Phase imaging using focusing polycapillary optics

Sajid Bashir

*University at Albany, State University of New York, sbashir@albany.edu*

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Phase Imaging using Focusing Polycapillary Optics

By

Sajid Bashir

A Dissertation
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the Requirements for the Degree of
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Abstract

The interaction of X rays in diagnostic energy range with soft tissues can be described by Compton scattering and by the complex refractive index, which together characterize the attenuation properties of the tissue and the phase imparted to X rays passing through it. Many soft tissues exhibit extremely similar attenuation, so that their discrimination using conventional radiography, which generates contrast in an image through differential attenuation, is challenging. However, these tissues will impart phase differences significantly greater than attenuation differences to the X rays passing through them, so that phase-contrast imaging techniques can enable their discrimination.

A major limitation to the widespread adoption of phase-contrast techniques is that phase contrast requires significant spatial coherence of the X-ray beam, which in turn requires specialized sources. For tabletop sources, this often requires a small (usually in the range of 10-50 micron) X-ray source.

In this work, polycapillary optics were employed to create a small secondary source from a large spot rotating anode. Polycapillary optics consist of arrays of small hollow glass tubes through which X rays can be guided by total internal reflection from the tube walls. By tapering the tubes to guide the X rays to a point, they can be focused to a small spot which can be used as a secondary source.

The polycapillary optic was first aligned with the X-ray source. The spot size was measured using a computed radiography image plate. Images were taken at a variety of optic-to-object and object-to-detector distances and phase-contrast edge enhancement was observed. Conventional absorption images were also acquired at a small object-to detector distances for
comparison. Background division was performed to remove strong non-uniformity due to the optics. Differential phase contrast reconstruction demonstrates promising preliminary results.

This manuscript is divided into six chapters. The second chapter describes the limitations of conventional imaging methods and benefits of the phase imaging. Chapter three covers different types of X-ray photon interactions with matter. Chapter four describes the experimental set-up and different types of images acquired along with their analysis. Chapter five summarizes the findings in this project and describes future work as well.
1 Introduction

Traditional X-ray imaging relies on the attenuation properties of the object being imaged to generate a two dimensional map of intensities.\(^1\) Such images produce high contrast between objects where the electron density differences are substantial, for example, bone and flesh, but result in poor contrast for soft tissue imaging where the tissues exhibit similar attenuation. Soft tissue imaging is of clinical interest in cancer detection, for example discriminating infiltrating ductal carcinoma from glandular tissue in mammography.

Figure 1-1 shows the comparison of three different soft tissues as a function of X-ray energy. The very small attenuation differences between glandular and cancerous tissues make it difficult to distinguish at higher energies. Therefore, conventional mammography imaging techniques are commonly performed at low energy to detect any suspicious mass, for example, at 10 KeV to 25 KeV.\(^2\)

![Attenuation of Breast Tissues](image)

*Figure 1-1. Linear attenuation coefficients for fat, glandular, and carcinoma tissues as a function of X-ray energy. As the energy of the x rays increases, the attenuation coefficient differences decreases with the increase in energy of X-ray photon [Figure from Ref.2].*
Figure 1-2 (a) shows a sketch of an ACR (American College of Radiology) mammography accreditation phantom on which different objects of varying sizes have been embedded in wax. The phantom simulates a 4.2 cm thick compressed breast of average density with 50% glandular and adipose tissue composition. There are six fibers, six specks groups and five masses. These objects simulate fibrous structures, microcalcifications and tumors of varying sizes respectively. Figure 1-2 (b) is a typical image of Figure 1-2 (b) taken using conventional radiographic techniques. Most of the low attenuating objects are barely visible on the image of the phantom, which reveals the limitation of conventional imaging in discriminating low z objects, for example small tumors surrounded by normal tissue.

Figure 1-2. Diagram of mammography accreditation phantom. The figure on the right side shows the conventional image of the phantom. Because of low contrast, most of the objects on the phantom are barely visible showing limitations of conventional radiography for soft tissues. [images are from Ref.4].

With the aim of overcoming the intrinsic limitations of attenuation-based X-ray imaging, a number of phase-contrast techniques have been developed. Unlike conventional X-ray methods, these techniques are not only sensitive to the attenuation, but also to the phase shift that X rays experience when passing through matter. The differences in the real refractive indices are
much larger than the attenuation differences. If the X rays, after traversing through the object, are given sufficient distance to interfere by placing the detector far enough downstream, the result is an increased edge-enhancement across the borders where sharp changes in refractive index occur. Propagation-based imaging is relatively simple, robust and is implemented in this project to obtain phase-contrast images.

Figure 1-3 is the image of the ACR phantom in phase-contrast mode. The image was taken at a tube voltage of 40 KV, 250 μA current and 30 second exposure time. The focal spot size was 7 μm. The image shows greater detail of specks groups, masses and fibers as compared to the conventional image of the same phantom.

![Figure 1-3. Phase-Contrast image of the ACR phantom. The specks group, masses and fibers are more clearly visible than that in conventional mode image of the same phantom][Figure from Ref.4].

Usually, as for Figure 1-3, propagation-based phase contrast methods require a small X-ray source along with a high resolution detector. In this work, the source size limitation was relaxed by using a large spot size X-ray source with polycapillary optics to create a small, intense secondary source.
2 Theoretical Background

2.1 X-ray Interactions with matter

Because detectors measure the intensity of X-ray beam, it is necessary to understand how the intensity pattern arises from the interaction of the X-ray beam with atoms in the sample (patient). X-ray photons can be absorbed or scattered out of the beam to create contrast. In addition, phase contrast mechanisms may also be manipulated to create image contrast.

2.1.1 Elastic Scattering

Elastic scattering occurs when a photon interacts with an electron and a scattered photon of the same energy is emitted.

2.1.2 Inelastic Scattering (Compton Scattering)

Compton scattering occurs when a photon interacts with a nearly free electron and the electron recoils. The incident photon is absorbed and a new photon of low energy is radiated. The energy of this scattered photon equals the energy of the incident photon minus some of the kinetic energy gained by the recoil electron and its binding energy. Scattered photons travel in all directions. The higher the energy of the scattered photon, however, the greater the probability that the secondary photon will be forward-directed. The probability of Compton scattering is directly proportional to the electron density. For incident energy, $E_r$, the differential cross-section is,

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 r_e^2 P(E_r, \theta)^2 [P(E_r, \theta) + P(E_r, \theta)^{-1} - 1 + \cos(\theta)^2]}{2}$$

(1)
where $\frac{d\sigma}{d\Omega}$ is the differential cross section, $d\Omega$ is the infinitesimal solid angle element, $\alpha$ is the fine structure constant, $\theta$ is the scattering angle, $r_c = \frac{h}{m_e c}$ is the reduced Compton wavelength of the electron, $m_e$ is the mass of the electron ($\approx 511\text{KeV}/c^2$) and $P(E_\gamma, \theta)$ is the ratio of photon energy before and after collision,

$$P(E_\gamma, \theta) = \frac{1}{1 + \frac{E_\gamma}{m_e c^2 (1 - \cos(\theta))}}. \quad (2)$$

The number of electrons in bone is greater than in water, therefore the probability of Compton scattering is correspondingly greater in bone than in tissue. In clinical setting, since Compton scattering occurs over a range of angles, some portion of the scattered radiation continues in the forward direction and reaches the detector thus decreasing contrast of the image.

### 2.1.3 Photoelectric Absorption

Photoelectric absorption occurs when an incident photon is absorbed by an electron that is ejected it from its shell. The free electron becomes a recoil electron (photoelectron). The photoelectric phenomenon is primarily responsible for absorption contrast in conventional radiography.

### 2.1.4 Refractive Index

The refractive index of the tissue describes both coherent scatter and photoelectric absorption in a material. The index is

$$n = \sqrt{\frac{\varepsilon}{\varepsilon_0}} = 1 - \delta - i\beta. \quad (3)$$
Where ‘n’ is the index of refraction, $\varepsilon$ and $\varepsilon_o$ are the dielectric constants of material and vacuum respectively, the imaginary term $\beta$ is related to the absorption of X rays and real decrement $\delta$ determines the phase shift and is proportional to the cross-section for coherent scattering.

### 2.1.5 Reflection

Because the index of refraction is very close to one, high angle reflectivity is negligible. However, the reflectivity is nearly unity at grazing incidence. For example, X rays are guided down the length of polycapillary optics through total external reflections.

The real part of the index of refraction is

$$n = 1 - \delta \equiv 1 - \frac{\omega_p^2}{\omega^2} \quad (4)$$

$\omega_p$ is the plasma frequency of material and $\omega$ is the photon frequency. Since the plasma frequency, $\omega_p$, for glass is small compared with the photon frequency $\omega$, $n$ is slightly less than unity. X rays traveling in vacuum or air can be totally externally reflected from smooth surfaces since the refractive index of X rays in material is less than that in air i.e., $n_2 < n_1$. Using Snell’s law,

$$n_1 \sin \theta_1 = n_2 \sin \theta_2 \quad (5)$$

the angle in the medium is just grazing the surface as shown in Figure 2-1.

$$\sin \left( \frac{\pi}{2} - \theta_c \right) = n \sin \left( \frac{\pi}{2} \right) \quad (6)$$
The angle $\theta_c$ is termed the critical angle. Using eq.4 and eq.6,

$$\theta_c \approx \frac{w_p}{w}. \quad (7)$$

Figure 2-1. Sketch showing reflection and refraction of waves.

The incident, refracted and reflected wave vectors are $k_{\text{in}}$, $k_{\text{tr}}$ and $k_r$, respectively. In a homogeneous medium, the electric field derived from Maxwell’s equation leads to a propagation equation,

$$\nabla^2 E + k_j^2 E = 0 \quad (8)$$

where $k_j$ is the wave vector in medium $j$. The solution of the Helmholtz equation, for perpendicular polarization, the incident (in), reflected (r), and transmitted (tr) plane waves are

$$E_j = A_j e^{-i(\omega t - k_j \cdot r)} \hat{e}_y \quad (9)$$

with $j = \text{in}, \ r$ or $\text{tr}$, $k_0 = |k_{\text{in}}| = |k_r| = 2\pi/\lambda = |k_{\text{tr}}|/n$, and $\hat{e}_y$ the unit vector along the $y$ axis, the direction perpendicular to the plane of incidence.
The reflection coefficient is then

\[ r(\theta) = \frac{\sin \theta - \sqrt{n^2 - \cos^2 \theta}}{\sin \theta - \sqrt{n^2 - \cos^2 \theta}}, \tag{10} \]

and the reflectivity, \( R(\theta) = r(\theta)^* r(\theta) \), is approximately

\[ R(\theta) = \frac{\sqrt{\theta^2 - 2\delta - 2i\beta}}{\sqrt{\theta^2 - 2\delta + 2i\beta}}. \tag{11} \]

Using simple complex manipulation, equation (11) becomes

\[ R(\theta) = \frac{(\theta - \theta_1)^2 + \theta_2^2}{(\theta + \theta_1)^2 + \theta_2^2}, \tag{12} \]

where

\[ \theta_1 = \frac{\{(\theta^2 - 2\delta)^2 + 4\beta^2 \}^{\frac{1}{2}} + (\theta^2 - 2\delta)}{\sqrt{2}}, \tag{13} \]

and

\[ \theta_2 = \frac{\{(\theta^2 - 2\delta)^2 + 4\beta^2 \}^{\frac{1}{2}} - (\theta^2 - 2\delta)}{\sqrt{2}}. \tag{14} \]

### 2.1.6 Absorption

The absorption of radiation takes places as the rays traverse through the object. In conventional, absorption-based imaging, this passage of radiation can be described by

\[ I = I_0 e^{-\mu z}, \tag{15} \]
where $I_0$ is the intensity incident on the object and $I$ is the intensity detected by the detector after traversing through an object of thickness $z$. The attenuation coefficient can be expressed as

$$\mu = \mu_a + \mu_s$$  \hspace{1cm} (16)

where $\mu_a$ is the absorption coefficient and $\mu_s$ is the Compton scattering coefficient. The absorption coefficient is related to the $\beta$ term of the refractive index ‘$n$’ as

$$\mu_a = \frac{4\pi}{\lambda} \beta$$  \hspace{1cm} (17)

### 2.1.7 Contrast

Consider the case of a smaller object with an attenuation coefficient $\mu_2$ and thickness $t$, is embedded inside a larger object of thicknesses $d$. Intensity incident upon the detector after traversing the object is

$$I_1 = I_0 e^{-\mu_z}.$$  \hspace{1cm} (18)

$$I_2 = I_0 e^{-\mu_1(d-t)} e^{-\mu_2 t} = I_1 e^{-(\Delta \mu)t}.$$  \hspace{1cm} (19)
Figure 2-3. Schematic diagram for a conventional absorption imaging technique. The smaller object with attenuation coefficient $\mu_2$ embedded in a larger object of attenuation coefficient $\mu_1$.

The contrast is the normalized intensity difference,

$$\text{Contrast} = \frac{I_1 - I_2}{I_1} = 1 - e^{-(\Delta\mu)t} \quad (20)$$

The attenuation coefficient of an image is a function of energy, therefore, as the energy of X-ray photon increases, the contrast decreases.

2.1.8 Blur

The smallest detail that can be visualized by in an image is largely determined by the amount of blur produced by the imaging procedure. It reduces the visibility of details such as small objects and structures. Every image has some amount of blur. Some methods, however, produce images with significantly less blur than others. For example, mammography uses small source to produce high resolution images.
Blur (unsharpness) is especially noticeable at the edges within an image produced by X-ray tube focal spots, as shown in Figure 2-4 (a). A smaller focal spot produces less blur than the larger focal spot. Even the smaller focal spot can be considered as the collection of a large number of point sources from where the X rays are originating. These rays strike the edges of the object at different angles thus producing the blur. Figure 2-4 (b) and (c) show the simulated images of the plastic phantom by displacing the detector at two different places. As the detector is moved away from the source, the blur also increases because of the magnification.

Figure 2-4. (a) As the focal spot size increases, the blur on the edges of the images increases. (b) The blur also increases with increasing distance as illustrated with the computed (matlab) image of an edge with the detector placed at a large distance from the step edge. (c) the detector paced close to the step edge.
2.1.9 Absorbed Dose

Absorbed dose is the amount of energy absorbed per unit mass by an object or person,

\[
\text{Absorbed dose} = \frac{E}{M}
\]

where \(E\) is the energy and \(M\) is the mass. The usual unit of absorbed dose is milligray (Gy), 1 milligray = 1mJ/1Kg. In medical imaging, the overall approach is to reduce the dose delivered to patient without compromising the quality of the image because of the biological risk associated with these ionizing radiations.

2.1.10 Phase

X rays after passing through an object exhibit phase change. The propagation of X rays through matter can be described by the complex index of refraction, \(n = 1 - \delta - i\beta\), where the real part \(\delta\) causes a phase shift relative to an X ray in vacuum. The accumulation of phase after propagating a distance \(x\) is

\[
\phi = \frac{2\pi}{\lambda} \int \delta(x) \, dx
\]

For X-ray energies of up to 10 to 100 KeV, \(\delta\) is about \(10^{-6}\) to \(10^{-8}\) and \(\beta\) is about \(10^{-9}\) to \(10^{-11}\) as shown in Figure 2-5. The phase shift term is approximately 1000 times greater than the absorption term for soft tissues. Two soft tissues presenting similar attenuation may have quite different refractive indices. If the change of phase of the X rays is captured on the detector, these two soft tissues could be differentiated. Additionally, since \(\beta\) diminishes more rapidly than \(\delta\) with the energy, phase imaging can be performed even at higher energies where conventional methods provide little information. This feasibility of phase imaging at higher energies could lead to a reduction in dose as the absorption is reduced.\(^{10}\)
Figure 2-5. The variations of $\beta$ and $\delta$ curves with photon energies for breast tissue. $\beta$ varies more rapidly than $\delta$ [Figure from Ref.10].

Figure 2-6 is an image of a 3 mm acrylic phantom taken with two different system geometries. Image (a) was taken with the detector placed in close proximity to the phantom and image (b) with the detector displaced at a distance of 41 cm from the phantom. Edge enhancement is visible between two portions of phantom in the form of a distinct line when the detector was away from the phantom. The corresponding line profiles showed an increase in intensity of the peak (edge-enhancement) as the distance of detector to phantom was increased.

Figure 2-6. Images of the edge phantom. (a) Image obtained with the detector close to object. No edge enhancement is visible. (b) Image obtained with detector distance of 410 mm. Phase-contrast edge enhancement is visible as bright and dark lines running. (C) Three profiles taken at different detector to phantom distances. As the distances increases, the edge-enhancement, shown by the peak, increases [Figure from Ref.10].
Several phase contrast methods have been investigated to visualize the refractive properties of the sample. Often they require specialized optical elements and sophisticated alignment in addition to high resolution detectors.\textsuperscript{12} Propagation-based methods are simple and easy to implement since they rely on measuring the intensity at some displaced distance from the patient and are therefore well suited for the clinical environment.

The successful implementation of propagation-based phase imaging requires sufficient lateral coherence width, \( w \),

\[
w = \frac{\lambda d}{s},
\]

where \( \lambda \) is the wavelength, \( d \) is the distance of detector from the object and \( s \) is the source size. Propagation-based phase contrast methods require a small intense radiation source and a large distance between the object and the detector in order to allow the x rays of different phase shifts to interfere, as shown in

Figure 2-7. A high resolution detector is used to capture these fine interference fringes. Small sources are low power and therefore require relatively long time to get adequate intensity on the detector plane. We used a polychromatic rotating anode Mo X-ray source aligned with polycapillary optics to create a secondary intense source of small size.
Polycapillary optics are arrays of small hollow glass tubes. X rays are guided down these curved and tapered tubes by multiple reflections in a manner analogous to the way fiber optics guide light. X rays can be transmitted down a curved hollow tube as long as the tube is small enough, and bent gently enough, to keep the angles of incidence less than the critical angle for total reflection $\theta_c$, as shown in Figure 2-8. The critical angle for borosilicate glass is approximately $^{14}$

$$\theta_c \approx \frac{h \omega_p}{2 m \epsilon} = \frac{30 \text{ mrad}}{E} \text{ keV},$$

(24)

which is approximately $1.7^\circ$ for 1 KeV photons and $0.086^\circ$ for 20 KeV photons.
Polycapillary optics can be used to redirect X-rays to focus or collimate the beam if the capillary is not bent too much. The minimum allowable radius of curvature is $^{15}$

$$R \geq \frac{d\theta^2}{2}.$$  \hspace{1cm} (25)

For a given radius of curvature, the requirement that the incident angles remain less than the critical angle necessitates the use of small channel sizes, $d$, typically between 5 and 30 μm.$^{16}$

![Figure 2-8. Transmission of X rays through a bent capillary tube. The top ray entering at grazing incidence reflected back. The X ray entering at the bottom hits at a larger angle [Figure from Ref.16].](image)

A focusing optic receives radiation from the divergent source and redirects it to an intense small spot, as shown in Figure 2-9. Pinholes, after the focusing optic can reduce the size of the beam. We used two pinholes of 100 μm and 25 μm diameter. Pinholes spatially limit the source, increasing lateral coherence, while polycapillary optics converge the beam to a point resulting in increased intensity through the pinhole.

![Figure 2-9. Focusing mode of polycapillary optics where the rays are made to converge [Image from Ref.16].](image)
Although a pinhole without an optic would also produce the necessary coherence, the large increase in intensity due to optic will result in a reduction of the imaging time. This time reduction will reduce artifacts due to patient motion.

Incorporating the polycapillary optic in the beam path increases the intensity on the output focal point as compared to a pinhole. The gain of the optic is the rate of the intensity with and without the optic \(^1^4\)

\[
\text{Gain} = \left( \frac{d_{\text{in}}^2}{f_{\text{in}}} \right) T \left( \frac{L}{\sigma} \right)^2,
\]

where \(d_{\text{in}}\) is the input diameter of the optic, \(T\) is the transmission, \(f_{\text{in}}\) is the input focal length, \(L\) is the distance from the source and \(\sigma\) is the diameter of the pinhole.

For alignment purposes, the optic is placed near the source and is translated in the two dimensions perpendicular to the optic axis in small steps, producing a measurement of intensity versus relative source position. In order to determine the focal length of the optic, scans are repeated at increasing distances from the source. The plots are symmetric and Gaussian near the focal point, which indicates good alignment of the source, optic, and detector.

The minimum expected value of the scan width is given by the FWHM\(_{\text{calc}}\) at the input focal point is approximately \(^1^7\)

\[
\text{FWHM}_{\text{calc}} = \sqrt{s^2 + (1.5f_{\text{in}}\theta_c)^2},
\]

where \(s\) is the source size of 300 µm, \(f_{\text{in}}\) is the input focal length and \(\theta_c\) is critical angle given by equation 23. For an X-ray energy of 20 KeV, the critical angle is 1.5 mrad. Assuming a typical
polycapillary transmission of 10%, and using the anode-to-window distance of 60 mm to set a minimum value for L, the gain is about 40 for the measured spot size of 93 μm and would be several hundred for a smaller output spot. Alternatively, a smaller electron spot can be used on the anode. However, rotating anode systems typically are about a factor of 10 brighter (allow more current per millimeter of electron spot), but have spot sizes which are too large to employ in phase systems.

2.3 Phase Retrieval

The propagation of electromagnetic waves in the forward model along the z-axis can be described by the following equation

$$E(x,z) = E_0(x,z) e^{i\omega z} = E_0(x,z) e^{\frac{2\pi}{\lambda} z} e^{-\frac{2\pi}{\lambda} \delta z} e^{-\frac{2\pi}{\lambda} \rho z},$$

(28)

where the first term shows the vacuum propagation, second term shows the phase change and the third term shows the change in absorption as the waves propagates through the object. x is the point in the object plane perpendicular to the z-axis and $E_0$ is the incident electric field. As we are mainly interested in extracting phase, the other term will be factored out.

The propagation of a monochromatic wave can be described by a paraxial wave equation

$$\frac{\partial}{\partial z} E(x,z) = \frac{i}{2k} \nabla^2_x E(x,z),$$

(29)

where $\nabla^2_x$ denotes a 2D Laplacian in the transverse is plane and $k = \frac{2\pi}{\lambda}$ is the wave number.

Propagation-based phase-contrast images are edge-enhanced images where the intensity contrast can be described by propagation equations in terms of the object phase and attenuation. Phase
retrieval, a computational method for reconstructing phase from the measured intensity can be used to solve these equations for phase. The transport of intensity equation (TIE) is a commonly employed method for the phase retrieval. This equation provides a relationship between intensity and phase in the near field region. Substitution of equation 29 in equation 28 yields,

\[
\frac{\partial}{\partial z} I(x, z) + \nabla_x \cdot \left\{ \frac{1}{k} [I(x, z) \nabla_x \phi(x, z)] \right\} = 0, \tag{30}
\]

which is a linear differential equation for phase \( \phi \) provided \( I \) is measurable. The transport direction is the beam direction and \( \vec{x} \) is a vector in the transverse (x,y) direction. In order to solve this equation the derivative \( \frac{dl}{dz} \) can be approximated from closely spaced images using finite differences. The intensity changes generated by absorption and the intensity changes generated by phase will propagate differently, so if images are taken at different distances, the phase and absorption properties can be untangled. As an example, a contact image taken just after the sample contains only the contributions from absorption, and once the absorption is known the phase information can be retrieved from an image taken further away from the object. So with two intensity measurements at different distances from the source, the phase can be retrieved.\(^{18}\) For propagation-based X-ray phase imaging, the TIE relates the intensity \( I(x, z_d) \) at a distance \( z_d \) to the intensity \( I_o(x) \) at the exit face of a sample,

\[
I(x, z_d) \approx I_o(x) - \frac{z}{k} \nabla_x \left[ I_o(x) \nabla_x \phi_o(x) \right]. \tag{31}
\]
The second term on the right hand side of the equation 31 is associated with phase contrast due to the deflection of rays at an angle $\nabla_x \phi_o/k$, from which it is clear that phase contrast will be strongest at sharp edges in the object.

Practically, computing phase by taking two images displaced in z is problematic. As the phase features in the images are fine structures, the detector’s lateral shift must be kept less than one pixel (~10 to 50 $\mu$m) as it is displaced by ~0.5 m. Image registration may be attempted on shifted images but a shift of a single pixel can produce severe artifacts in the reconstructed image even if the registration can be solved to that level. There are several phase retrieval techniques that circumvent shift artifacts by reconstructing phase from a single image by making strong assumptions on the relationship between attenuation and phase. We employed two, weak attenuation and phase attenuation duality.

In the weak attenuation model, it is assumed that the object uniformly attenuates the incident intensity. The TIE simplifies to a Poisson equation for phase in terms of the uniform exit intensities $I_o$,

$$-k \frac{I(x,z) - I_o}{Z I_o} \approx \nabla^2 \phi.$$  \hspace{1cm} (32)

The phase retrieval for a single-plane is then

$$\varphi = \mathcal{F}^{-1} H_{wa}^{-1} \mathcal{F} g$$  \hspace{1cm} (33)

where $\mathcal{F}$ and $\mathcal{F}^{-1}$ are Fourier and inverse Fourier transforms respectively. Equation 29 assumes a plane wave incident on the object, but in practice cone beam geometry was utilized in which the radiation was coming from a point source. In this case the measured intensity is
\[
g = \frac{2\pi \left( M I(Mx, Z) - I_0(x) \right)}{\lambda Z}, \tag{34}
\]

M is the geometric magnification of the object on the detector plane, Z is the propagation distance, and \( I(Mx, Z) \) is the measured intensity at the detector plane, and the Fourier transform function is

\[
H_{WA}^{-1} = \frac{1}{(2\pi)^2 u^2}, \tag{35}
\]

where ‘u’ is spatial frequency.

The transfer function is the Fourier transform of the point spread function which relates the input and output of linear time-invariant systems. Its function is to scale the spatial frequencies. Because \( H_{WA}^{-1} \) diverges at \( u=0 \), any noise signal at low spatial frequencies will be amplified, causing low frequency contamination in the retrieved phase image as shown in Figure 2-10.

![Figure 2-10. The solution diverges at low spatial frequencies as the left images shows. The Tikhonov regularization parameter is used to suppress the low spatial frequencies around the origin as shown in the right image. [Figure from Ref.16](#)](image)

To handle this situation and to suppress the low frequency component around the origin, Tikhonov regularization is used, as shown in Figure 2-10. The transfer function is modified to include a regularization parameter, \( \alpha \),
The Tikhonov regularization parameter is used to suppress the low spatial frequencies around the origin as shown in the right image. $\alpha = 0.3$ mm for image (a) and $\alpha = 0.1$ mm for image (b). [Figure from Ref.16]

An alternative method to get phase from a single image is the phase attenuation duality method. This assumes that Compton scattering dominates the attenuation and both Compton scattering and phase are proportional to each other and to the projected electron density. The electron density is derived from a single phase-contrast image \cite{21} is

$$\rho_{e,p}(r) = \frac{-1}{\sigma_{KN}} \log_e \left( \int \frac{F^{-1}\left\{ \frac{\bar{F}(M^2I)}{I_o \left[ 1 + 2\pi \frac{\lambda^2 r_e R^2}{M \sigma_{KN} u^2} \right]} \right\}}{u} \right).$$

(37)

where $I_o$ is the intensity at the exit face of the object plane, $I$ is the intensity on the detector plane, $u$ is the spatial frequency vector in the object plane, $M$ is the magnification, $r_e$ is the classical electron radius, and $\sigma_{KN}$ is the Klein-Nishina total cross-section for Compton scattering from a single electron.

The transfer function for this case is

$$H_{pa} = I_o \left[ 1 + \frac{\pi z y(\lambda)}{M} u^2 \right].$$

(38)
where $\gamma(\lambda)$ is a constant and is equal to $\gamma(\lambda) = \frac{2\lambda^2 \tau_e}{\sigma_{k,n}}$. In terms of this transfer function, the Eq. (33) can be written as,

$$
\rho_{ep}(r) = \frac{-1}{\sigma_{kn}} \log_e \left( F^{-1} \frac{1}{h_{pa}} F^M \right).
$$

(39)

The low frequencies components around the origin still amplify but they do not diverge. Using the Tikhonov regularization, the low frequencies can be damped down as shown in Figure 2-12.

![Graph of the transfer function for the phase-attenuation duality.](image)

**Figure 2-12.** The graphs of the transfer function for the phase-attenuation duality. The low frequencies amplify but do not diverge.
3 Experimental Set-Up

A 5 KW Molybdenum (Mo) rotating anode X-ray source was used for this study. The source has a spot size of 300 µm. This source size is too large to visualize any phase effect at a reasonable displacement. From equation 22, for a spatial coherence width of about 100 µm, which is equal to the size of the detector pixel used in the project, the distance has to be about 5 m to observe any phase fringes. Because of the inverse square law for intensity, the X-ray intensity reaching the detector surface would be unacceptably small. As shown in Figure 3-1, the polycapillary optic collects radiation from a divergent source and focuses it onto a small point. Two optics (690 and 1759), one at a time, were used in this project. The input and output focal lengths of these optics are given in Table 3-1. A solid state CdZn AmpTek XR-100 CR X-ray detector was used for alignment. For imaging, a Fuji Computed Radiography plate M1040019 with an active area of 10 cm × 12 cm and 50 µm resolution or a phosphor coated CCD camera with an active area of 3 cm × 4 cm and a pixel size of 22 µm were employed. A 1.6 mm polyethylene rod was imaged to test edge enhancement. Insect images were also taken to determine the feasibility of phase imaging on a biological specimen.
<table>
<thead>
<tr>
<th>Manufacturer</th>
<th>X-ray Optical System Inc.</th>
</tr>
</thead>
<tbody>
<tr>
<td>ID Number</td>
<td>690</td>
</tr>
<tr>
<td>Input Focal Length (mm)</td>
<td>81</td>
</tr>
<tr>
<td>Output Focal Length (mm)</td>
<td>5.5</td>
</tr>
<tr>
<td>Channel Diameter (µm)</td>
<td>9.4</td>
</tr>
</tbody>
</table>

### 3.1 Alignment of optic with source

In order to align the optic for maximum transmission, it was first placed as close to the source as possible and translated iteratively in the horizontal and then in the vertical directions to achieve the maximum intensity on the detector, as shown in Figure 3-2. The optic was moved away from the source along the Z-axis and same procedure was repeated. At each distance, the data of the scan along the x and y axes was fitted separately to a Gaussian distribution,
as shown in Figure 3-3 using Origin 6.0 software. The variation of beam width with distance is shown in Figure 3-4. At the input focal point, the FWHM= $w\sqrt{2\ln(2)}$, was minimum with a value of 0.40 ± 0.05 mm. The expected scan width as calculated from Equation 25 is 0.35 ± 0.02 mm, which is in agreement with the measured value.

$$y = y_o + A e^{-\left(\frac{x-x_c}{w}\right)^2},$$

(40)

The same procedure was repeated with the second optic (1759) and the corresponding results are shown in Figure 3-5 and Figure 3-6. The beam width at the input focal point was 0.47 ± 0.03 mm. The calculated beam width was 0.45 mm which is close to the measured value.
3.2 Focal spot measurement

3.2.1 By Image plate

An image plate was used to estimate the output spot size. The image plate (Fuji M1040019) was placed close to the optic focal spot and the resulting image was then read in a Fuji BAS-1800 reader using Image Gauge software. The smallest resolution of 50 µm was selected during reading of the image plate.

Figure 3-7 (a) shows the magnified view of the spot at a distance of 5.5 mm from the optic. The profile across the focal spot size drawn in Figure 3-7 (b) gave a Gaussian fit with a FWHM of 110 ± 50 µm at the output focal point. The variation of spot size with the distance from the optic is shown in Figure 3-8.
The expected output focal spot size of the optic is

$$w_{\text{calc}} = \sqrt{c^2 + (1.5f_{\text{out}} \theta c)^2},$$ \hspace{1cm} (41)

where $c$ is the channel size of a single fiber and $f_{\text{out}}$ is the output focal length of the optic. For the optic 690 used here, the expected value of the spot size is 15 µm. The difference between the measured and expected focal spot size is likely due to the manufacturing errors in the optic as it is difficult to make the final cut of the optic at the plane which would ensure that all the channels are aligned to a point. An error in cutting the optic can result in a large focal spot. The optic was
actually designed to collimate the rays at a large focal point but in this project it was used in the reverse direction to get small spot size.

The graph of spot size FWHM variation with the distance from the source for optic 1759 is shown in Figure 3-9. The spot size was measured to be 93 µm ± 0.03 while the value as predicted by equation 25 was 20 µm.

![Figure 3-9. Spot size as a function of distance for optic 1759. The spot size is a minimum value at the output focal point.](image)

To ensure a small enough secondary source for phase imaging, 25 µm and 100 µm diameter pinholes were placed, one at a time, at the output focal point of the optics 690 and carefully aligned to maximize the output. After aligning the optics and pinhole with the X-ray source, the output beam had high enough spatial coherence at a reasonable detector displacement for phase imaging and was used to form phase contrast rods and insects.
3.2.2 Knife-Edge Scan

The knife-edge method is a beam profiling method for determination of the size of the X-ray beam focal spot.

![Diagram of knife-edge scan](image)

Figure 3-10. The schematic diagram of knife-edge scan. The knife-edge moves perpendicular to the direction of the beam propagation.

The knife-edge method requires a sharp edge, in the form of a razor blade, a computer-controlled translation stage and a detector that records the intensity of X-rays as shown in Figure 3-10. The knife-edge is translated perpendicular to the direction of propagation of the beam. As the blade moves across the beam, X-rays are blocked from reaching the detector and the measured intensity continuously decreases to zero as shown in Figure 3-11.

![Intensity vs. scan range graph](image)

Figure 3-11. Knife-edge scan of the X-ray beam at a distance of 5mm from the optic 690. The long tail leads to broad diameter of the beam.
The intensity was then differentiated to obtain the beam profile. The derivative of the data was taken at any point by averaging the derivative of two adjacent data points using the equation,

\[ \frac{dl}{dx} = \left( \frac{y_{i+1} - y_i}{x_{i+1} - x_i} + \frac{y_i - y_{i-1}}{x_i - x_{i-1}} \right), \]

(42)

Where \( \frac{dl}{dx} \) is the rate of change of intensity perpendicular to the beam direction. The FWHM found using Gaussian fit was 100 μm which agrees with the image plate measurement.

![Graph](image)

*Figure 3-12. The differential graph of Figure 1 using equation 1. The broad width is a result of the long tail as shown in Figure 3-11.*

As the knife-edge scan results do not agree with the theory, computed radiographic image plates then used to measure the spot size.

### 3.3 Image Measurement

Several images of a 1.6 mm diameter polyethylene cylindrical rod and a cricket were taken to show phase contrast edge enhancement using optics 1759 and 690. The latter was also aligned with 100 μm and then 25 μm pinholes to reduce the spot size.
3.3.1 Image of Rod

Several images of the rod were taken by changing source-to-rod and rod-to-detector distances and with different pinholes and detectors as described in Table 3-2.

Table 3-2. Description of different parameters used to image 1.6 mm diameter polyethylene rod.

<table>
<thead>
<tr>
<th>Optic</th>
<th>Pinhole</th>
<th>KV</th>
<th>mA</th>
<th>R1 (cm)</th>
<th>R2(cm)</th>
<th>Time(minute)</th>
<th>Magnification</th>
<th>Detector</th>
<th>EE/N</th>
</tr>
</thead>
<tbody>
<tr>
<td>1759</td>
<td>None</td>
<td>30</td>
<td>20</td>
<td>11</td>
<td>24</td>
<td>4</td>
<td>3.2</td>
<td>IP</td>
<td>4.8</td>
</tr>
<tr>
<td>1759</td>
<td>None</td>
<td>30</td>
<td>20</td>
<td>14</td>
<td>76</td>
<td>4</td>
<td>6.4</td>
<td>IP</td>
<td>5.6</td>
</tr>
<tr>
<td>690</td>
<td>100</td>
<td>30</td>
<td>30</td>
<td>14</td>
<td>25</td>
<td>26</td>
<td>2.8</td>
<td>IP</td>
<td>2.1</td>
</tr>
<tr>
<td>690</td>
<td>100</td>
<td>40</td>
<td>40</td>
<td>17</td>
<td>64</td>
<td>2.5</td>
<td>4.7</td>
<td>IP</td>
<td>3.1</td>
</tr>
<tr>
<td>690</td>
<td>25</td>
<td>30</td>
<td>40</td>
<td>25</td>
<td>55</td>
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<td>3.2</td>
<td>IP</td>
<td>2.5</td>
</tr>
<tr>
<td>690</td>
<td>25</td>
<td>40</td>
<td>40</td>
<td>25</td>
<td>37</td>
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<td>4.2</td>
<td>IP</td>
<td>2.7</td>
</tr>
<tr>
<td>690</td>
<td>25</td>
<td>40</td>
<td>10</td>
<td>37</td>
<td>45</td>
<td>15</td>
<td>2.6</td>
<td>CCD</td>
<td>4.8</td>
</tr>
<tr>
<td>690</td>
<td>25</td>
<td>40</td>
<td>10</td>
<td>37</td>
<td>90</td>
<td>15</td>
<td>3.4</td>
<td>CCD</td>
<td>6.5</td>
</tr>
</tbody>
</table>

3.3.1.1 Rod Images with Optic 1759

A rod image was taken using optic 1759, as shown in Figure 3-13. The rod was placed at a distance of 11 cm from the output focal point of the optic (R₁) and the image plate was placed approximately 24 cm from the rod (R₂) giving a magnification

\[
M = \frac{R_1 + R_2}{R_1}, \quad (43)
\]

of 3.2. The rod was exposed for 4 minutes with voltage of 30 kVp and a current of 20 mA. The bright white spot in the center of the images might be due to the direct rays coming from the polycapillary optics that did not converge to the focal point, but rather cut straight through the glass of the optic.

The presence of noise in the image diminishes the strength of the phased enhanced signal. The conspicuity of the edge enhancement can be described by the edge-enhancement to noise ratio (EE/N) ratio.
Figure 3-13. Image of a 1.6 mm rod taken with optic 1759. The white line along the length of rod shows edge enhancement.

\[
\frac{EE}{N} = \frac{P - \overline{B}}{\sigma_B},
\]

where \( P \) is the peak intensity, \( \overline{B} \) is the average intensity over a region of background labeled B as shown in Figure 3-14 which is a graph of profile taken perpendicular to the length of the rod in the image of Figure 3-13. This line was created by summing five neighboring pixels along the length of rod. The peaks on the both sides of the valley show edge-enhancement. The edge-enhancement-to- noise ratio was 4.8 ± 0.2.

![Graph of profile taken in figure 3.14. The two peaks on both sides of the valley show edge-enhancement.](image)

Figure 3-14. The graph of profile taken in figure 3.14. The two peaks on both sides of the valley show edge-enhancement.

The second image with the optic 1759 was also taken by changing the objects to detector distance to 76 cm with the rest of the parameters kept constant. The white line shows edge enhancement. The image looks grainy because the current and exposure time were not increased.
to compensate the reduction in intensity of radiation with the increase of object-to-detector distance. The objective was to get adequate quality images with the lowest dose to phantom.

![Image of a 1.6 mm rod taken with optic 1759. The white line along the length of rod shows edge enhancement.](image1)

Figure 3-15. Image of a 1.6 mm rod taken with optic 1759. The white line along the length of rod shows edge enhancement.

Figure 3-16 shows a graph of profile taken in the image of Figure 3-15. The peaks on the both sides of the valley show edge-enhancement. The edge-enhancement-to-noise ratio was 5.6. There is substantial variation in intensity across the object due to optic non-uniformity.

![Graph of profile taken in figure 3.15. The two peaks on both sides of the valley show edge-enhancement.](image2)

Figure 3-16. The graph of profile taken in figure 3.15. The two peaks on both sides of the valley show edge-enhancement.

### 3.3.1.2 Rod Images with Optic 690

#### 3.3.1.2.1 100 µm Pinhole

A cylindrical 1.6 mm diameter polyethylene rod was imaged using a 100 µm diameter pinhole. The rod was placed 14 cm from the pinhole ($R_1$) as shown in Figure 3-1 and the image
plate was positioned at a distance of 25 cm from the rod \( (R_2) \) was 4.7. The tube voltage was 30 KV and the current was 40 mA. The exposure time was increased to 26 minutes, but that did not substantially increase the detected intensity due to erasing of intensity on the image plate in the presence of visible light during the exposure.

The image captured at detector is shown in Figure 3-17 and presents a dark region within the body of the rod due to attenuation, as well as a white band at the rod edge which is edge enhancement due to phase contrast.

![Figure 3-17](image1.png)

*Figure 3-17. Image of the 1.6 mm diameter rod taken with a 100 µm pinhole. The white lines show edge enhancement.*

To make the white line in Figure 3-17 more prominent with less exposure time, the detector was displaced to a distance of 64 cm from the object. The tube voltage was increased from 30 to 40 KV since the detector was displaced at larger distance from the object but the current was decreased from 40 to 30 mA to keep the power constant.

![Figure 3-18](image2.png)

*Figure 3-18. Image of 1.6 mm diameter rod taken with a 100 µm pinhole. The white line is more prominent than the image in Figure 3.10.*
The exposure time was two and a half minutes. The product of current and exposure time is higher than the normally used for soft tissue imaging in clinical settings, however, the intensity loss was due to optic, pinhole and large distance between object and detector and would not create additional patient dose. In addition, an optic which is designed for focusing would produce a small spot without need for a pinhole and thus would have higher intensity and shorter exposure time.

For quantitative analysis, a profile of the rod was drawn perpendicular to the length of the rod and plotted against length of the profile, as shown in Figure 3-19. A background division by a profile taken through a no object image was performed to remove the effect of background intensity variation due to the optic structure in the image. Still both the intensity dip due to attenuation and the intensity peak due to phase are evident in this plot although the peak is somewhat obscured by noise.

![Figure 3-19. Profile of the 1.6 mm diameter rod taken with a 100 µm pinhole. The small peaks on both sides of the valley are due to edge enhancement](image)
3.3.1.2.2 25 µm Diameter Pinhole

The 1.6 mm diameter polyethylene rod was then imaged using a 25 µm diameter pinhole with the result shown in Figure 3-20. The rod was placed at a distance of 25 cm from the output focal point of the optic \( R_1 \) and the image plate was placed approximately 55 cm from the rod \( R_2 \) giving a magnification of 3.2. The rod was exposed for 2.5 minutes with voltage of 30 kVp and a current of 40 mA as shown in Figure 3-19, and \( \sigma_B \) is the standard deviation of intensity over region B. The edge-enhancement to noise ratio was 0.9 ± 1. The uncertainty is high because of the high noise and low signal.

![Image of a 1.6 mm rod taken with a 25 µm pinhole. The white line along the length of rod shows edge enhancement.](image)

Another image of the same rod was taken with different system geometry as shown in Figure 3-21. The image plate was placed approximately 110 cm from the rod \( R_2 \) keeping \( R_1 \) constant. The distances were increased to increase the beam coherence. The magnification in this case is 5.2. The rod was exposed for 6 minutes with voltage of 40 kVp and a current of 30 mA. The exposure time was increased to accommodate the loss of intensity in using the 25 µm diameter pinhole. The edge-enhancement to noise was 2.5 ± 1.
3.3.1.3 Charged Coupled Device (CCD) Camera

The previous rod images were taken with the image plate that had a pixel size of 50 µm. Edge enhancement tends to be a very narrow bright fringe and can be better imaged with smaller pixels. A higher resolution digital detector (CCD camera) with a pixel size of 22 µm was used to test whether the edge enhancement could be improved. Flat-field and dark field images were acquired with no rod in place and were applied to correct images. The dark field image is taken with no illumination. Dark image is subtracted from both the object and no object image before division. Physically, the dark field image is due to the random generation of electrons and holes within the depletion of the device.
Flat-field correction is used to remove any non-uniformities that are caused by variations in the pixel-to-pixel sensitivity of the detector and non-uniformity in the illumination. The image taken with no object is used to normalize object images. Hence a second image with no object would be completely uniform flat image. The flat-field correction image taken with no object for optic 690 with 25 μm pinhole is shown in Figure 3-22.

Figure 3-23 shows image of the rod with different system geometry. The image was taken with the optic 690 aligned with 25 μm pinhole. The rod-to-pinhole distance was increased to 37 cm, and an image of the rod was taken with a detector-to-rod distance of 45 cm, for a magnification of 2.6. As the exposure time per frame was limited to 30 seconds in the camera setting, average of 30 frames was taken to increase the exposure time to 15 minutes. The tube voltage was 30 kVp with a current of 40 mA. The edge-enhancement to noise ratio was 4.8 ± 1.

![Figure 3-23. A 1.6 mm diameter rod image taken with CCD camera after flat field correction. Two bright lines along the length of rod are clearly seen.](image)

Images were taken with the rod-to-pinhole distance of 37 cm, and the detector-to-rod distance was increased to 90 cm, for a magnification of 3.4. A bright line along the length of the rod could be seen on both sides of the rod as shown in Figure 3-24. The edge-enhancement to noise ratio (EE/N) for this image was 6.5 ± 1.
Additional images were taken at varying object-to-detector distances as shown in Table 3-3. The effect of increased magnification on edge enhancement is shown in Figure 3-25.

Table 3-3. The EE/N ratio increases linearly with the detector to object distance. The optic was 690 with 25 μm pinhole.

<table>
<thead>
<tr>
<th>R_2 distance (cm)</th>
<th>EE/N ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>17</td>
<td>1.8 ±1</td>
</tr>
<tr>
<td>25</td>
<td>2.1 ± 1</td>
</tr>
<tr>
<td>27</td>
<td>2.6 ± 1</td>
</tr>
<tr>
<td>40</td>
<td>5.5 ± 1</td>
</tr>
<tr>
<td>64</td>
<td>6.6 ± 1</td>
</tr>
</tbody>
</table>

The edge enhancement increases approximately linearly with increasing distance of detector to object as expected from the work of other investigators\(^6\) using a small spot microfocus source with no optic. The secondary source provided by a polycapillary optic and pinhole performed as a microfocus source.

Figure 3-24. A 1.6 mm diameter rod image taken with CCD camera after flat field correction. Two bright lines along the length of rod are clearly seen.
3.3.2 Image of Insect

Insect images of a dead cricket were taken using optic 690 with 25 μm pinhole to demonstrate the feasibility of phase contrast in biological tissues. A conventional absorption image, shown in Figure 3-26, was also acquired with the insect at a distance of $R_1 = 40$ cm from the source, with the image plate placed just behind the object at a distance of $R_2 = 2$ cm. The phase contrast image shown in Figure 3-27 was taken at a distance of 67 cm from the insect keeping $R_1 = 40$ cm. The absorption image was taken with tube voltage of 30 KV, current 20 mA and time 2 minutes. The phase-contrast image was taken with voltage of 40 KV, current 30 mA and time 6 minutes.
The phase contrast image in Figure 3-27 shows not only the internal structures (due to absorption) but also shows clearly defined boundaries evidencing edge enhancement. Some of the improvement is due to the smaller effective pixel size due to the magnification; however there is visible edge-enhancement.

### 3.4 Quantitative Phase Retrieval

In addition to simple edge-enhancement, significant information about the material in the object could be extracted if a quantative phase can be calculated. Quantative phase retrieval was performed for rod image shown in Figure 3-24 using the weak attenuation model. Each reconstruction (from image B to D) in Figure 3-28 shows the phase at each pixel, calculated by assuming that the phase is proportional to electron density as described in equation 36.

Tikhonov regularization was used with a range of regularization parameters $\alpha = 15.8, 0.3$ and 0.1 mm$^{-1}$. Decreasing the parameters increases the different effects on low frequencies and
therefore on the quality of the retrieval phase images. The background in phase imaging is comprised of low frequencies while the higher frequencies contain the sharp features of interest. For $\alpha = 15.8 \text{ mm}^{-1}$, the corresponding size of the features is about 63 $\mu$m and therefore any feature that has size of more than 100 $\mu$m can be distinguished. For $\alpha = 1 \text{ mm}^{-1}$, the minimum feature size is 1 mm as in Figure 3-28 (D). In that case the low frequency structure dominates.

![Raw image (A)](image)

![B](image)

![C](image)

![D](image)

Figure 3-28. Image A is the phase contrast image of the rod (raw image). Images B & C and D were processed with the value of regularization parameter $\alpha=15.8, 0.3$ and $0.1 \text{ mm}^{-1}$. As the value of $\alpha$ decreases, the low frequencies damp down and higher frequencies (showing edges)

The pure phase,

$$\phi = \frac{2\pi}{\lambda} \int \delta(x) dx,$$

(45)

The value of $\phi$ calculated for polyethylene rod of 1.6 mm, assuming an average energy of 20 KeV, an average density of 0.90 g/cm$^3$ and a refractive index decrement $\delta \approx 5.74 \times 10^{-7}$, was 80 radians while the measured values are in the range 0 to 40 radians. The difference between the calculated and experimental phase values may be due to the underlying assumption made
during the calculations of phase as well as the discrepancies in the model used to reconstruct the phase. The weak attenuation model assumes uniform object with uniform attenuation.

The lowest value of the regularization parameter does the best job of actual phase, but at the expense of losing spatial resolution.

The same rod, image of Figure 3-24, was also processed using the phase attenuation duality (Equation 38 and 39) model with the results shown in Figure 3-29.

Phase attenuation duality assumes that phase and attenuation can be considered proportional to each other. The calculation also assumed that imaging was performed at 60 KV where Compton scattering is the dominant process. Noise in the image because of the polycapillary, extensive regularization and misregistration of images with and without object may also be the causes of deviation of the phase values from both models.

The insect images were also reconstructed using the same two approaches. Figure 3-30 (A) is the raw phase contrast image of the insect same as shown in Figure 3-27. Figure 3-30 (B) is the phase reconstructed images using weak-attenuation duality image and Figure 3-30 (C) is the phase reconstructed image using phase attenuation duality approach. The value of Tikhonov
regularization used was $\alpha = 0.03 \text{ mm}^{-1}$. Both processed images show an enhanced image contrast along with improved noise reduction. The image obtained using phase–attenuation duality (image C) can be perceived as being smooth while that obtained using weak-attenuation (image B) has higher resolution features because of the large value of regularization parameter for the weak attenuation model removes all of low frequency features.

![Figure 3-30](image)

Figure 3-30. The image A is the phase contrast image (edge-enhanced image) of the insect. Image B is the pure phase image using weak-attenuation duality model. Image C was obtained using phase-attenuation duality approach. Image D was obtained by applying Gaussian filter on image C.

As we are mainly interested in recovering features that correspond to sharp variations of phase, a Gaussian filter of width 20 pixels wide was applied on the image C. The Gaussian filter removed the low frequency components present in the image and retaining the high spatial frequencies. The Gaussian filter acts like a high pass filter.
4 Conclusion & Future Work

4.1 Future Work

During this project insect images were taken to observe the feasibility of phase contrast images in biological sample. This can be extended to image mice to provide a mammalian model. Most insect images were taken with a maximum of X-ray tube potential of 45 KV. Higher energies can be used to reduce dose. Background subtraction to remove the non-uniformities in the images was hindered by uncertainties in image registration. This might be overcome by using a reference marker. Additional optimization of the optical system is required to reduce the exposure time.

4.2 Conclusion

Propagation-based phase imaging was successfully tested with a large spot rotating anode X-ray source aligned with polycapillary optic. The polycapillary optic accepts radiation from a large X-ray source and converges it to create a small spot. A rod of 1.6 mm diameter was imaged with different geometric configurations using an image plate and CCD camera. Edge-enhancement to noise ratios up to a value of 6.5 were obtained. Conventional absorption and phase images of an insect were acquired to determine the feasibility of phase imaging in biological tissues using this technique. A pure phase object was obtained using phase attenuation duality and weak attenuation approaches by taking only a single image.
5 References


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