left( \frac{\psi + \delta + n}{1/(1-\omega_h)} \right) \quad (6.43)$$

Here we drop the time subscript to denote the steady state. Note that along the balanced growth path, $y_t/A_t$ is the detrended output per capita and thus is a constant.

Next, substitute $k/y$ and $h/y$ in equation (6.41) with equations (6.42) and (6.43), and rearrange to yield:

$$y_t = A_t^{\frac{(1-\omega_h)(1-\theta_k - \theta_h - \phi)}{Y}} (\psi + \delta + n)^{-\frac{\theta_h + \phi}{Y}} \left( \frac{\omega_e (\theta_h + \phi)}{Y} \right)^{\frac{1}{Y}} \left( \frac{1-\omega_e - \omega_h}{Y} \right)^{\frac{1}{Y}}$$

$$\text{(6.44)}$$

where $Y = (1 - \omega_h)(1 - \theta_k) - \omega_e(\theta_h + \phi)$. Next, take the logarithm:

$$\log y_t = \frac{(1 - \omega_h)(1 - \theta_k - \theta_h)}{Y} \log A_t - \frac{(1 - \omega_h)\theta_k + \theta_h + \phi}{Y} \log(\psi + \delta + n) + \frac{(1 - \omega_e - \omega_h)(\theta_h + \phi)}{Y} \log n_h + \frac{\omega_e (\theta_h + \phi)}{Y} \log(1/y)$$

$$\text{(6.45)}$$

Following Mankiw, Romer, and Weil [53], assume $A_t$ is randomly distributed across countries. Then the equation above enables us to run a single cross-country regression on per capita GDP with respect to each country’s investment-to-GDP ratio, education expenses-to-GDP ratio, education enrollment, and the sum of exogenous rates. We use our largest country sample, countries excluding those major oil producers, which has 78 countries over 1995 - 2007. Details of the cross-country regression are elaborated in Chapter 2. Then parameters $\omega_e, \omega_h$ can be calculated from the estimated coefficients. The results imply $\omega_e = 0.11$ and $\omega_h = 0.61$. The regression results are summarized in Table 6.2.
There exists a discrepancy between the empirical regression and this model in that I allow hours worked entering the production function. Then I need to examine the magnitude of such discrepancy and the deviation of the estimated coefficients due to differences in the assumption. To check this I add hours per worker, $n_{w,t}$ into the aggregate production function:

$$Y_t = z_t K_t^{\theta_k} H_t^{\theta_h} (n_{w,t} L_t)^{1-\theta_k - \theta_h} h_{a,t}^\phi$$

The absence of leisure in this model yields $n_{w,t} = 1 - n_{h,t}$. Given the physical/human capital production technology in (4.2) and (4.3), the logarithm of per capita output is

$$\log y_t = \log A_t - \frac{(1 - \omega_h)(\theta_k + (\theta_h + \phi))}{\Upsilon} \log(\psi + \delta + n) + \frac{(1 - \omega_h)\theta_k}{\Upsilon} \log(i/y) + \frac{\omega_c(\theta_h + \phi)}{\Upsilon} \log(e/y) + \frac{(1 - \omega_c - \omega_h)(\theta_h + \phi)}{\Upsilon} \log n_h$$

$$+ (1 - \omega_h)(1 - \theta_k - \theta_h) \log(1 - n_h).$$

Write the log labor hours as $f(n_h) = \log(1 - n_h)$, and approximate it at $n_h = \overline{n}_h$:

$$\log(1 - n_h) \approx \log(1 - \overline{n}_h) - \frac{\overline{n}_h}{1 - \overline{n}_h} (\log n_h - \log \overline{n}_h).$$

Substituting it for the last term in (6.46) yields:

$$\log y_t = \left[ \log A_t + \frac{(1 - \omega_h)(1 - \theta_k - \theta_h)}{\Upsilon} \log(1 - \overline{n}_h) - \frac{(1 - \omega_h)(1 - \theta_k - \theta_h)}{\Upsilon} \frac{\overline{n}_h}{1 - \overline{n}_h} \log \overline{n}_h\right] - \frac{(1 - \omega_h)\theta_k}{\Upsilon} \log(i/y) + \frac{\omega_c(\theta_h + \phi)}{\Upsilon} \log(e/y) + \frac{(1 - \omega_c - \omega_h)(\theta_h + \phi)}{\Upsilon} \log n_h$$

$$+ (1 - \omega_h)(1 - \theta_k - \theta_h) \frac{\overline{n}_h}{1 - \overline{n}_h} \log n_h.$$

$\overline{n}_h$ is the sample average of education enrollment rate across countries, so $\log(1 - \overline{n}_h)$ and $\log \overline{n}_h$ can be consolidated into the constant. This implies that the regression is the same as the one without labor hours since the dependent and independent vari-
ables are identical. Based on the estimated coefficients on four regressors summarized in Table 4, the implied \( \omega_e \) and \( \omega_h \) can be calculated. As a result, I find that \( \omega_e = 0.11 \) and \( \omega_h = 0.61 - 0.1 \times (0.62 - \theta_h) \). \( \theta_h \) cannot be calculated from the regression results since the number of unknown parameters is larger than the number of equations. Given that \( \theta_h > 0, \omega_h > 0.55 \). When I use the benchmark value \( \theta_h = 0.18 \), then \( \omega_h = 0.57 \). Therefore, when adding labor hours, \( \omega_e \) doesn’t change and \( \omega_h \) is only slightly smaller. So I conclude that the discrepancy will not cause noticeable changes to the results.
### Table 6.2: Single Cross-Country Regression

<table>
<thead>
<tr>
<th>Dependent Variable:</th>
<th>log $y$ in 2007</th>
</tr>
</thead>
<tbody>
<tr>
<td>Countries:</td>
<td>78</td>
</tr>
<tr>
<td>Constant</td>
<td>2.80</td>
</tr>
<tr>
<td></td>
<td>(2.02)</td>
</tr>
<tr>
<td>log($i/y$)</td>
<td>0.83***</td>
</tr>
<tr>
<td></td>
<td>(0.28)</td>
</tr>
<tr>
<td>log($\psi + \delta + n$)</td>
<td>-3.96***</td>
</tr>
<tr>
<td></td>
<td>(0.58)</td>
</tr>
<tr>
<td>log $n_h$</td>
<td>0.86***</td>
</tr>
<tr>
<td></td>
<td>(0.17)</td>
</tr>
<tr>
<td>log($e/y$)</td>
<td>0.35</td>
</tr>
<tr>
<td></td>
<td>(0.23)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.77</td>
</tr>
</tbody>
</table>

**Implied Coefficients:**
- Implied $\omega_e$: 0.11
- Implied $\omega_h$: 0.61

---

* The standard errors are in parentheses immediately below.

* * * indicate significance at 10%, 5% and 1% level, respectively.

$^a$ log $y$: log real GDP per working-age population in 2007, calculated by dividing real GDP by working-age population (15-64), from Summers-Heston PWT 7.1 and World Development Indicators, respectively.

$^b$ $n_h$: the ratio of secondary and tertiary school enrollment to the working-age population, from UNESCO.

$^c$ $e/y$: public spending on education-to-GDP ratio, World Bank Data


$^g$ $n$: working-age population growth rate, from World Development Indicators.

$^h$ The investment, working-age population growth rates and education enrollment rates are averaged over 1995-2007; ($\psi + \delta$) is assumed to be 0.05.
BIBLIOGRAPHY


