Dismantling the birdcage: adolescent girls' attitudes towards learning mathematics with a relational pedagogy in a problem-based environment

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DISMANTLING THE BIRDCAGE:

ADOLESCENT GIRLS’ ATTITUDES TOWARDS LEARNING MATHEMATICS
WITH A RELATIONAL PEDAGOGY IN A PROBLEM-BASED ENVIRONMENT

by

Carmel Schettino

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Dismantling the Birdcage:
Adolescent Girls’ Attitudes towards Learning Mathematics with a Relational Pedagogy in a Problem-Based Environment

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To Anne and Frank Schettino, my first and last teachers
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As everyone who has supported me through this process knows, the work that is in this study is much more than a dissertation project for me. It has been a passion for bringing my love of teaching and learning to the classroom and helping all students who have ever craved more from their learning to attain that goal. I could not have reached this accomplishment without the care, support, guidance, and yes, comfort, from a huge group of people that I see as my own learning community in my life. For this and so much more, I must take a moment and acknowledge the gratitude I have for the part they all played in this truly extraordinary feat.

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ABSTRACT

Although the Gender Achievement Gap is closing in mathematics, the “interest gap” in pursuing STEM fields is not. Mathematics education research has discussed constructivist, student-centered and inclusive methods of teaching that have been found to encourage students that have underachieved and been underrepresented in STEM fields. One recommendation for encouraging young women to continue in their mathematical studies is to find instructional methods that allow them to feel included in the learning process and “unfix” the dichotomous, gendered socio-cultural view of mathematics and the mathematics classroom fostering positive attitudes such as self-confidence, empowerment and agency.

In this study, the journeys of five adolescent girls currently studying secondary mathematics were explored. These girls were identified to be learning in classrooms led with a pedagogical approach that I am calling Relational Problem-Based Learning (RPBL) which seeks to encourage such attitudes. I chose to explore the relationship between the pedagogical approach and the girls’ attitudes of value in the subject of mathematics, enjoyment of study, self-confidence in their ability in mathematics and their own senses of empowerment and agency in the classroom. Their experiences in this relational, discourse-driven world of mathematics were often compared to their past experiences in more traditional, direct instruction classrooms and shared through multiple narrative methods.

This study found that although each girl’s journey was quite individual to their own interests and past experiences, the RPBL classroom made an extremely positive
impact on her attitudes towards herself as a mathematics student. Specifically, in every case each girl’s feeling of self-confidence in and enjoyment of studying mathematics, as well as their senses of empowerment and agency in the classroom were described as positively impacted by the RPBL pedagogy. Although the value of the material taught was not as impacted, they did see the value in the “soft skills” that came with mathematical problem solving skills and the discourse and communication skills that came along with the RPBL method giving further credence to this instructional approach and making mathematics class even more meaningful in more contexts.

A theoretical framework for the RPBL Classroom is described in order to support the mathematics classroom that may positively impact a whole population of students in our country that are not getting their needs met in their learning of mathematics. Research shows this population is not just young women and if we want to create true equity in STEM fields we must listen to that call for change before too many of them are left behind.
If you look very closely at just one wire in the cage, you cannot see the other wires… You could look one wire up and down the length of it, and be unable to see why a bird would not just fly around the wire any time it wanted to go somewhere… There is no physical property of any one wire… that will reveal how a bird could be inhibited or harmed by it except in the most accidental way. It is only when you step back, stop looking at the wires one by one and take a macroscopic view of the whole cage, that you can see why the bird does not go anywhere; and then you will see it in a moment. It is perfectly obvious that the bird is surrounded by a network of systematically related barriers, no one of which would be the least hindrance to its flight but which by their relations to each other, are as confining as the solid walls of a dungeon. (p.5)

INTRODUCTION

I will begin with a story. It is the story of a young girl excited and interested in learning and doing in all aspects of her elementary education. Luckily, her parents were always encouraging and supportive of her learning goals and her initial schooling included “enrichment” class for which she was chosen to receive out-of-class group instruction in advanced topics – including mathematics and science. The girl was confident, motivated and eager to move forward in her exploration of new topics and share these ideas with her friends and family. As middle school approached, it became clearer to the girl that categorizing students by ability became more important and she realized that her work and grades in her classes, as opposed to her interest in mathematics, would begin to determine her path through her education. The pressure of this realization, and possibly other determinants, affected her performance and she was placed in a pre-algebra course in the eighth grade, which she knew, even then, would set her on a trajectory that somehow indicated less success.

However, the following year, the girl’s work in algebra was so successful that her teacher that year recommended that this adolescent girl now double-up in her mathematics courses and take geometry and a second year algebra course concurrently. Reinvigorated and more confident in her abilities, she regained her momentum and faith in herself as a mathematics student, although the fun with her peers and connections with the teacher from her “enrichment” classes were now a thing of the past. Mathematics seemed made up of a set of disjointed courses that needed to be passed sequentially in order to fulfill the requirements for graduation.
Finally, the ultimate course in mathematics came during her senior year of high school where she would be able to truly show that she had made it to the top – Advanced Placement Calculus. However, difficulties arose when little interaction occurred between the teacher and the students surrounding mathematics in the classroom. Utilizing a textbook that was published almost 25 years earlier, the now young woman felt isolated and alone in a class where asking questions seemed to signify weakness and demanding an explanation also showed that a student was incompetent. Students who could easily and quickly replicate the mathematical exercises performed by the teacher were praised and favored whereas those with difficulties were dismissed and even asked not to take the Advanced Placement exam at the end of the year. Sadly, our young lady was among those disinvited to be part of the elite exam takers. This was a turning point in her desire to continue with mathematics as an intellectual endeavor. She vowed to never take a math class again and moved on to college to pursue music as a major field of study.

On arriving at her chosen college in the fall, the young woman was required to take a mathematics placement exam in order to fulfill her natural science portfolio requirement. Begrudgingly, she took the short test and a few days later she was told she could register for Calculus III. How was this possible? She did not even take the AP exam in May and barely passed the course in high school. Would this roller coaster ride of messages of encouragement and discouragement ever end? Who did they think were, telling her to move into Calculus III? She would show them and just retake Calculus I and be done with it – get that natural science requirement out of the way and move onto much more interesting and meaningful courses so that she could leave mathematics in the dust.
However, something surprising happened in that basic Calculus I course that fall. The young woman had an interested professor that saw her potential and talents. The professor engaged her in conversation about mathematical justification and questioning. Citing the young woman’s exceptional ability in Calculus, the professor questioned why she was even in the class. At the end of the term, the professor had convinced the young woman to continue on and even elect to take a computer programming course to see if she liked it. “Why not?” the professor said, “and it’s required just in case you decide to be a math major someday.” The young woman laughed out loud.

One by one mathematics courses came and went. The smaller seminar style of the upper level mathematics electives worked extremely well for her learning style. Although she was often one of two, or the only female in the course, the girl believed that she was supported and encouraged by the professors she met. There was a community of mathematicians who allowed her to grow and develop her skills, as opposed to suppress and discourage them. Abstract courses like Linear Algebra, Number Theory and Topology connected much of the mathematics that for too long seemed discrete and disconnected. After serving as a teaching assistant for much of the department and receiving honors on her senior thesis, the girl was encouraged to apply to graduate programs.

In graduate school, once again the girl found isolation among a male-dominated community of academics and senior mathematics professors seemed to look differently at her, wondering why she was not in the Master of Arts in Teaching program with the women. After two years of struggling with the environment, but quite enjoying and thriving in the teaching classroom, the young woman realized her gift and decided to find
a way to make her journey complete. Combining her love of mathematics and her talents for teaching was the way to make a life worth living while also bringing the consistent support and encouragement to students that she so greatly needed all those years. Although it took her 20 years to realize this direction, ultimately it became a passion and lifelong commitment.

At this point in stories like these it is generally tradition to state “The End.” However, at this point, I would change the phrase and say “The Beginning.” Yes, it was just this story that has led me to this place and passion in my research for gender equity in mathematics education. Now, 22 years into my teaching career I can look back and see how it began with this personal experience, but when I started my teaching and my doctoral program, I am not sure I was as aware of the implications my own story had for my research and teaching interests. As my career brought me in and out of single-sex schools, my research interests led me towards a relational pedagogy. As individual students that I crossed paths with shared their own hopes and fears about their relationship with mathematics, it began to be clear to me that it was more than a coincidence that my dissertation research, and perhaps my life’s work, would be centered around finding ways to improve the education of marginalized students in mathematics education, if possible.

And so it was the beginning – the beginning of a long journey with this question about how it might happen - how to improve the learning of students who feel marginalized in the world of mathematics as I once did. But I would begin with one group of marginalized students in the mathematics classroom to whom I could relate most readily; adolescent girls.
CHAPTER 1

Educational Research Problem

The Call for Equity

During the past few decades, there has been a call for change in the way mathematics is taught in American schools to provide equity and accessibility for all (Leder, 2003), including changes specifically focused on underrepresented and underperforming students in our society due to gender, race/ethnicity, class or socio-economic status (SES) (McGraw, Lubienski, & Strutchens, 2006). For years before any headway was made in the area of equity in achievement, the social justice and gender equity movements in educational research called for a movement away from the “banking” concept of education to one of more dialogue so that all groups could be heard equally (Boaler, 1994; Elizabeth Fennema & Peterson, 1985; Freire, 1970). The realms of constructivism and feminist mathematics pedagogy overlapped to provide intersections that brought into focus clear characterizations of the instruction needed to help support equity and inclusiveness for women and girls in mathematics classrooms (Mau & Leitze, 2001; Meece & Jones, 1996; Solar, 1995). Correspondingly, others have recognized the importance of alternative instructional methods for other marginalized groups in the discipline of mathematics and ways in which they might best be served in the classroom (Gutstein, 2007; Ladson-Billings, 1995; Lubienski, 2007; Spielman, 2008). The overall goal here being to increase the number of total underrepresented students interested in continuing with future mathematical studies and increasing the diversity, both in people
and ideas, in the field of mathematics, and STEM (Science, Technology, Engineering and Mathematics) fields in general – creating a better field for America and the future.

Some studies found safety and equity in mathematics classes especially to be issues for underrepresented groups like females and racial or ethnic minorities, or those with lower ability or interest levels (Boaler, 2008; Kellermeier, 1996). So it was even more revolutionary for the National Council of Teachers of Mathematics (NCTM) to include the “Equity Principle” in their standards which states that “excellence in mathematics education requires equity – high expectations and strong support for all students” (NCTM, 2000). This helped propel the already forward-moving gender equity and social justice movements in mathematics education even farther ahead. Educational research is coupling the two recommended standards of improved mathematics communication and equity by looking at how discussion and more relationally-based teaching methods are often preferred by, and improve learning for, marginalized groups in mathematics classrooms (Boaler, 2008; Lubienski, 2007).

Closing the Gap

It is true that for the past decade, the achievement gap in mathematics education has been closing remarkably. Females have higher grades in math and science courses in secondary schools than males, scores between male and female students on the NAEP exams have grown closer than ever (although males are still slightly higher) and the number of women choosing certain mathematics and science undergraduate majors has increased annually (Hill, Corbett, & St. Rose, 2010). However, it has also become evident that men still outnumber women in STEM fields in graduate degrees and professional work in STEM fields. In the 2011 research study *Generation STEM,*
conducted by the Girl Scout Research Institute including approximately 852 teenage girls, of girls who categorized themselves as STEM-girls, only 31% saw themselves as choosing a career in mathematics and of those not categorizing themselves as STEM-girls only 5% (Modi, Schoenberg, & Salmond, 2012). In their 2010 report, *Why So Few?*, one of the American Association of University Women researchers’ conclusions is that girls’ achievements and interest in mathematics are “shaped by the environments around them” (Hill, Corbett, & St. Rose, 2010). For girls especially, it seems the mathematics classroom environment has a great influence on their attitudes towards learning and it is greatly affected by the relationships and beliefs that are forged in those classrooms. The GSRI study also stated that learning the STEM classrooms needs to “tap into girls’ inquisitive thought processes” in order to keep their interest in those classes and draw them to those fields (Modi, Schoenberg, & Salmond, 2012). A 2007 U.S. Department of Education report recommended encouraging girls in mathematics by “embedding word problems” in the daily routine, giving access to “narrative texts” as they participate in class and having them “solve a series of problems” with their peers (Halpern et al., 2007).

Another issue pervasive in our culture is the idea of the implied masculinity of the field of mathematics and STEM fields in general. This may seem like an outdated idea, but the antiquated stereotypes are embedded in our culture from the all types of media messages to the textbooks of today to implicit gender bias in STEM fields that is extremely difficult to name. An interesting quantitative study published in 2012, did comparative advantage analysis on pre-existing data from three longitudinal educational studies regarding student grades and college majors (Riegle-Crumb, et. al, 2012). There was a rather enlightening finding that came out of this study relevant to the question of
gender equity in STEM fields. Even though for years, educational research has focused on girls’ high school achievement in mathematics, it seemed that it was not the boys’ “comparative advantage in math test scores over reading test scores that works to chip away a little at the gender gap in choice of major,” but more importantly it is the girls’ advantage in their English grades over time (1982-2004) that are leading them to not choose the STEM fields (Riegle-Crumb, et al, 2012). Because of this, it seems that the cultural issue of the dichotomous male-female roles that have existed in our culture continue to in part dictate, whether explicitly or implicitly, the decisions young women are making. Riegle-Crumb goes on to state:

U.S. culture…still strongly endorses the idea that the genders are innately “equal but different” and therefore continues to encourage the enactment of distinct affinities….Our cultural and institutional logics include the maintenance of gender essentialist beliefs, noting that a large part of women’s progress in the educational and occupational sectors is in domains that do not violate traditional gender roles; and even when they do enter male-typical domains, women are more likely to choose those subfields within them that seem “consistent with their (tacitly gendered) notions of their interests and their ‘true selves.’ From this perspective, girls may improve their math and science achievement…because such measures of performance are generally important for college matriculation…Yet because of societal pervasiveness of gender essentialist beliefs and the accompanying socialization and micro-level interactions that support them, gendered patterns in choice of major will not shift accordingly. (p.1067)

So although there has been an improvement in the Achievement Gender Gap, when analyzing standardized test scores and GPAs, the need to achieve in high-level mathematics classes for college admissions in the past 20 years could possibly account for this. Once young women reach college and are asked to truly follow their passions and find pursue their own course of study, there could be more at play that has to do with the subject matter, the classrooms they are in and the interactions contained in both.
Some gender theorists and educational researchers claim that the “level of interaction and exchanges” in social and interpersonal learning relations is “perhaps the least studied and most potentially informative area of research on gender equality” (Riegle-Crumb et al, 2012). It is time that we looked at how young women view learning mathematics and the subject of mathematics in their secondary education and whether or not the method of learning plays a part in that experience. In my view, the instructional methods that are employed in mathematics classrooms would allow all students, regardless of gender, race/ethnicity or SES, the safe, secure space to build those relationships and beliefs that would make their learning experience optimal. Therefore, it should be a goal of mathematics educators to find instructional approaches that satisfy the relational needs of a diverse group of learners and improve the experiences of those learners in mathematics classrooms. However, given the inequities that persist in STEM fields, the problems that exist in retaining women in STEM careers, it remains of crucial importance to examine girls’ learning and paths to STEM fields of work and study.
CHAPTER 2
The Theoretical Framework and Literature Review

To situate this study, and hence my own framework for mathematics education, I must put forth the following two premises as stated by Burton (2002):

- Learning in the mathematics classroom is social, not individual
- Coming to know mathematics depends upon active participation in the enterprises so valued in that community of mathematics practice that they are accepted within that community

Within this view, mathematics knowledge is understood to be constructed within the classroom community in which it exists and a learner “knows” mathematics based on the values that are prescribed within that community. For many, this is a very different view of mathematics learning and knowledge. For example, a traditional lecture-based mathematics classroom that many adults today presume as the typical mathematics classroom, has been found to have teacher lecture or demonstration of methods followed by individual practice take up 84% of classroom time (Boaler, 2008). This method of instruction implies a philosophy that values one version of the truth of knowledge (stemming from the instructor), that the learning of mathematics is mostly individual (since students learn from the instructor and then practice themselves) and that their passive interaction by listening to the teacher allows them to learn the information they need to know. If a learner “knows” mathematics based on the values prescribed within such a learning environment, I put forth that in a traditional mathematics classroom, a learner comes to “know” mathematics in a very individual, superficial, rote way.
Further, and in contrast to the context described above, I situate mathematical learning, and learning in general, within the context of the greater relational approach to knowing – where “knowers are social beings-in-relation-to-others”, and these relationships must be built on respect and care, not oppression and power (Thayer-Bacon, 2004). According to this view, education has a relational character and it is just that relationship between the teacher and the student, and even possibly the student and her classmates, that affords the community the opportunity for the interaction in education (Biesta, 2004). The communication in these interactions between individuals is not about the transport of meaning but about the participation in and co-construction of meaning between individuals and those members of the community in relationship to each other which in turn allows “education [to] exist only in and through the communicative interaction between the teacher and the learner” (Biesta, 2004, p.21). In this relational world of knowing, learners improve their knowledge and further develop understanding by making greater connections – with material, concepts and others (Thayer-Bacon, 2004). This is consistent with the definition of mathematical learning for understanding which has been widely encouraged and supported in the mathematics teaching community:

A mathematical idea or procedure or fact is **understood** if it is part of an internal network….The degree of understanding is determined by the number and the strength of the connections. A mathematical idea, procedure or fact is understood thoroughly if it is linked to existing networks with stronger or more numerous connections…Understanding involves recognizing relationships between pieces of information (Hiebert & Carpenter, 1992).

The task then is to craft a pedagogical framework for mathematics instruction that facilitates relational learning and construction of knowledge and it should incorporate the
ideologies that enable as many students as possible the freedom to create those connections and relationships.

**Relational Trust**

In the greater workings of a school, relationships are extremely important for success in communication, motivation, morale and many other interpersonal beliefs in the community. Viewing trust through a relational lens can help support that success (Bryk & Schneider, 2003). However in the microcosm of the classroom, this relational view of meaning-making could also be seen in the collaborative learning experience between the members in a learning community, which inherently implies a level of trust between those members. Creating that connection in the classroom is not always easy and does not always come naturally for all individuals – both teachers and learners. However, it can be nurtured if an environment of trust is established based on relational ideals that are generally led by teacher beliefs and behaviors. I focus my definition of Relational Trust on the aspects that pertain most directly to classroom interactions between members of the learning community.

The first two facets of Relational Trust that stem from the teacher are somewhat intertwined. They link the teacher’s ability to connect to the learners (and hence the learning community as a whole) and her ability to actualize the “genuine interest” she has in the students’ own ideas (Raider-Roth, 2005). This “connectedness” can be interpreted as a willingness to question further, a sincere interest in the well-being of the student or a mindfulness of the holistic nature of the individual. At one point in educational theory this concept of “connectedness” was specifically formalized to support women’s and
girls’ ways of knowing and learning, and specifically in mathematics education (Becker, 1995; Belenky, Clinchy, Goldberger, & Tarule, 1986). However, more recently opponents of gender difference theory in mathematics education promote an “unfixing” of the differences “to see mathematics as an opportunity to develop relations with others and re-make themselves” (Mendick, 2005b). Mendick goes on to say that

By aligning separate-ness with masculinity and connected-ness with femininity, these approaches feed the oppositional binary patterning of our thinking and in the final analysis reiterate it. (p 163)

Supporters of this more humanizing approach to the multiplicities of student relationships with mathematics agree that rethinking gender differences in a larger framework would benefit both boys and girls. It may be possible to do this if mathematical learning were viewed in less of an oppositional way (male vs. female, objective vs. subjective, etc.) and in more of a interhuman relational way – appreciating all of the various needs of connection including being “authentic” and “feeling seen” by the other (Raider-Roth, 2005).

Within this connected relation of trust, the teacher would also be able to sincerely express interest in listening to and following up on students’ original ideas. In order for this expression to come through in the classroom, the teacher needs to attend to being ‘present’ – as defined in terms of relational connections to self, students, pedagogy and subject matter:

A key aspect of being present to students’ experience means assuming a connected stance. In this stance students must have a sense that their teachers can see them and their learning, their strengths and their weaknesses. Not only do they see but they also accept what they see without judging it as good or bad. It is mutuality that strengthens the vision…They[the students] know that they can extend themselves to the
very edges of their learning, to the borders of their known world, because they know that someone will be there to meet them....In short, a teacher who is ‘present’ is a real learning partner. (Rodgers & Raider-Roth, 2006, pp. 278-279)

Allowing the teacher to be seen as a partner in collaboration builds trust in the classroom and also helps to redefine the vision of classroom authority and dissolves the traditional structure of hierarchy in relational and feminist ways. This helps to build an environment of safety and risk-taking that empowers student agency and encourages student voice – both furthering the relationships that will enable learning to take place.

Relational Authority and Relational Equity

Considering that learning is a relational enterprise, one must also consider that traditional classrooms in the United States and mathematics classrooms especially, are fraught with problems of equity. Authority is often described as something that one single person holds and possesses. Although many authors describe the concept of “sharing” authority, it is difficult to get away from the concept of authority being held by one person who is the sole leader and wielder of the “influence over another” (Biesta, 2004). Gadamer’s philosophy of authority is elaborated on here:

For authority to succeed in its aim of educating the student, the student must acknowledge that there is an important insight to be gained from the teacher. The student has an active role of authorizing the teacher by following the teacher’s pedagogical lead. To learn thus entails the authorization of the teacher by the student. (Bingham, 2004, p. 31)

This concept of relational authority is at the heart of a pedagogy of relation. If education happens relationally in the interactions between individuals in the community of learning, then there must be an acceptance that all members of the community have authorized the learning to take place. It is that respectful and reflexive interaction that allows for the
opportunities to arise in order for learning to happen. Connected to this construct of authority is a similar view of equity. The term relational equity in the classroom (Boaler, 2008) has been used to describe classroom relations between students, and I would extend that to teachers and students, where respect for others’ ideas is held as a priority, as is treating different viewpoints fairly. There is also a commitment to learning from others’ ideas, and this mutual respect and common commitment leads to positive intellectual relations (Boaler, 2008).

Voice and Agency

In theory, relational authority and equity in the classroom is a very idealistic notion, with goal of fostering an environment that allows students to freely express ideas, grapple with learning tasks openly and question not only authority, but knowledge in general. Those of us who strive for these ideals in our practice know the realities of the obstacles that encumber the development of student voice and agency in the learning process. We are all too aware of the hidden curriculum, the unspoken social prescriptions that govern the classroom and the habits of learning that have been subconsciously taught for years through the traditional educational process. Especially for those students who consider themselves in underrepresented groups because of gender, race, ethnicity, sexual orientation or other categorization, including opportunities for dialogue in the classroom by itself might not be enough. Taylor and Robinson state:

Student voice…may not currently have the practical or theoretical tools…to explain, or to contend with, the multifarious ways in which power relations work within school…processes. As a consequence, it may find itself implicated in reproducing, rather than unsettling or transforming, the hegemonic-normative practices it sought to contest. In addition, it may remain bound by the presumption that…such dialogue is itself a manifestation of a classed, gendered and ‘raced’ form of cultural capital (2009, p.169).
In other words, if not done in a deliberate and careful way, dialogue, even when attempting to be emancipatory, can simply perpetuate the hierarchy that already exists in the community of practice. Voices that were silenced can remain silenced and those that have been heard will continue to be heard. One view of student voice work is geared towards action, participation and change (Taylor & Robinson, 2009). These are worthy goals that need to be focused towards allowing the individual student to use that action, participation and change to move towards their own agency in their learning process. Taylor and Robinson (2009) discuss the focus of postmodernist theory on reflexivity – transparent and open sharing of thoughts - and the production of knowledge in the context of student voice. It is important that the dialogue move individuals towards growth in their agency in the educational process. In addition, one must keep in mind the multiplicities of identities that students construct as they move through the process of belonging to a community of practice (Maher & Thompson Tetreault, 2001), which can make the formation of student voice even more complex. Therefore, any empowerment that is promoted in dialogue needs to also consider the awareness of the subtleties of the race/class differences in students’ identities. In the context of creating a relational learning environment, empowering student voice and agency is facilitated by creating a safe environment, further demonstrating the interdependence of the relational framework on each of its parts.

**A Pedagogy of Feminist Relation in Mathematics**

My theoretical framework, which includes relational trust, relational authority, relational equity, voice and agency, has at its roots what was historically known as Feminist Mathematics Pedagogy, stemming from the gender difference movement of the
‘90’s (Becker, 1995; Boaler, 1997; Burton, 1995; Solar, 1995; Willis, 1996). The intersections and overlaps of these constructs are not coincidental and therefore, I cannot ignore the feminist influence which initiated the concept of valuing learning from a humanistic standpoint and appreciating individual differences and voices within the mathematics classroom. Solar (1995) posited an inclusive pedagogy based on a postmodern epistemology and identified concrete attributes that characterized the “four dialectical aspects” of feminist pedagogy: (a) passivity and active participation, (b) silence and speech, (c) omission and inclusion, and (d) powerlessness and empowerment. The framework is also corroborated by another model of a feminist mathematics classroom (Anderson, 2005) in which empowerment, agency, development of authority, valuing of intuition, and honoring of voices were key components. The characteristics described in both of these models are consistent with the main tenets of postmodern feminist epistemology that resists dichotomous thinking and focuses on subjective thought and multiple perspectives (Hesse-Biber & Leavy, 2007). Further, it is well known that most traditional pedagogies in mathematics include process-driven and rather objective perspectives of mathematics which create “environments in which most students surrender agency in order to follow predetermined routines” (Boaler & Greeno, 2000). I therefore posit that a feminist mathematics classroom should be situated in a theoretical framework that is consistent with goals that allow for a sincere environment where the interhuman connectedness of relational learning takes place.

The purpose of this study is to investigate the effects of the pedagogical and instructional approach that I define as Relational Problem Based Learning (RPBL) in the secondary mathematics classroom on a group of girls’ attitudes towards learning.
mathematics. Specifically, do the relational aspects of the classroom affect the attitudes at all and in what ways? For the purpose of this study, I am defining RPBL as an approach to curriculum and pedagogy where student learning and content material are (co)-constructed by students and teachers through mostly contextually-based problems in a discussion-based classroom where student voice, experience, and prior knowledge are valued in a non-hierarchical environment utilizing a relational pedagogy.

I am defining the attitudes to be investigated in this study as enjoyment, self-confidence and value while I will also investigate the students’ senses of empowerment and agency – the last two of which may need specific definition. I feel that empowerment is the intellectual and emotional energy for the agency which enables an action to occur. Empowerment has many definitions, but I would agree with a commonly held view in social science that empowerment is a “multi-dimensional, social process…that fosters power (that is, the capacity to implement) in people, for use in their own lives…by acting on issues that they define as important” (Page & Czuba, 1999). So in education, a student that comes to feel empowered in the classroom is brought to a feeling that they have power and a sense of directing their own learning. Agency, with respect to learning, can be seen as the ability to act in a willed or voluntary way. It can also be seen as having the capacity to behave in an autonomous or independent way on one’s behalf while learning in the classroom (Biesta & Tedder, 2006). These attitudes and senses will be addressed in this study in both the realms of the subject matter of mathematics itself and the learning experience of the mathematics classroom.
Gender Equity in Mathematics Education

The growing racial, cultural and overall diversity of our student body in the U.S. has caused a surge of concern for the inequity in mathematics education for underrepresented groups like African Americans, Hispanics, Latinos and those of lower SES Status. Many researchers have stated that similar to females, these students are not served by the traditional ways that mathematics has been taught in many school systems (Ladson-Billings, 1995; Lubienski, 2007; Vithal, 2002). Researchers have studied the needs of students when controlling for race, ethnicity and SES in mathematics classrooms and have found that valuing their cultural perspective and their need for political empowerment, encouraging reciprocity and responsibility, and promoting equity in experience are common values that help improve success for marginalized groups of students (Boaler, 2008; Frankenstein, 1983; Gutstein, 2007). The concept of Culturally Relevant Pedagogy (Ladson-Billings, 1995) defined good teaching for African American students as a pedagogy that allowed them not only to achieve academic success, but also to develop a critical cultural consciousness and remain connected to their culture. Lower SES and racially diverse mathematics classes were also found to have great success with classrooms that exhibited “relational equity” (Boaler, 2008).

Since females can be considered a specific subcategory of all of these marginalized groups, it seems prudent to consider the intersections and comparisons of the literature in mathematics education. When looking at the research on gender equity in mathematics education, there is evidence that the “gender gap” in mathematical ability is closing, but that there is still evidence of and concern about performance, an interest gap at the secondary level, and in a lack of females choosing to enter math and science-
related fields (Hanna, 2003; Hill, Corbett, & St. Rose, 2010; Lloyd, Walsh, & Shen, 2005; Modi, Schoenberg, & Salmond, 2012; Mullis, Martin, & Foy, 2005). Much of the minimizing of the gender gap in the past two decades has been attributed to “female-friendly” teaching techniques that have been motivated by the realms of mathematics and gender research (Belenky, Clinchy, Goldberger, & Tarule, 1986; Boaler, 1997, 2002; Jacobs, 1997). Many educational philosophers and researchers integrated these ideas and connected them to feminist perspectives and epistemologies and argued against the “deficit model,” positing that perhaps it was not a problem with girls’ ability to learn mathematics, but with the way the teaching of mathematics was being delivered to girls, not matching with their learning styles in mathematics (Boaler, 2002). In a discussion of a feminist mathematics pedagogy several authors have explored a means by which gender equity might occur in mathematics classes with a different instructional approach (Anderson, 2005; Burton, 1995). The major characteristics of a feminist mathematics classroom that have emerged from research over many years are consistent with Belenky’s (1986) research on women’s ways of connected knowing and learning. They included equity and power sharing, valuing prior knowledge and experience, cooperation and collaboration, valuing intuition and emotion, allowing room for authorship and ownership of the material, and making space for discussion-based learning that values all voices (Kellermeier, 1996; Mau & Leitze, 2001; Weiler, 2001).

Once the “deficit model” was dismissed it became acceptable to view mathematics and its learners in a broader way. Research began to focus less on females as a broad category of mathematics learners, but on the differences between groups of females – African- American girls or Hispanic girls or white girls’ attitudes towards
learning mathematics, the mathematics classroom or the subject of mathematics (Hoang, 2008; Lim, 2008a, 2008b). Feminist Standpoint Theory, which is rooted in the concept that all perspectives, and thus knowledge, are situated in the individual’s personal life experience standpoint, informs research methods so that investigators place their participants at the center of the research process and consider the unique perspectives from which they come. Taking a lesson from Standpoint Theory, researchers became wary that for too long they had been generalizing about the issues surrounding gender equity in mathematics, making assumptions about all types of girls by looking through too unfocused a lens. Looking through the filter of Culturally Relevant and Relational Pedagogies, what seems clear is that most mathematics classes in the U.S. even today are still “fundamentally grounded in separate, procedural, individual and competitive work” which is often opposing young women’s cultural and social inclinations (Lim, 2008b). Communication characteristics like free verbal expression and talking aloud are often considered disruptive behavior in a typical mathematics classroom. The preferred learning and pedagogical characteristics of holistic and relational interdependence (Ladson-Billings, 1995) are generally substituted by distant, objective interactions. This poses problems for holding interest and maintaining positive attitudes for many young women and specifically, young women of color. Lim (2008b) found that in general adolescent girls of color struggle with accepted norms in traditional mathematics classrooms to which their cultural and learning communication behavior norms do not conform. These struggles may even go as far as purposefully repressing natural behaviors such as excited discussion and emotional relationships in order to fit the norms in these classrooms.
Because of this, many researchers, including Meece & Jones (1996) and Zohar (2006), have noticed the overlap between the constructivist teaching movement and feminist pedagogies. Both the National Council of Teachers of Mathematics and the U.S. National Research Council have prepared documents citing new standards and principles of mathematics learning that coincided with the values of feminist mathematics pedagogy (Donovan & Bransford, 2005; NCTM, 2000). In order to find ways in which teachers could better prepare students for these new outcomes, problem solving as an instructional outcome became the focus of a number of studies (Kurz & Batarelo, 2005; Lampert, 2001; Renkl, Atkinson, & Maier, 2002).

The Practice of PBL in Mathematics Education

One method of instruction that incorporated all of these principles that had been around for some time, but used at the professional schooling level, is known as Problem-Based Learning (PBL) (Savery, 2006). Unlike a traditional classroom that might have practice problems that follow a lecture, PBL classrooms are places where communication skills, prior knowledge, metacognitive skills and lifelong learning skills are practiced by focusing on problems prior to explicit instruction. The classroom practice is based on student presentation of solution ideas that are partially complete or not necessarily known to be fully correct at times. According to researchers, discussing the problems in PBL is a form of Anchored Instruction in which the learning is anchored in the problem - the focus for “acquiring knowledge and reasoning strategies” (Hmelo-Silver, 2004) which has important implications for student learning and agency in mathematics (Boaler, 2003). Outcomes of PBL classrooms include improved problem-solving, collaboration, and reflection skills, but more research has been called for at all educational levels.
(Strobel & van Barneveld, 2009). These outcomes of PBL all coincide with the standards and principles of learning for understanding supported by major educational organizations in the U.S. such as NCTM Process Standards of Communication and Problem Solving and the National Research Council’s Principles for Student Learning of Mathematics (Donovan & Bransford, 2005; NCTM, 2000). More recently, one could argue that PBL outcomes support the Common Core Curriculum State Standards for Mathematical Practice that have been recommended not only at the secondary level, but for K-8 grades as well (Common Core State Standards for Mathematics, 2010). These standards include:

1) Making sense of problems and persevering in solving them.
2) Reasoning abstractly and quantitatively
3) Constructing arguments and critiquing the reasoning of others.
4) Looking for and expressing the patterns used in reasoning (i.e. generalizing).

By taking part in regular, non-simulated, authentic problem solving in the context of a PBL curriculum, learners are asked to persevere on a regular basis, as well as decide for themselves in a group which arguments make the most sense. This practice, with a teacher’s guidance, can lead to improved outcomes in these areas.

Looking at both the theoretical frameworks of Feminist Mathematics Pedagogy and Critical Pedagogy, it was clear from the literature that researchers have found their intersections to inform best practices for instruction for underrepresented groups of students, including females. Some feminist teachers promote ‘lecturing with questions’ as a way to instruct in the hard sciences with a more feminist pedagogy since a more active pedagogy is often used in these disciplines “without considering how feminist epistemologies might engage us to think differently about what is science.” (Crawley, 2008). From a more critical perspective, others promote focusing on complex word
problems in context with further similar exercises (Lubienski, 2007). Many, though, take the stance that educators must treat mathematics as an “integral component of a larger system creating educated citizens” (Spielman, 2008) and it is only through this larger lens that we can truly come to a sense of equity – adapting the outcomes of the instructional practice as broader, more universal skills as opposed to specific skills valued by some elite class.

The Intersections

In comparing the literature on the desired outcomes for these pedagogical frameworks and Problem-Based Learning, it is interesting to note the intersections of the two. For example, group work, which is a foundational part of PBL, when done collaboratively and with respectful discussion, would be supporting Feminist Mathematics Pedagogy - valuing all voices and thereby creating a non-hierarchical group setting. In Critical Pedagogy, the concept of respect goes one step further and reaches towards reciprocity and responsibility for others’ learning. In PBL, discourse in community is foundational for construction of learning – between teacher and students and students and students – because in order for construction to be truly owned by the whole community, all voices must take part. This discourse also is foundational in both pedagogical practices because the methods used to exhibit the values of the theories need to ensure that all voices are heard, fairly and without bias. These intersections also resemble the theoretical framework of the Feminist Pedagogy of Relation in which I am framing my study. Unfortunately, there is little to no literature on connecting the mathematics classroom and relational pedagogy. Database searches that include the key words ‘pedagogy’, ‘relational’, ‘relation’, ‘mathematics’, ‘instruction’ and similar terms
only seem to turn up past studies that have interpreted Culturally Relevant Pedagogy or Critical Pedagogy in a relational way (Cobb & Hodge, 2002).

It also seems that to optimize the PBL learning environment, the teacher must make the classroom environment as open and safe as possible when it comes to the potentially risky practices of conjecture and stating one’s perspectives and opinions. From a feminist perspective, belonging and becoming, in terms of ‘learning in community’ are key agents in an individual’s practice in that community (Griffiths, 2008). In other words, how one enters that community of practice helps not only define who they are individually, but it also defines the practice of that community. Using a pedagogy of relation and focusing on respectful learning sets the tone for individuals to be who they are and to support one another as a community of learners.

Because the PBL method requires students to eventually become more and more responsible for their own learning, the teacher’s scaffolding of the learning and discourse fades as students become more expert in their discourse strategies and capable of moving forward in discussion (Hmelo-Silver & Barrows, 2006). In many ways, this type of instructional approach is a model of cognitive apprenticeship, as the teacher is constantly modeling problem-solving, conjecturing and risk-taking, while coaching the student in learning (Hmelo-Silver, 2004). Although some research has been done showing the effectiveness of PBL in teaching problem-solving skills and self-regulation (Savery, 2006), it is true that much more research needs to be done, especially at the secondary level (Strobel & van Barneveld, 2009). Most recently, research has shown that problem-centered approaches improve both achievement and attitudes of students regardless of perceived ability level (Ridlon, 2009).
At the same time, it is clear that a PBL teacher needs to be a facilitator of discussion and utilize strategies that allow for student learning in this complex situation. Some master PBL facilitators have been found to utilize strategies such as (a) probing students for deep explanations (b) using open-ended metacognitive questions (c) revoicing and (d) summarizing (Hmelo-Silver & Barrows, 2006). The discourse in a PBL classroom has been found to be very different from the typical teacher-directed instruction and more than half of the questions are generally student-initiated stemming from the ideas under investigation (Hmelo-Silver & Barrows, 2008). A PBL classroom can be run in many different ways but to foster the values of equity and social justice, if that were one’s goal in the classroom, this must be done deliberately and with a pedagogical philosophy in mind. One instructional method known as Dialogue, Participation and Experience (DPE) (Chow, Fleck, Fan, Joseph, & Lyter, 2003) states facilitator strategies such as voicing student views, placing learners on center stage, focusing on interdependency and reducing frustration to diffuse tension as means to help create a classroom climate that helps students participate in active dialogue. Orchestrating that dialogue is a major component of the role the instructor plays in the classroom community, making sure to “build on students’ ideas…and develop an understanding of increasingly powerful perceptual operations that underscore” the students’ thinking and construction of knowledge (Ridlon, 2009). Researchers and theorists who are more cognitively inclined have posited that PBL and other instructional approaches that include “minimal guidance” are ignoring important cognitive architectural structures that make them less effective (Kirschner, Sweller, & Clark, 2006). However, PBL instructors maintain that the scaffolding on which they focus in their
directed questions to guide student learning separates PBL from mere free-form “discovery learning” and enables students to make use of existing cognitive structures (Hmelo, Golan Duncan, & Chinn, 2007). Further, PBL supports growth of the “soft skills” of collaboration and communication that direct guidance does not.

The instructor must also be cognizant of student engagement at all times and the social interactions that are occurring that are allowing (or not allowing) mathematical meaning to be made in the community. Situating PBL within the context of a Relational Feminist Mathematics Pedagogy has done just that, in my experience, and allows students the inclusive and relational learning environment within which a diverse group of learners can learn and thrive.

**Situating this Study in the Current Research**

Although there has been some research done recently on PBL as an instructional approach at the secondary level, it has mostly been in the area of science, economics or technology instruction (Chin & Chia, 2004; Goodnough & Cashion, 2006; N. L. Maxwell, Bellisimo, & Mergendoller, 2001). Interestingly, mathematics has not been an area in which this type of instructional approach has been investigated at this level, although there seems to be great interest in PBL among practitioners. Most recent studies that have compared traditional curricula to reform or NCTM standards-based curricula (defined as NSF-research funded, but not necessarily RPBL as defined in this study) have “generally not attended to issues of pedagogy in a careful way” and so would not deal with the ideas of relationality or encouraging student voice (Smith & Star, 2007). Other research reports that have asked students about their attitudes towards standards-based curricula or the transition between two types of curriculum have found “substantial
diversity in the differences that students perceived” between the programs (Star, Smith, & Jansen, 2008), which may give credence to the need for a more qualitative perspective while still appreciating the thoroughness of the 2008 study. The NCTM recognized the interest in PBL at the secondary level when its focus issue of *Mathematics Teacher* on geometry for 2011, specifically called for articles on teaching geometry using problem-based learning (Schettino, 2011/2012).

Much of the recent research surrounding gender issues in mathematics education is intertwined with the brain-based research on decisions to make some mathematics classrooms single-sex that many public schools are facing (Eliot, 2009; Shapka, 2003; Spielhofer, Benton, & Schagen, 2004). Although it seems that concern about the gender achievement gap is closing, there are still a few researchers looking at girls’ attitudes towards the subject of mathematics and the mathematics classroom in order to help attract and retain more girls and women to the field of mathematics – specifically the ways in which mathematical identity is formed and our society’s attitudes towards women who do mathematics (Lim, 2008a; Mendick, 2005a)

It is my hope that this research study will add to the current body of literature on girls’ attitudes towards the subject of mathematics and learning in the mathematics classroom in both the affective and cognitive domains by looking at the girls’ narratives of their experiences of learning through the instructional approach of RPBL. By looking at both their feelings towards 1) the subject of mathematics (which is typically seen as a more masculine and disconnected subject matter) and 2) experiencing learning in the mathematics classroom with PBL situated in a Feminist Pedagogy of Relation (which is traditionally taught in a rather objective and non-relational instructional way), I hope to
add new information to the knowledge we have about how girls, and by implication, women, view mathematics. It is also my hope that through the lens of girls’ views, we may open up the discussion of other marginalized groups of students and perhaps future research will incorporate those students’ attitudes towards mathematics with RPBL as the method of instruction.
CHAPTER 3
Methodology

Overview

My purpose in conducting this qualitative study was to explore the nature of adolescent females’ experiences learning in a classroom utilizing a relational problem-based pedagogy (RPBL). I sought to explore the question of how adolescent girls experience a mathematics classroom situated in a Pedagogy of Feminist Relation that intends to foster a different type of learning environment potentially positively impacting adolescent females’ (and other underrepresented groups of students in our country) feelings about and success in the field of mathematics. To investigate how the use of RPBL relates to young women’s experiences of mathematics, I endeavored to address the following question:

What is the nature of the relationship between girls’ attitudes towards mathematics and their learning of mathematics during and after experiencing it in an RPBL environment? How do they describe their experiences?

Because of the descriptive nature of this question and the relational aspect of both the pedagogy of the classroom and my theoretical framework for this study and I chose a feminist relational voice-centered methodology to guide the collection of the data. Because it was the voices of the adolescent girls that I felt was most central to the research question, I hoped to record their narratives from a perspective where their voices would be most heard. Although there is a great deal of educational research literature about adolescent girls, the work that focuses on data analysis methods for research with
girls is quite limited (Mitchell & Reid-Walsh, 2008). The data analysis method, I chose to use, The Listening Guide, was one of the first data analysis methods that was seen as specifically capable of picking up on the complexities of the ways in which girls viewed their lives relationally (Mitchell & Reid-Walsh, 2008, p.19). Because of its four-step process, it pays close attention to the multiple layers of girls’ voices and attends to the stories about relationships (Mitchell & Reid-Walsh, 2008). For my purposes, I was looking for the relationships in the classroom between the material, classmates, teachers and even themselves for each of the girls in the study. The Listening Guide (Gilligan, Spencer, Weinberg, & Bertsch, 2003) offers so much more than coding because it has a way to “trace the movement in girls’ understanding of themselves” while at the same time “illuminates my positionality and regulatory power with respect to the girls” (Brown, 1997). In this chapter, I describe the feminist narrative inquiry method in terms of data collection and analysis. I provide detail as to how the relational feminist voice-centered approach of The Listening Guide functions and how I ensured for rigorous analysis and appropriate ethical considerations in using this method.

Narrative Inquiry

In keeping with the epistemological views of my theoretical framework, it seemed clear that a relational approach to investigating these research questions would be most valuable and natural. Qualitative inquiry, and more specifically, narrative inquiry, would give me a way of looking at a student’s learning experience in both a personal and social way, but from the individual perspective in relation to the social context (Clandinin & Connelly, 2000). Also, narrative inquiry allowed me to be as consistent as possible with the feminist relational approach by viewing participants through their “narrated selves” –
one perspective of their own identity. Some qualitative researchers claim that the 

“narrative self” is intrinsically relational:

“Self” does not exist in isolation or inter-relationship with other “selves” and other lives and is grounded in the material reality of everyday life; and a key part of this material reality is formed by the narrations of selves and others. [This] underlines a view of narrated subjects …[who] are constituted in relation to other subjects and to the ‘material reality of everyday life.’ (Doucet & Mauthner, 2008)

Because the concept of a narrative self does not only exist in the transcript, story or interview from which it is being told, but it is truly grounded in the actual experience that is being spoken about, it is important that the relationships from which the narrative self have developed are not only between individuals or other selves. The narrative self was developed through the experience with the world and other material subjects, values and social structures that affected that experience. I believe this statement characterizes the extent to which an individual may have the ability to describe themselves and their experiences in relation to their world. An individual may have the capacity to know those relationships and their feelings as one view of their own self, which is what I had hoped to capture through the inquiry process.

However, taking a pragmatist worldview stance (Creswell & Plano Clark, 2007), I wanted the perspective offered by the multiple realities that exist (especially those that may not be my own) as well as the practicality of the research situated in real-world practice. Therefore, I carried out a qualitative study that combined multiple data collection methods to compare intersections and differences as well as gain a “broader understanding” of the responses and data through triangulation (J. Maxwell, 2005). The importance of triangulating the data during the collection phase was paramount especially
in this study where I was intimately connected to many aspects of the research as a
teacher, curriculum writer and other roles. As Denzin has stated:

> All researchers take sides, or are partisans for one point of view or
> another…This is the case because every researcher brings preconceptions
> and interpretation to the problem being studied…This means that scholars
> must state beforehand their prior interpretations of the phenomenon being
> investigated. Unless these meanings and values are clarified, their effects
> on subsequent interpretations remain clouded and often misunderstood.
> (Patton, 2002).

**My Complex Relationship with RPBL**

As a teacher at the school, and the past department chair, I was an integral part,
and perhaps the driving force, in the implementation of the RPBL curriculum, and a
writer of the original materials. My passion for encouraging girls in the field of
mathematics has stemmed not only from my own experiences of studying mathematics as
an undergraduate and being the only female in my masters program, but from my twenty
year teaching career. At times in my career I have been curriculum writer, teacher,
professional developer, teacher educator, department chair, researcher and writer with
respect to problem-based learning, and continue my explorations with this progressive
method of instruction. My past research on teacher classroom behaviors that support
RPBL (See Appendix A) has allowed me to train teachers in the main tenets of a
Pedagogy of Feminist Relation (Schettino, 2011) and has fueled my interest in and
pursuit of knowledge about RPBL as an instructional approach not only for girls, but for
all students. As a postmodern feminist, I feel that my research and teaching should strive
to deconstruct gender norms instead of regulate them, and I hope to teach in a way that
allows students to resist dichotomous thinking (gendered and mathematical) and focus on
their own ideas, constructing their own truths, even in mathematics class.
In 1995, I had the good fortune to be hired into a tremendously gifted mathematics department at an independent school where they were writing their own curriculum, and through that experience became well-versed in the development of PBL and inquiry-based methods for secondary mathematics. I became extremely interested in how different students transitioned to the PBL environment, how different teachers utilized the PBL curriculum and how they facilitated discussion and problem solving. Over time and with research and experience, it became clear to me that the relational approach of learning in the context of good problems, scaffolded for learning, was optimal.

During the time of this study, although I was an integral part of the curriculum writing team at the institution where the research took place, a teacher of the course, and trained the other teachers in the pedagogical approach, I took steps to minimize the participants in the study who are my own students. I also was not supervising the teachers of the remaining sections of the course. Although I was deeply invested in the success of the program at the institution itself, my investment in the research at the time had larger implications for the overall exploration of girls’ attitudes in general with respect to this method and not specifically the advancement of those girls who were my students or the advancement of the institution that was my place of employment.

Research Design

Because of my complex relationship with RPBL, from a research standpoint I was keenly aware of the need for rigorous techniques to increase the credibility of my data collection and findings. To minimize the possibility of my own bias affecting the data collection (and later on, analysis), I attempted to create a research design that triangulates
all aspects of the study’s fieldwork by cross-checking and cross-validating sources (Patton, 2002). I looked for other avenues of data collection that would allow me to remain true to the relational theoretical framework and narrative inquiry methodology, while at the same time allow for a number of different types of data to be collected that would let me hear that participants’ voices in a number of ways.

Research Site Selection

In order to study the attitudes of adolescent girls with respect to mathematics in an environment that was governed by relation and a pedagogy of feminist relation, it was crucial that this pedagogy be consistent as possible and that I be able to sample participants from as large a population as possible. Since I was familiar with the development of the curriculum, the training of the teachers and the history of the commitment to feminist pedagogies at Greystone, it was natural for me to choose it as a research site. To ensure the anonymity of the school and the participants in the study, I have changed the name of the school and have not cited the web site that holds the curriculum that was used at the school. Student and teacher participants have been given pseudonyms although their relationship to me is apparent.

Greystone Academy is an all-girls independent boarding and day school with approximately 60% of its students who are boarders and 26% of its students who are international. The sample of participants from Greystone is of course limited in that students at this selective private school are not fully representative of the general population since this is a tuition- and admission-based school and students are generally more academically motivated and may not reflect the diversity that would exist more widely in a public setting. However, with almost 18% students of color in the student
body and 53% of the student body receiving some form of financial aid, the diversity of the school, (race, ethnicity, SES), allowed for a diverse selection of the students in the study.

*Teacher Participant Selection*

The mathematics department at Greystone Academy had decided to change their geometry curriculum to a problem-based one three years prior to this study, the premise being that incorporating more discussion and deliberate problem solving would allow students to foster the 21st Century Skills needed to develop independent and higher order thinking (McCain, 2005). The three teachers of the course during the year in which the study was conducted were myself, Ms. Brown and Ms. Johnson – all three of us were the original collaborators on the department’s curricular RPBL project. Ms. Brown and Ms. Johnson had both been at Greystone for six years and teaching with RPBL for three. Ms. Brown was a mathematics

<table>
<thead>
<tr>
<th>Teacher</th>
<th>Number of sections taught</th>
<th>Education</th>
<th>Years at Greystone</th>
<th>Years of Teaching Experience</th>
<th>Years of Teaching RPBL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ms. Brown</td>
<td>1</td>
<td>B.A., Math</td>
<td>6</td>
<td>13</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>M.A.T, Education</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ms. Johnson</td>
<td>2</td>
<td>B.S. Physics</td>
<td>6</td>
<td>8</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>M.S., Physics and Engineering</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ms. Schettino</td>
<td>2</td>
<td>B.A. Math</td>
<td>10</td>
<td>19</td>
<td>15</td>
</tr>
<tr>
<td></td>
<td></td>
<td>M.A. Math</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Table 3.1 Participant Teachers’ Information*

educator at mid-career and was the chair of the department at the time of the study, while Ms. Johnson was a younger teacher with a background in physics, and newer to the classroom. There was one other teacher teaching a section of the course during that
school year, but she was being initiated into teaching RPBL and was not considered to be capable of being part of the study for that reason. All of us were truly dedicated to the idea of relational learning and found the professional development we did together over three summers extremely fulfilling both personally and professionally.

As a team throughout the academic year, we met weekly to discuss classroom management issues, the intricacies of the problems and scaffolding learning through discussion in weekly meetings, as well as assessment and grading policies. We often observed each others’ classes in order to keep up our professional development and to compare our practices. We shared assessments of all types throughout the year and often shared alternative methods of solving problems that students thought of that we thought were creative and interesting (See Appendices B & C for curricular and assessment information). All in all, we attempted to use a pedagogy of feminist relation with each other as much as we did with our students. In contemplating the diversity of our classrooms, we were cognizant of the fact that the three of us were all white and middle-class, so we often discussed the idea of dissolving the hierarchy of the classroom, and as I observed over the years, we strived for consistency in that regard. Our cooperative methods helped to keep our classroom as consistent as possible while allowing for our own relationships with students that made our individual classes unique.

In any given year, there were usually five or six sections of the course that over a period of four years had come to be taught with RPBL. It was entitled Integrated Algebra and Geometry: M210 and generally enrolled students from grades 9-11, each class with an average size of 13 students. The curriculum utilized was adapted from the self-authored materials available online by Phillips Exeter Academy and which are revised
every year (PEA Math Department, 2012). These curricular materials contain a set of decompartmentalized problems that are scaffolded in such a way that topics from any college-prep algebra or geometry curriculum could be found within the text. However, it relies heavily upon students’ prior knowledge as they work through the problems on nightly assignments, and class discussions center on the attempts that students make at connecting ideas from problem to problem in order to move forward in their knowledge. Student presentation is part of the daily routine up at the board, instead of teacher lecture and student practice, and students become facile with the idea of sharing their ideas publicly and working off of each others’ knowledge. This curriculum, although problem-based, was not necessarily written with a pedagogy of feminist relation in mind, but when coupled with the tenets of this pedagogy, PBL seems to work optimally (See Appendix D for Course Expectations).

Student Participant Selection

At the heart of this study lies the narrative inquiry that allowed me to “gain insight into the world of [my] respondents” and “gain a new perspective on their lives” (Hesse-Biber & Leavy, 2007). This design was selected to best capture the essence of the experience of the female students in order to address the research questions and optimally analyze students’ reactions and feelings about the pedagogical approach utilized to teach mathematics. It was important to have a range of students that captured the diversity of the current students enrolled in the course. To obtain the key participants for the in-depth interviews, my hope was to recruit a maximum of approximately 8 students from the total number of girls (n=46) who were enrolled in M210 in that academic year. The recruitment of participants began with my short class visits to each of the five M210 class
during which I read from a Student Recruitment Script in order to personally introduce them to concept of the study.

Initially, 14 students expressed interest in becoming participants and returned an assent form, and at that time I emailed the Parent Consent Form to those parents. Once assent and consent were attained, I obtained the metacognitive writing journals from the RPBL class of those 14 students who had shown interest becoming participants. My main goal was to be sure there was diversity among the final participants in the study over a variety of categories. In Figure 3.1, I have attempted to

![Figure 3.1 Desired Demographic for Student Participants](image)

graphically represent the diversity of variables I hoped to achieve among the population of students taking M210. I read through these students’ journals to ascertain whether
their written communication would be helpful in telling the story of their experience by giving snapshots of their problem solving or explaining their processes in detail. Some students started the year out in a less articulate way and grew, which was helpful, and some students were skilled in this method of communication from the start of the year. Other students’ journals did not give helpful insight into their experience in the classroom because they had not learned about writing mathematically or been able to use the journal as a tool to describe their problem solving usefully at that point in the year. At times I found it difficult to ascertain from the many varieties of writing styles at that point in the year which students might be the most suitable candidates for participation. However, I used the range of grades on the journal entries, student capability to articulate mathematical ideas and processes, and also their expressiveness in their writing as guidelines to help decide who would be interviewed. I do believe that in the end it was most important for me to include a variety of demographic information, to be sure that all of the participants were not my classroom students, and to allow for a range of interest and ability.

I identified a set of eight students to participate in the interviews and obtained student assent and parental permission. I had the wonderful experience of conducting initial and final individual interviews with all eight young women that examined their experiences with this pedagogical approach. After completing all data collection, however, I decided to narrow down the eight participants to five to make the amount of data to deal with more manageable. Although not always optimal, I found ways to balance the diversity in all seven categories as best I could. The five final participants can be seen in the diversity of their characteristics in Table 3.2.
Table 3.2 Student Participant Information

Data Collection

Because I held honoring the girls’ voices in their retelling the nature of their relationship with their mathematical learning as paramount, listening to individual stories allowed me to remain true to the theoretical framework in which the study is situated. I planned then to support the individual stories by the collection of classroom observations, teacher interviews and journal writings of the key participants. The overall research design included:

Spring Semester 2011

- Read Journals
- Interview Students
- Interview Teachers
- Triangulation of Data
- Observe Classes

Figure 3.2 Overall Research Design
design is shown in Figure 3.2. This study included classroom observations of the students’ experiences while learning mathematics, as well as interviews of the teachers for their perceptions of the students’ learning experiences in their classes.

Journal Entries

The students’ mathematics metacognitive journals gave insight and added another lens through which to view the students’ learning of mathematics. The students were required to write journal entries (one entry per week, collected every two weeks). These entries were geared toward writing about a problem that the class has discussed and how the student came to understand that problem in the larger context of the curriculum. Many students grew to become better metacognitive writers as the year progressed, so it was important to analyze the year’s writing. I analyzed these entries for evidential writing that connected with learning outcomes of RPBL. Specifically, I read for thoughts about attitudes towards problem solving and mathematics learning and any corroborating comments related to their interview statements. Comments about growth in their individual empowerment and agency or growth in their self-confidence throughout the year were identified. Any statements about enjoyment of the class or how their journey of valuing the class or learning mathematics had changed were also noted. Entries that were identified as valuable were transcribed electronically. At times, if there were signs of pronoun use consistent with changes in a student’s senses of empowerment or agency I noted those as well.
Classroom Observation

Classroom observations allowed me to contrast these experiences with the interview data and the journal reflections, allowing for triangulation. As much as possible, the classroom observation served to record the actual observed experience of the students’ learning of mathematics and in many instances triangulated the data that was collected in the student interviews and the journal entries. I took notes in the form of questions the student asked, actions they took in class (volunteering solutions, helping other students, writing on the board, etc.) and I also documented the timeline of the class and interactions that occurred. It was important to note the possible difference in a student’s perception of their learning and my perception of their learning. Fenstermacher makes the distinction between ‘studenting’ and learning whereas ‘studenting’ is “to be engaged in a series of performances whose form and substance are primarily shaped by the institutional and organizational properties of the school setting” (2006, p.102). In this way, it is clear that what I observed in the classroom could easily be misinterpreted and therefore should be viewed as interpretation. However, in this form of naturalistic inquiry, the relatedness of the observer, the observed and the phenomenon being observed comes across in the data through the observer’s point of view, or as Carini states it:

The observer has a point of view that is central to the datum and it is in the articulation – in the revelation of his point of view—that the datum of inquiry is assumed to emerge (Carini in Patton, 2002, p. 328).

It was through the recording of observed behaviors of the students’ actions during mathematics class and their observed learning that my point of view of their learning emerged, and ultimately was compared with the students’ point of view from interview
data and journal data, as well as the teacher’s perspective. For participants who were students in my own class, I kept a record of notes in a separate notebook at the end of specific class periods. I also used the summary comments I wrote at the mid-term and end of year to corroborate my “observation.”

Participant Interviews

Although many data sources were used, my main focus for information and the voices of the participants were the one-on-one interviews with the students. In order to keep our discussions semi-structured, I adapted the well-validated and reliable Horizon Research Interview Protocol (Horizon Research, Inc., 2000) of a student interview about the student experience in the mathematics classroom. I added questions concerning their experiences in learning mathematics and the subject of mathematics. I used a similar interview protocol in a pilot study with a smaller sample at the same school in order to hone the questions to be sure they were focusing on the attitudes and senses chosen for this study - enjoyment, self-confidence, value, empowerment, and agency – and on both feelings about the subject of mathematics and the classroom experience. The protocol was semi-structured allowing for variation in questioning due to the diversity in student experience. The plan was for two interviews of 45-60 minutes per student, scheduled 8-12 weeks apart spanning the second semester, from February – May. This allowed the students to gain different experiences in the course and share their reactions to their successes and failures in the interviews.

In order to corroborate data from the participant interviews, classroom observations of the key participants and any connections with journal entries, I conducted
interviews with the classroom teachers of the key participants. This occurred sometime after the participants had already completed their first interview and after at least one classroom observation. The purpose of these interviews was to gain a description from the teacher of the students’ attitudes towards mathematics and learning mathematics in the classroom in order to get another perspective on the experience that they are describing in their own interviews. Interview prompts stemmed from actual occurrences observed in the classroom for teachers’ reactions and views on the students’ perceived attitudes towards learning at those moments or patterns in the student’s behavior at similar moments.

**Figure 3.3 Summary of Purposes of Collected Data**

In summary, the goal of the design of the study was to obtain the most holistic view of the girls’ stories of their relationship with their mathematics learning and the subject of mathematics as possible, while at the same time realistically gaining as many multiple perspectives as possible (see Figure 3.3). This triangulation of data helped to
validate and cross-check data in order to keep the data collection methods as rigorous and credible as possible.

Reciprocity and Ethics

Because of the narrative inquiry and the qualitative nature of the data collection, clearly correlation and/or causation was not my goal. Capturing the participants’ experience and the nature of the development of their attitudes – telling their stories and gaining insight into their world was the more fundamental goal. In keeping with feminist beliefs, I was interested in the realities that often go unarticulated and came out through the in-depth interviews, acknowledging the awareness of my relationship as a teacher in the community and researcher on this project (Hesse-Biber & Leavy, 2007). The interview protocol gave an in-depth description and narrative account of the students’ experiences and rich details of their feelings with this type of pedagogical approach, allowing insight into ways in which they appreciate learning mathematics and feel empowered in the classroom.

One concern that was evident in the participant interviews was the question of equity in the interviewer/interviewee relationship. As the principal investigator of the study and also a teacher of the course, students could easily have perceived me as an adult in a power position. It was of utmost importance that I attempted to share equity with the participants at every step of the process and assure them that I was attempting to tell those stories not usually heard (Seidman, 1998). In the recent study done by Raider-Roth described in her 2005 book, Trusting What You Know, she describes her interview process at a school where she formerly taught and with students who were once hers...
Recognizing the relational context of the interview process is a core element of the research methodology used in this study. That is, the knowledge communicated in the interview process is embedded in the quality of relationship between the interviewer and interviewee. (2005, p.181)

To balance out the extreme closeness and potential casual nature that the familiarity might have created, it was also important that I described how I wanted the interview to proceed. I stated that I was interested in the reconstruction of stories, not just sharing of memories, that I would try to interrupt them only when necessary, but attempt to keep them focused, and that it should really be a conversation between the two of us, not just a question/answer session (Hesse-Biber & Leavy, 2007; Seidman, 1998). It was important for the participants to feel that I was listening intently and sincerely for their story without judgment and if they sensed my sincerity, they would be more inclined to openly share information not only that they think I want to hear, but that was truly their own story. To that end, I shared with them initially this hope and let them know that I was interested in all of their feelings related to their classroom and learning experience, not just those they thought I might want to hear in the interview. I intended to be as reciprocal as possible in the interview process, possibly sharing my own comfortable and uncomfortable experiences as a student learning in mathematics class, and as a teacher as well. Many interviewing relationships can be affected by unspoken issues of race, gender, class or positions of power. However, in my situations and framed in the feminist relational context, I was most concerned with race, class and the question of power. I decided to focus the interviews with the students on their own stories and not to
allow the questioning to be pervaded by the questions of class or race (Seidman, 1998, p.87). If the parts of their stories dealt with each student’s individual race and class, I viewed it as helping to portray their story in a more detailed and honest light. However, I can remember times when I did hold back sharing specific memories as I felt it was inappropriate to focus too much attention on my own experiences. Overall, I believe I balanced my professional judgment and reciprocal sharing in the context of my theoretical framework. To help keep the power relationship as equal as possible at all times, in the student assent form, it was clearly stated, that the participants continued to have the right to withdraw from the study without penalty. Even with all of those pieces in place, there was still the possibility with adolescents that their interpretation of my intentions was different from mine.

Data Analysis

Once the data collection was complete, I could then begin analyzing the complete set of data. In keeping with the theoretical framework of education as a relational phenomena, I used the Listening Guide (Brown & Gilligan, 1991, 1992; Gilligan, Spencer, Weinberg, & Bertsch, 2003), a voice-centered, relational approach to narrative data analysis. With this method, a researcher employs multiple readings, or “listentings,” of interview transcripts, In each reading a different perspective of the participant’s voice is identified and “listened for” (Doucet & Mauthner, 2008) because one’s discourse has multiple layers. The first reading is done while listening for the plot. The basic story of what the interviewee is telling and how the reader has responded to that story. During the second reading, the voice of the self should be listened for and it is in this stage where phrases that are described in the first person (with the pronouns “I” and “we”) and
contrasted with phrases described in the second person (with the pronoun “you”). These I-Poems, as they are called, are an alternative way of viewing the interview text in a poetic form. In each consecutive reading thereafter, what Gilligan called “contrapuntal voices” are read for which bring out voices that seem to be in potential contradiction with each other. With this method, it is important for the researcher to respect the participants’ experiences without judgment and as she navigates the often coded, indirect language of girls and women (Beauboef, 2007).

The Listening Guide

My journey with the Listening Guide was complex and through many different experiences I have come to an intimate comprehension and full appreciation of the intricacies of its power. Initially, from my readings of the literature, it seemed that there existed many different interpretations of how to use the Listening Guide for data analysis (Balan, 2005; Beauboeuf, 2007; Doucet & Mauthner, 2008), including the ways in which researchers physically formed the “I-poems” with quotes from the participants’ transcripts or choose to create “member-checking kits” with statements in each of the different voices. It seemed like the traditional use of the ordered pattern of listening for plot, creating the I-statements, then listening for the Contrapuntal Voices was sometimes replaced with a variety of different choices of readings. Depending on the study, there was a great deal of variety in the types of voices that were “read for,” which confused me even more. For example, in Beauroe’s study of African-American women and strength (2007) she was led to choose her readings as “Voices of Strength”-“Voices of Self” (I-Statements focusing on strength) – “Perspectives on Strength (subvoices)”. These discrepancies led me on a path of attempting to understand more about the Listening
Guide before I felt comfortable beginning my data analysis phase of my dissertation work. I knew it was the appropriate method to use since it was so clearly consistent with my theoretical framework and wanted so deeply to analyze the voices of my participants. However, I did not feel that I had enough of an analytical framework to guide me.

I decided to use the qualitative and mixed methods analysis program, MaxQDA, for all of my analytical work including overlap code maps (See Appendix E). I had all of the transcribed interviews of the participants and teachers imported, as well as the journal entries and class observation notes. As I began the readings (or “listenings”) for my first participant, I struggled greatly with differentiating between the listenings and then the “voices.” As I completed the first reading, I had created many codes that clearly represented the story she was telling. I grappled with the data in the second reading of the voice of self, and created some I-Poems, summarizing the message sent by the substructure of the pronoun phrases. Surprisingly, many of the same codes emerged that also were recognized in the first reading. The I-Poems themselves would often summarize almost prophetically what I had been seeing in the first listening. One researcher I read stated that the “I-Poem attends to an associative stream of consciousness carried by the first person voice running through a narrative rather than being contained by the full structure of sentences” (Kiegelmann, 2007, p.77). In other words, the I-Poem allows you to see a secondary structure to the story that is brought out by focusing on only the pronouns (I, you and we) and how the participant is describing her experience through those voices alone. For example, with one participant, a clear theme in her story was the idea of having more control over her learning process in the RPBL classroom (or being more agentic in this type of mathematics classroom). This young woman told
stories of seeing herself differently and feeling like she made an impact on her and her classmates’ learning process many times.

I feel accomplished that I get to…not influence, but in a way influence others and at the same time receive influence from others, because…then I feel accomplished like I’ve done something [that] not only affects myself as a learner, but others as well.

During the second reading, the following I-poem came out in the secondary structure through the voice of the self.

<table>
<thead>
<tr>
<th>I</th>
<th>You</th>
</tr>
</thead>
<tbody>
<tr>
<td>I know</td>
<td>You’re given that opportunity</td>
</tr>
<tr>
<td>I know I do</td>
<td>Who you are as a person</td>
</tr>
<tr>
<td></td>
<td>How you learn</td>
</tr>
<tr>
<td>I independently learn</td>
<td></td>
</tr>
<tr>
<td>I know</td>
<td></td>
</tr>
<tr>
<td>I have control</td>
<td>Control over your life</td>
</tr>
<tr>
<td>How I perceive</td>
<td></td>
</tr>
<tr>
<td>Why wouldn’t I want that</td>
<td></td>
</tr>
</tbody>
</table>

They both give evidence to the point that this student grew in her feeling of control over her learning, but in different way – one explicit and the other implicit. One leaves nothing to interpretation, but the other is more of an art form in the poetry of the first- and second-person voices or the self describing the new feelings learning and growing in this type of learning environment. Both the first and second readings agreed in the themes and told the same story, but in different ways.

After creating and coding the I-Poems, sometimes they would start showing patterns of alternating voices, but upon the third or fourth reading, I could actually hear the voices in tension with each other. The idea of counterpoint in music was used quite often by Bach in his compositions of fugues and inventions and is understood by
musicians and composers to be the technique of combining two or more melodic lines at the same time. Often the lines move in patterns like voice-crossing, parallel motion or contrary motion (Smey, 2005). On the written page, it is easy to visually notice the different voices in the music.

With the Listening Guide, the purpose of the third and fourth readings are to listen for voices that may be simultaneous but speaking in different patterns as can happen in contrapuntal music. The voices can be sending different messages and also speaking separately, for if I silenced one I would still hear the other voice speaking (Raider-Roth, 2012). This was so apparent at times there were almost moments of joy for me. I could clearly hear the conflict within the girl whom I could also remember sitting with and sharing in conversation. These voices of tension were also consistent with the themes that came up in the first two readings, but allowed me to go deeper into what they were saying. The contrapuntal voices made the codes more complex and spoke more deeply about them, relating them to each other.

Gilligan says that the voices in tension don’t necessarily have to negate each other, but the research needs to describe the relationship between the “inner and outer dialogue” (2012). This implies that oftentimes when there are voices in tension there are questions unresolved or lack of reconciliation in a person’s true narrative. These tensions
would be important to explore and include as part of the whole experience although not explicitly part of the first reading of the story. A wonderful example of this was with one participant who clearly had conflicting voices of how she saw herself as a math student. At one point in the interview, the student told me that she like algebra better than geometry as a subject because it was more procedural (“I like steps). But later on, she could be heard talking about what she enjoyed about the RPBL classroom was the discussion and the multiple ways in which they could see how a given problem was done (“If more classes were taught like this, I...I might like them a lot more”). She characterized people who were “good at math” as people who liked steps (“they like really like steps, and like “OK, this, this, this, go for it”). However, she did not think that those types of people would like this type of classroom unless they enjoyed discussion. It was difficult for this young woman to see herself as “good at math” if she couldn’t fit into her preconceived notion of what that person looked like and at the same time enjoyed this type of teaching. It was an interesting dilemma.

What I was seeing in other researchers’ work as disparity between the original Listening Guide method was sometimes just that, some researchers who had perhaps strayed just a bit from Gilligan’s original intent of the structure and purpose in the different phases of listening. But sometimes what I was interpreting as disparity was really just the researchers’ creativity in the interpretation of the voices in the third and fourth listening or in the authorship of the I-poems. Hearing from other researchers that the synthesis of the listening was a creative act that I would need to embrace in order to see the full picture of the story the participant was telling, and in most cases it might not be what I was looking for (Raider-Roth, 2012) was truly enlightening and helped me
move forward in this phase of my work. It helped me to realize that this type of analysis is never an either-or conclusive statement, rather so much more descriptive, or as Raider-Roth put it, a “rendering of the complexity of human relationships” (2012).

In the end, the procedure for the Listening Guide that emerged from my work took on the form in Table 3.3.

<table>
<thead>
<tr>
<th>Reading</th>
<th>Theme</th>
<th>Questions</th>
</tr>
</thead>
<tbody>
<tr>
<td>First</td>
<td>Plot/ Reader Response</td>
<td>What is happening? What has occurred? What actions are described? What stories are told? What are my interpretations of the story? How do I hear the story? How did I respond to what the participant was saying at the time? Now?</td>
</tr>
<tr>
<td>Second</td>
<td>Voice of the self (are there subvoices?)</td>
<td>Who is the actor? Can I engage with the speaker? Can I identify “I statements”? Are there multiple voices speaking? What is the first person meaning-making going on? What are the other voices heard? “you” statements, “we” statements, what story is told in the different voices or perspectives?</td>
</tr>
<tr>
<td>Third/Fourth</td>
<td>Contrapuntal listenings for attitudes in research question</td>
<td>Which voices seem to speak out about the experience in mathematics class? What are the juxtapositions of the experience? Where do they happen and how they relate to each other? Do they balance each other or is one stronger? How are they coming out and when?</td>
</tr>
</tbody>
</table>

*Table 3.3 Outline of The Listening Guide*

In the second reading, I listened specifically for phrases, both cognitive (“I think…”, “I know…”, “I do…””) and the affective (“I want…”, “I like…”, “I feel…”) and what those phrases might imply or how they might make meaning about the attitudes or attitude changes for the participants. It was important for me to listen for the potential for
multiple voices of the same young woman – for example, voices of scholars as well as adolescent schoolgirls. Also, for statements in the first, second or third person or for the associative or dissociative stance – all which can be implied in pronoun use of the speaker. This is the importance of the formation of the I-poems and the creativity that can come writing them (Brown & Gilligan, 1992). In the contrapuntal readings, I looked back at the original attitudes and senses from my original research question and allowed those to guide my “ear” while keeping in mind the themes that emerged from the coding of the first reading. I used the dichotomous attitudes (see Table 3.4) as a starting point, although this is not necessarily to say they were the actual voices that I heard. The voices I heard were often related to these attitudes in some form either contrapuntally - discussing a feeling of both enjoyment and aversion in spite of a lack of ability in mathematics or descriptions of opposing views of agency in very different types of classrooms.

<table>
<thead>
<tr>
<th>Attitude/Sense from Research Question</th>
<th>Dichotomous Attitude/Sense for Contrapuntal Listening</th>
</tr>
</thead>
<tbody>
<tr>
<td>Self- Confidence</td>
<td>Self-Doubt</td>
</tr>
<tr>
<td>Value</td>
<td>Worthlessness</td>
</tr>
<tr>
<td>Enjoyment</td>
<td>Aversion</td>
</tr>
<tr>
<td>Agency</td>
<td>Lack of Agency</td>
</tr>
<tr>
<td>Empowerment</td>
<td>Disempowerment</td>
</tr>
</tbody>
</table>

*Table 3.4 Contrapuntal Listenings - Guiding Voices*

In answering the questions in the interview, the participants shared accounts of experiences that described their learning through the RPBL method and how it affected
their attitudes towards mathematics and mathematics class. It was important to hear all of this through their own voice, using each listening to find evidence to help answer the research question.

A Continuum of Girls’ Learning in Mathematics

To prepare myself for the readings and to use the Listening Guide, I felt I needed a framework through which I could view the main attitudes that I would be investigating my research question attitudes: enjoyment, self-confidence, value, and a student’s sense of empowerment and agency. I also felt I needed a framework for understanding adolescent girls’ mindset and perhaps the lens through which they might be sharing their narrative with me. Much qualitative research has been done in recent years on adolescent girls concerning identity formation in mathematics. In these studies, however, the framework used for data analysis of the transcripts pertained more to identity creation in discourse related to mathematics in a sociocultural way. I desired a framework based on girls’ experiences and their perceptions of learning and also connected to the Pedagogy of Feminist Relation in which the girls were learning. I needed another lens through which to analyze the data from the learning perspective. However, this learning perspective needed to remain consistent with my theoretical framework, truly consider the audience which I was studying, and its relational learning and communicative needs. Brew (2001) formulated such a framework from three theorists’ work – Perry (1970), Belenky et al, (1986) and Baxter Magolda (1992) - with respect to women’s ways of knowing applied specifically to learning mathematics. Both Perry’s and Baxter Magolda’s models were based on a continuum of development from a dualistic mindset of truth in knowledge to a vision of the multiple ways in which we make meaning for ourselves. However, with a
feminist approach, Brew desired to include within that continuum the domains of knowledge that Belenky et al. concluded were gender-related (and Baxter Magolda supported as not “gender-dictated”) leading to a convergence of meaning making in context that allowed for both the listening of the inner voice and the voices of others (Brew, 2001, p.18). This framework struck me as most consistent with a relational educational theory while still attempting to “unfix” the gendered feminine assumptions of Belenky’s research. It also left open.

**Figure 3.5 Theoretical Framework for Analysis of Girls’ Learning in RPBL (based on Brew, 2001)**

- **Absolute Dualism**
  - No voice
  - Absolute orientation towards knowledge and truth
  - Each piece of knowledge is discrete

- **Transitional Multiplicity**
  - Listening to others’ voices/Receptive
  - Emerging acceptance of multiple perspectives in areas where knowledge is considered uncertain
  - Directed awareness of connected knowledge

- **Independent Relativism**
  - Listening to the voice of reason
  - Rejection of absolute truth where context plays an important part in assessing knowledge
  - Independent awareness of Connected vs. Separate knowledge

- **Contextual Commitment**
  - Integration of the two procedural voices
  - Perception that meaning-making once expected to come from outside themselves, from authority, emanates from within
  - Ability to perceive the complexities of interconnectedness of knowledge
the possibility for describing the diversity in the participants’ learning experiences and viewing those experiences on a continuum in order to envision any potential growth that might have occurred on it. Figure 3.5 attempts to incorporate visually the three separate theories (Perry, Belenky et al, & Baxter Magolda) into one (Brew).

In the diagram, I attempted to show the integration in the continuum of a female learner’s growth. It combines Belenky’s domains of knowledge (from silence to integration of internal and external voices) with Perry’s schema of ways of knowing (from absolute knowing through to contextual knowing). Brew aligned these two schema hierarchically via intellectual development beginning with Absolute Dualism and transforming into Contextual Commitment (Perry, 1970, Baxter Magolda, 1992). As I read through Brew’s ideas, I attempted to look for each part of the three theorists and where each of their ideas would lie on the continuum. I placed the ideas from each of their theories in the level that seemed appropriate. The first bullet point on the continuum relates to the girl’s level of awareness of voice in learning, whether it is internal, external or non-existent. The second bullet point describes the girl’s developmental level with respect to their own view of absolute truth in knowledge. The final bullet point expresses a continuum of the girl’s ability to view ideas as separate or connected or someplace in between.

I planned to analyze my research data by situating the participants’ voices (from the Listening Guide Method) at the level that seemed appropriate for each girl both at the beginning of the school year (pre-RPBL engagement) and at the end (post-RPBL engagement). This will allow me to reflect on how the girls described their learning
experiences within the mathematics classroom and how I interpreted the themes that emerged from the analysis of the readings.

Since mathematics is traditionally taught in a way that often suppresses voice (and particularly girls’ voices), it seemed prudent to see where the participants oriented themselves with respect to voice in the mathematics classroom and whether the RPBL pedagogy allowed them to notice their attitudes (namely enjoyment, value, self-confidence) and their sense of empowerment or agency and any connection to that student voice. Student voice is an important part of the Pedagogy of Feminist Relation. Also, since mathematics is traditionally a discipline that has a rather dualistic nature (answers are right or wrong), it also seemed important to view the participants’ stories in a way that allowed them to describe their perspective on the nature of truth and process in problem solving and whether the RPBL pedagogy affected their views on that nature. This continuum also helped me to place each participant’s expectations for her own agency in the mathematics classroom in context. Specifically, I anticipated that if their conceptualization of student voice was only at the Absolute Dualism level, their expectation, desire or need for their own agency would probably be lower as well.

Because I viewed knowing as a relational experience, I also took into consideration “the larger social context in which both student and teacher are embedded” (Thayer-Bacon, 2004). In addition, I found connections between this framework and the observed experience of the student within the classroom setting. One of my participants could easily be viewed listening to others in classroom observation, but growing in the skill of listening to the voice of reason. I made observations of engagement, agentic action and other behaviors that connect directly to one of the framework’s five learning
domains (silent, receiving, subjective, procedural or constructive (Brew, 2001) or that show evidence of one of the four levels of intellectual hierarchy (Absolute, Transitional, Independent or Contextual (Brew, 2001)). As well, I coded connections between the teacher interviews that signified any transformation between the levels of hierarchy (growth in student agency or empowerment) and the classroom observations. The less interpretive data pieces of the students’ journal entries helped to support the voices from their narratives, and gave strength to their voices from classroom observations. I anticipated that they would attribute some of their attitudes towards mathematics to some of the parts of the continuum (e.g. multiplicity or integration), or find themselves at one end because of a desire for more absolute dualism through their attitudes because of the RPBL. This is how this theoretical framework helped guide my thinking when it came to helping to give an overall system of structure.

**Trustworthiness and Rigor**

As previously stated, because of the descriptive nature of the study, any generalizability to other similar contexts is questionable. The purpose of this study was to obtain a description of these girls’ experiences of learning mathematics in this particular setting and observe if there are any patterns in their attitude of enjoyment, self-confidence, value, or their sense of empowerment and agency towards their mathematics learning. There should be no assumptions made of correlation or causality between problem-based learning, feminist relational pedagogy, students’ attributes and the attitudes they hold towards mathematics and their learning. It may be possible through the patterns that emerge from the analysis to offer some context for the journey the girls’ move through in their attitudes throughout their experiences with RPBL in this course.
and how it changes their feelings towards learning mathematics in general. However, once again, no generalizations have been made from that, only a holistic picture of the experiences of these girls’ views of learning mathematics with this approach.

Finally, when all of the coding of the journals, class observations and interviews were complete, I carried out member-checking. I made documents that included all I-poems with the general themes of the poems that I created along with my interpretations of them. I sent these via email to the girls, who were all still students at Greystone Academy. I received interested and helpful feedback from all five girls.

My five participants were a diverse group of young women who had much in common in terms of their overall characteristics, but individually had interesting and unique stories to tell. Let us now turn to the wonderful worlds of Sarah, Leona, Isabelle, Alanna and Kacey – with whom I had the honor and true privilege to work over the past three years.
“You have to connect a whole bunch of different things that you’ve learned in the past to what you’re learning now. I think it’s, yeah, the whole class is pretty much about connections, I think.” – Sarah, 14

CHAPTER 4
Sarah’s Story: Mending the Past

The campus was uncharacteristically quiet as I walked into the beautiful stone building where I was to meet Sarah for our interview. It was Greystone Academy’s winter vacation and Sarah was the first participant I was interviewing for the study. As a day student, Sarah lived nearby and was able to easily come back to campus for her initial interview so we decided that a friendly place like a classroom would be a good place to meet. Sarah was a new student to Greystone the year of the study, a freshman from a white, upper-middle-class family from a surrounding suburban town. Her mother was an aerobics instructor and her father owned his own marketing business. Sarah was a very artistic young woman who had already illustrated a published children’s book and to me, proclaimed herself “not a math person.” She had spent time at both public and private schools, but most recently she had attended her town’s public middle school from sixth through eighth grades where she was in an integrated algebra class which was considered to be the honors level class. She described being with all the “smart kids” but definitely did not describe herself that way (“even in, in elementary school I never liked math, it was always like my least favorite subject”). I asked Sarah to describe her experience in the classroom prior to coming to Greystone Academy.

When I was at my old school, I went to a public school beforehand, and um, the teacher would just stand at the board and she’d just like read off
notes and how to do the problem, so you never actually got to figure them out with each other.

Sarah’s description of a rather traditionally taught mathematics classroom with a direct instructional approach described a classroom that was not open to questioning or inclusive of the students’ ideas (“she’d just read off notes”). She felt a lack of freedom, and even condescension, in her learning in the way that the teacher did not consider the students a part of the problem solving or capable of productive contribution. Sarah expressed sincere disappointment with her past mathematics classroom experiences, but in middle school especially. There was a lack of satisfaction with her learning in that classroom in particular because of the interaction with the teacher and the fact that it was an “honors” level class whose culture she did not see herself fitting into.

And in my class there’s some really, really smart kids, the ones that, like, study math in the summer and everything. So, I think the teacher more directed her talks and stuff to them, because she, like… she even called some of us, like, boneheads and stuff.

Clearly, there were some residual emotions from her experiences in this class that had affected Sarah negatively. Even her teacher, Ms. Brown, had noticed that Sarah had negative feelings towards mathematics and she believed that it stemmed from her past mathematical experiences (“I would actually say her self-confidence is what is still being mended from whatever happened in middle school. So she’s hesitant”, “I think she had a really challenging middle school experience with a teacher that was um… kinda tuned her out a little bit”). Sarah spoke about her difficulties in learning in a this type of classroom mostly because she did not feel free to ask questions or that the classroom culture was open to questioning (“There were so many more people - it’s like they would all be judging you almost”, “I was like so afraid to ask a question”, “It was harder to learn
in that type of environment"). Sarah’s insights about her lack of learning didn’t stop there. She also viewed the teaching style as one that did not allow for retention or depth of thought to say nothing of questioning. I asked Sarah her opinion on being in an accelerated or honors class in middle school.

*Ms. S.*: You don’t think that even though you were in an accelerated class last year that the teacher was pushing you hard?

*Sarah*: Well, I think she was pushing us hard but she wasn’t giving us the right tools that we could use to solve the problems.

*Ms. S.*: Hmm…

*Sarah*: Because she would move on even if the whole class didn’t understand what she just said.

When Sarah spoke about her past work in mathematics and this class specifically, there was a clear voice of frustration and disappointment. Sarah’s feeling of being left behind and confused stemmed not only from a feeling of inadequacy when comparing herself to the other students in the class (possibly unfounded) but also in the pace the teacher took and how inconsequential the students’ questions or confusion seemed to the teacher. Her RPBL teacher, Ms. Brown noticed that in the first half of the year Sarah’s attitude towards mathematics changed and Ms. Brown believed it had to do with the pedagogy itself. Sarah needed the freedom to ask questions and be free from the fear of judgment and being wrong. She said to me that in this course Sarah’s “not being told ‘here’s the answer and now figure out how we got it’ as opposed to ‘go try to figure this out and even if you’re wrong it’s ok.’” Just a simple switch in the message being sent to the class about how the practice of mathematics is viewed seemed to give Sarah permission to move outside her comfort zone and make mistakes safely.
It was clear that Sarah and Ms. Brown had a very strong teacher-student relationship, which I’m sure helped Sarah’s perspective become more positive. Ms. Brown admitted that Sarah’s parents even attributed Sarah’s success in the class to Ms. Brown (“Her parents implied it that way. I really helped her love math, but I don’t know how much is the teacher”). Just to be sure, I asked Sarah to clarify for me what she saw as the difference between her middle school experiences and her experience so far in ninth grade.

*Ms. S.*: Like if Ms. Brown just stood up at the board and lectured the whole time do you think you would still feel this way or is it really the, like this sitting around the table and sharing at the board and that kind of thing?

*Sarah:* Um, I think it’s definitely a lot of both.

*Ms. S.*: Yeah?

*Sarah:* Because, I think Ms. Brown is a great teacher, and... but I also, if she were to just stand at the board and just tell us how to do it all, I think it would be a really dull class – I’m not saying that she’s dull [both laugh] but, just the way it would be taught… Well I think …Ms. Brown like tries to get us all involved.

Ms. Brown’s experience utilizing the RPBL pedagogy was one of her strengths and it definitely made an impact on Sarah. It was that fundamental involvement in learning that drew Sarah in and captured her interest and enthusiasm for the rest of the year. Ms. Brown said it best when we spoke together as she tried to describe Sarah’s intrinsic curiosity and excitement for learning mathematics now. Ms. Brown said, “She was waiting to be shown something and as soon as she saw it, [the interest] like sparked in her to go see [the mathematics].” Through just being involved, it was as if something was awakened in Sarah that she didn’t even know was there. Being given the freedom to
ask questions and feel as if the questions mattered and had relevance allowed her to see the richness of inquiry in mathematics.

**Inquiry for Deeper Learning**

As Sarah found herself experiencing mathematics differently for the first time, she started looking at the subject through a different lens. Sarah expressed deep appreciation for the set-up of the classroom and the practice of sitting around a table as well as students presenting their partially complete solutions (“I think it means more and you understand things better”). Her sincere interest in and curiosity about the material emerged because of the acceptance and outright encouragement of not only clarification questions but extension questions. Both in my class observations and in the teacher interview, there was evidence that Sarah had a natural inclination to look further than the question posed. Ms. Brown said, “She’s one of the few that thinks outside the box” and in their individual interviews both Sarah and Ms. Brown gave the same meaningful example of when this happened for them. Towards the end of the year in any traditional geometry course, the volumes of three-dimensional solids are taught in order for students to learn the formulas for the various solids – prisms, cylinders, cones, pyramids and spheres. Sarah was intrigued by the following problem.

As a spherical gob of ice cream that once had a 2-inch radius melts into the cone that it sits upon Sam wonders if the ice cream will fit. The cone also has a two inch radius and notices that the height of the cone is three times as big as the radius. Does the ice cream fit or does it overflow?

In class, a student presented the problem and everyone checked their work and the ice cream did overflow. But Sarah was intrigued and asked, “Well how big does it have to
be?” She wanted to know how big a cone you would need in order for the ice cream not
to overflow. I asked her in our time together about asking these types of questions.

Ms. S.: Do you um, do you feel yourself doing that in class or is it natural?
Is it…

Sarah: I don’t know, I didn’t [both laugh] I don’t think so, I think I just
was wondering and…[pause]

Ms. S.: Oh

Sarah: Then she makes it into like a whole ‘nother problem - we spent
fifteen minutes on it!

Ms. S.: Yeah? [laughs]

Sarah: She’s like “Oh this is awesome”. Happened again when we did um,
we were finding like the area of a sector of a circle.

Ms. S.: Mm…

Sarah: And it was a pizza question and then I asked how long would the
crust be and then we went into a long discussion on that.

Ms. S.: Yeah?

Sarah: And she’s like “great job!” I’m like, “Oh I was just wondering.”
[both laugh] I don’t know…

When I listened for the story, it was clear that Sarah was speaking with a voice of
inquiry. But further listening let me hear a voice of diminishing importance. What Sarah
casually states as “just wondering” is her downplaying the importance of Ms. Brown’s
interest in her extension question. Because of her prior statements regarding her fear of
being judged and her teacher moving on without concern of understanding, it is very
significant that in this classroom Sarah has found her voice that might have previously
been easily dismissed. In my classroom observations of her, I witnessed an example of
this type of extension question where Ms. Brown was having students work on a problem
where they were finding the area of the cheese on a piece of pizza. Students did not have a formula for the area of a sector of a circle at this point yet.

What is the area of the cheese on one piece of a 16” cheese pizza if it is cut into 12 slices?

This is a question that is written to lead up to students realizing the relationship between central angles and sector area as well as arclength in circles. Sarah and her classmates were at the board working on this problem and after discussing what it meant to have a 16-inch pizza, they easily realized that if they found the area of the pizza they could take one-twelfth of it to find the area of one slice. Quickly, Sarah thought of another question and said, “What if it only asked for the area of the crust?” and drew a diagram like this:

![Figure 4.1 Sarah’s Pizza Slice](image)

Suddenly, the class was very interested and Sarah went on to say that she wanted to subtract the Isosceles triangle’s area from the sector area. It was a great example of a moment where she was able to follow her curiosity and extend a problem into something that was more complex than the question asked.
Sarah shared with me how much she valued her ability to go deeper into her own questions and the questions of the group in this classroom ("I think it, it like helps you remember how to do the problem more and you understand it rather than just knowing the steps"). The other aspect of the class that seemed to foster Sarah’s sense of inquiry came through in her voice as well every time she spoke about being “at the board” ("I think going to the board helps me more, like it’ll, it’ll help me like remember how to do the problems."). In this classroom, student presentation of their ideas is a valued and focused part of the class discussion. In the following I-Poem, I could hear Sarah’s voice of appreciation for what she learned from being “at the board.”

<table>
<thead>
<tr>
<th>I</th>
<th>You</th>
<th>We</th>
</tr>
</thead>
<tbody>
<tr>
<td>I think going up to the board</td>
<td>you kind of have to learn</td>
<td></td>
</tr>
<tr>
<td>I find interesting</td>
<td>on your own</td>
<td>we sit around the table</td>
</tr>
<tr>
<td></td>
<td>help you along the way</td>
<td></td>
</tr>
<tr>
<td></td>
<td>you like</td>
<td></td>
</tr>
<tr>
<td></td>
<td>do your homework</td>
<td></td>
</tr>
<tr>
<td>I go up to the board</td>
<td></td>
<td></td>
</tr>
<tr>
<td>I always find mistakes</td>
<td>you’re up there explaining</td>
<td></td>
</tr>
<tr>
<td>I did the night before</td>
<td>you get a better understanding</td>
<td></td>
</tr>
<tr>
<td>I go up and do it</td>
<td></td>
<td></td>
</tr>
<tr>
<td>I think</td>
<td></td>
<td></td>
</tr>
<tr>
<td>I think</td>
<td></td>
<td></td>
</tr>
<tr>
<td>I think about it</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Although this poem starts with an inclusive “we” voice, Sarah alternates between the “I” and “you” voices later denoting more of a sharing between the first and second person. She wants to describe the experience from her perspective, but also share the views of a general student in the class as well. In the first person, she is sharing her own experience
of going up to the board and the mistakes that she has made herself. In the “you” voice, she is speaking as a student in the class and how, as a student “you” actually learn from those mistakes and the experience enables not only “you” but the others in the class to learn as well. It is quite telling that she starts with “we sit around the table” and end up with “I think about it” which shows the connectedness in the learning between the whole group, the individual and the material (we-I-it). This feeling of connectedness is part of the learning environment that is definitely something that Sarah felt was missing in her prior mathematical experiences.

Mathematics as a Unifying Experience

Throughout every listening I did, I kept in mind Sarah’s voice of disappointment with her middle school mathematics experience. I heard a voice of isolation and fear. In the following I-Poem, a stanza of hope is surrounded by two stanzas of disconnection and fear with regard to her prior experiences.

<table>
<thead>
<tr>
<th>I</th>
<th>You</th>
<th>We</th>
</tr>
</thead>
<tbody>
<tr>
<td>we’d fill in notes</td>
<td>we’d do homework</td>
<td>we still go over the homework</td>
</tr>
<tr>
<td>you can really</td>
<td>If you don’t understand</td>
<td></td>
</tr>
<tr>
<td>I think in my old class</td>
<td>you can definitely ask</td>
<td></td>
</tr>
<tr>
<td>I was so afraid to ask</td>
<td></td>
<td>all be judging you</td>
</tr>
</tbody>
</table>

The significance of Sarah discussing her old class in the first person (in both the “we” and “I” voices) is how personally she took that experience. All of the interactions in that class affected her a great deal. It was not only the mathematical activities, but the tone
set by the teacher, the classroom culture and the day-to-day non-mathematical activities (like filling in notes) that left an imprint on what she thinks of mathematical activities. Her perspective on mathematical thinking connects with the emotion of fear and judgment. Interestingly, the stanza that is in the second person, is where she is talking about the RPBL class and “you can definitely ask” in order to get your questions answered. It’s almost as if she doesn’t want to get too close or get her hopes dashed. However, the fact that she has hope at all is encouraging.

As stated previously, Sarah described the act of “going to the board” as a pivotal part of the class for her. The support that she garnered from Ms. Brown and her classmates seemed to surprise her and help her through some of her fear and anxiety.

And I was just like so, I didn’t understand it at all, but when I went to the board and I, I tried to show people what I did, [Ms. Brown] helped me through it, and I think, I don’t know, I think that’s when I was like “Oh, I… well, maybe I can do this.”

The big difference for Sarah was the support and where the information was coming from. Possibly for the first time, she was learning from knowledge that she and her classmates were constructing. They were the “team” that were in charge of working together to build the knowledge and this was making a real difference for her understanding and enjoyment of the material.

But I mean, we all really try, I think, to do the best we can and help each other out. I mean it’s kind of like a team thing almost. If you’re like at the board and everybody’s helping you out with how to do the problem – I mean everybody’s figuring it out together. I think when, uh, the whole class like helps you figure out a problem, you, I don’t know…It means more…You get to work off of like each other’s knowledge.
This is what Maher (2001) called the “duty of interpretation” in the feminist classroom where the teacher needs to foster an environment of sharing authority and deliberately positioning students as academic authorities (p.130) by using classroom practices like having them do presentations that might not be of fully formed solutions. When they share in the authority of the co-construction of knowledge it makes the classroom experience a much more unifying experience. Particularly in mathematics education, where the more traditional classroom tends to be isolating and disconnected, some students crave this type of learning. Sarah found what she needed in this RPBL classroom. Even her teacher commented on how much Sarah looked forward to sharing her ideas and not worrying about judgment with her classmates (“She loves coming in and checking her answers with her partners and the classmates and seeing if she got it the same answer or a different answer.”) The fear of other students judging her ability is no longer paramount in her mind because she sees a mutual respect for each others’ ideas and risk-taking. There is also a culture of cooperation and mutual responsibility for each others’ learning that has been set by the RPBL pedagogy from the principles of relational equity and authority.

Sometimes in my class, um I don’t know, when one person’s at the board the others are really listening to what they have to say. Because not everybody can solve all the problems on the homework, So, if one person’s at the board and they fully understand it and they explain it to the class I think it means more than just a teacher telling you how to do a problem…Yeah the more people that participate, the more you understand it.

In observing Sarah’s class I would concur with her statement that when a student was at the board, others were intently listening. They responded with questions and comments. There was a culture of respect and mutual esteem for the hard work and courage that it
took to be up in front of the class. More than that, they knew there was a responsibility to
care for each others’ understanding. The fact that the class manifested the principle of
Relational Equity is evidence that Ms. Brown was successfully practicing RPBL in the
classroom. Relational Equity as a tenet of a pedagogy of feminist relation promotes
holding others’ ideas as a priority, treating different viewpoints fairly, and committing to
learning from others’ ideas (Boaler, 2008). As stated previously, this mutual respect and
common commitment leads to positive intellectual relations between the students and can
only promote a more successful learning environment, as articulated by Sarah.

**Muddying the Waters of Aversion**

When listening for contrapuntal voices, there was consistently a tension between
Sarah’s strong voice of enjoyment of mathematics and her previous dislike. This tension
was very apparent in the following I-poem where there were clear voices expressing
enjoyment and aversion of mathematics throughout.

<table>
<thead>
<tr>
<th>Enjoyment</th>
<th>Aversion</th>
</tr>
</thead>
<tbody>
<tr>
<td>I like it</td>
<td>I’ve never really liked math</td>
</tr>
<tr>
<td>I like geometry</td>
<td></td>
</tr>
<tr>
<td>the only time I’ve ever liked math</td>
<td></td>
</tr>
<tr>
<td>I like the way</td>
<td></td>
</tr>
<tr>
<td>I’m doing a problem</td>
<td></td>
</tr>
<tr>
<td>I do like math class</td>
<td>I didn’t like that class</td>
</tr>
<tr>
<td></td>
<td>I didn’t like the stuff we did</td>
</tr>
<tr>
<td>I like it a lot</td>
<td>I always found math really boring</td>
</tr>
<tr>
<td>I feel accomplished</td>
<td></td>
</tr>
<tr>
<td>I like math</td>
<td>I had to struggle</td>
</tr>
<tr>
<td>you like math</td>
<td>I never liked math</td>
</tr>
<tr>
<td>I think it’s a great class</td>
<td>I just never liked math</td>
</tr>
<tr>
<td></td>
<td>I’m not really a fan</td>
</tr>
</tbody>
</table>
I love that
I also like journals

Although it is not evenly balanced, her main thoughts are about enjoyment, but it does seem that her associative stream of consciousness is speaking about her aversion to mathematics in the past. Those experiences are truly still a part of who she is and although her experiences with RPBL have been positive and have changed her outlook, there are still residual effects from her past. Observing Sarah in class, she worked with others, laughed and communicated about mathematics while remaining positive about problem solving. Ms. Brown was optimistic about her attitude towards mathematics and was certain that she had positive feelings towards both mathematics and the class (“she has one of the best attitudes about math - she loves it.”, “she came in here and suddenly just like looks forward to class every day.”, “she talks about how much she loves math.”). However, even in asking Sarah about her attitude towards math class, she responded with the tension between enjoyment and aversion from the past.

Well I, I don’t know. I think I’m a better math student now and I think this class has made me, like, have a better understanding of math and that I can actually do problems and… I think, I think it’s helped me learn a lot better and I have a, like, better respect for math class [both laugh] because before, even in, in elementary school I never liked math, it was always like my least favorite subject.

Even when Sarah was talking to me about how proud her parents are now about this change in her attitude, she became a bit modest and changed the subject to what she sees as good about enjoying mathematics now.

Sarah: Well I mean, I-, I tell my parents that I like math class, and they think it’s really great that I have a good teacher.
Ms. S.: Mm

Sarah: …and everything, like even when I bring home my journal entries, there’s like pages and pages of how to do centroids and orthocenters and I was trying to explain it to my dad one day.

Ms. S.: Oh wow

Sarah: [laughs] and he thought it was great but he thought I was a nerd [both laugh], but…

Ms. S.: Yeah. But they, they’re impressed?

Sarah: Yes.

Ms. S.: That you had this change?

Sarah: They’re definitely impressed [both laugh]


Sarah: Because I have like pages of how to, like in my… my um, journal I have like color-coded…It’s great to have, um, like not maybe a love for math, but if you understand it and you like math, I think it’s better and you can use it in like everything else.

Sarah’s switch to using “you” instead of “I” in the last statement indicates a disconnection from the idea of enjoying the mathematics as if she was not talking about herself anymore, but a student in general. There is a certain amount of pride that Sarah feels in her excellent work in her journal and also in her enjoyment of mathematics, but there’s something stopping her from taking total ownership of this part of her identity. It is clear from research that the formation of an individual’s identity in mathematics learning is a complex and subtle process (Lim, 2008a). Recent research points out that identity formation in mathematics for both boys and girls, often stems from a culture that relies upon gendered stereotypes and conceptions of a binary oppositional system of relationship with mathematics (Mendick, 2005a); you either get it or you don’t, you’re
either fast or you’re slow, you like math or you don’t, and often these dichotomous views are linked to specific genders, although sometimes they are not, depending on the experiences individuals have had. From the tension in Sarah’s voices, it sounds as though she still struggles with her mathematical identity. Perhaps this course helped break down those clear distinctions of dichotomous mathematical identity and muddied the waters for her in order to allow her to gain a different perspective to enjoy mathematical activity a bit more.

From Confidence to Empowerment

From her negative experiences in the mathematics classroom in her elementary and middle school years, it was very easy for Sarah to come into high school with unresolved feelings about mathematics as a subject. But interactions in a classroom with students that were spoken to differently because of their perceived ability (“I think the teacher more directed her talks and stuff to them, because she, like… she even called some of us, like, boneheads and stuff”) had really hurt her confidence in her own abilities in mathematics. She had chosen to be quiet and withdrawn as if her questions and ideas were not important. Even Ms. Brown could see this in Sarah’s work and stated, “I think she still struggles a little bit because it’s tied in with her self confidence.” When I observed Sarah in class, I noticed that there was a tension between knowing when she was right and doubting her abilities – always questioning if she was right or wrong. She would often answer questions correctly under her breath, or when she wrote something on the board that she thought was correct, she would shyly state, “Never mind” before her classmates would support her in her efforts. I asked Ms. Brown about this behavior of Sarah’s and she had this to say:
If I ask her a question, she sometimes thinks I’m sometimes maybe saying her question is wrong and she… I’m just asking her to explain more. And she’ll be like “Did I do it wrong then?” and I’ll say, “Oh no, you have the right work up on the board, I just want to understand where you got those numbers.” So she still needs some validation. When she’s at the board she needs a little assurance that her solution, either her process is correct or her answer is correct and I’m just questioning why she did something. She gets a little self-conscious when she thinks she made both errors.

The validation that Sarah is seeking from her teacher, and probably herself, is not only that of being right, but also in some ways, of being good. Sarah expressed the feeling of appreciating the praise that Ms. Brown gave when she did a good job in class (“if I’m doing a problem and Ms. Brown like gives you praise for doing it correctly, I think that really helps you learn better too”, “I think when you finish a problem and if she really thinks you did a good job, she’ll praise you for that – I think praise… like, I love that.”). It is the affirmation that she is a good student and is trying and doing a good job that is making Sarah feel good about herself. It may be that in a more traditional classroom the reliance on the teacher’s authority role and control over the learning did not allow Sarah to gain the confidence that she needed (“I think geometry, like, it does help me have, like, a better self-confidence because you’re up there at the board and, like, work through all the problems on your own.”). The importance of experiencing the mathematics and making connections for herself was something that Sarah had not realized until this course. When I listened for a voice of confidence, there always seemed to be a contrapuntal voice of self-doubt not far behind. Because Ms. Brown mentioned this theme and I was curious about Sarah’s unwillingness to shift toward greater confidence. In our conversations, I asked Sarah about this.

Ms. S.: So maybe it’s easier to feel confident when you know you are the one that um, kind of did the process?
Sarah: Yeah.

Ms. S.: It wasn’t somebody like showing you the process?

Sarah: Exactly

Ms. S.: You feel stronger? I don’t know what to say, but like, um…

Sarah: Yeah

Ms. S.: Huh [pause]. Do you feel like you were always confident in math?

Sarah: No [laughs]

Ms. S.: Oh no?

Sarah: Definitely not

This is where I could hear confirmation of the contrapuntal voices. Sarah uses such strong words such as ‘Exactly’ to agree with my statements about her confidence in math, but then when I ask her if she was confident in math, she says no, laughingly and uses just as strong language saying ‘Definitely not.’ However, at other times she uses a voice of confidence and clearly articulates the feeling that she knows she has a good ability in mathematics (“I think this class has made me, like, have a better understanding of math and that I can actually do problems”). There is a real revelation in her voice that she is quite capable, but something still keeps her from fully accepting her ability with complete confidence. Ms. Brown even senses it through her experiences with Sarah in class.

Yeah, it’s a kind of interesting duality for her. Um, but like when we did a review question for her on reflecting a point across a line, she was one of the first to remember what to do. So I think that when she understands something it’s really in there and she can really apply, but she gets kind of questioning herself when... if she’s not…especially in the really basic
things…like adding, subtracting and Pythagorean Theorem. Sometimes she’ll be, like, “I just did that” instead of thinking that that piece isn’t really worthwhile. But that’s really important especially given the make-up of the regular students who might not even understand how those numbers came to be.

What Ms. Brown is describing here is how easily Sarah will dismiss her contributions to class discussion especially if she thinks the mathematics was trivial (like arithmetic or the Pythagorean Theorem). But Ms. Brown’s view is that any contribution that a student makes is “really worthwhile” and tries to help Sarah see that, praises her and attempts to build her confidence. She knows that Sarah is one of the stronger mathematics students in the class and helping out with some of the foundational facts and skills actually is helping some of the weaker students in the class. Sarah may be growing in her confidence because of that. The subtle sense of accomplishment pushes her forward and encourages her confidence.

The following I-poem shows the contrapuntal voices of Self-Confidence and Self-Doubt that echo through Sarah’s narrative.

<table>
<thead>
<tr>
<th>Self-Confidence</th>
<th>Self-Doubt</th>
</tr>
</thead>
<tbody>
<tr>
<td>I don’t know</td>
<td>I didn’t know even</td>
</tr>
<tr>
<td>I don’t remember</td>
<td>I didn’t understand</td>
</tr>
<tr>
<td>I don’t know</td>
<td>I tried to show</td>
</tr>
<tr>
<td>I can use</td>
<td></td>
</tr>
<tr>
<td>I’m teaching myself</td>
<td></td>
</tr>
<tr>
<td>I think it</td>
<td></td>
</tr>
<tr>
<td>You understand it</td>
<td></td>
</tr>
<tr>
<td>I actually try</td>
<td></td>
</tr>
<tr>
<td>I’m trying</td>
<td></td>
</tr>
<tr>
<td>I’m doing a problem</td>
<td></td>
</tr>
<tr>
<td>I remember</td>
<td></td>
</tr>
<tr>
<td>I didn’t know even</td>
<td></td>
</tr>
<tr>
<td>I didn’t understand</td>
<td></td>
</tr>
<tr>
<td>I tried to show</td>
<td></td>
</tr>
<tr>
<td>I did</td>
<td></td>
</tr>
<tr>
<td>I think</td>
<td></td>
</tr>
<tr>
<td>I can do this</td>
<td></td>
</tr>
<tr>
<td>I took accelerated math</td>
<td>I was weak</td>
</tr>
<tr>
<td>------------------------</td>
<td>-----------</td>
</tr>
<tr>
<td>I taught myself</td>
<td>we never got taught</td>
</tr>
<tr>
<td>I understood</td>
<td>I didn’t know</td>
</tr>
<tr>
<td>I went further</td>
<td>I was lacking</td>
</tr>
<tr>
<td>I try to be a leader</td>
<td>I don’t always understand</td>
</tr>
<tr>
<td>we all felt so accomplished</td>
<td></td>
</tr>
<tr>
<td>I’m prepared</td>
<td></td>
</tr>
<tr>
<td>I can show</td>
<td></td>
</tr>
<tr>
<td>I feel accomplished</td>
<td></td>
</tr>
<tr>
<td>I’m a better math student</td>
<td></td>
</tr>
<tr>
<td></td>
<td>I had to struggle</td>
</tr>
</tbody>
</table>

It’s as if for almost every confident comment she makes about herself, she must balance it out with a statement that diminishes that growth that she has made. Some studies have shown that even girls with high achievement and evidence of strong ability in mathematics “were apt to openly reveal doubts over their abilities, to be overly concerned with image, and to display a lack of overall confidence” (Dentith, 2008, p.155). So although Sarah may be growing in her confidence, the voice of self-doubt may remain on display for reasons of image, identity or culture that Sarah has embedded in her memory of what a mathematics classroom is supposed to be. At least now she is capable of making statements of accomplishment, pride and ability that perhaps were not apparent to her from her past experiences. These have come out in confidence, but also in a sense of more of her own ability to direct her learning and to have a voice in the classroom – agency and empowerment.

Sarah’s journal was one of the most organized, well-done and creatively written journals that both Ms. Brown and I had seen in our time at Greystone. Ms. Brown said she had a “beautiful journal” in our interviews and when I went through Sarah’s book it
was evident she had put a great deal of time into it. She took pride in her ability to summarize her problem solving and made use of her writing in many different ways. As I read through all twenty of Sarah’s entries for the entire academic year, I noticed a change in her voice somewhere in the middle of the year. It sounded as if she became a bit stronger in the language she used as she spoke about what she did in the problem solving. For example, in the earlier entries she would say

If you have a 5x5 square and a 3x3 square, you can cut them into pieces that fit together to form a 3rd square. If you cut those two squares in a way like diagram 3, where there are two right triangles that fit together…

However, in the latter part of the year, Sarah was more likely to write her entries in the first person such as

First I drew the triangle depicted in the problem. I knew all angles = 60 degrees because it’s an equilateral triangle. I also knew when the equilateral triangle was split in half by an altitude it will create two congruent 30/60/90 triangles.

This switch from ‘you’ to ‘I’ in Sarah’s authorship voice indicates a movement to attachment and connection to the knowledge. It is understood in the mathematics register that the I-you contrast can be paralleled to the “process-object distinction” which associates the mathematical notion as an object independent of the action of the speaker (Rowland, 1999). However, the significance of the student’s use of ‘I’ can be thought of as the “recognition of the pre- or post-object cognitive state” suggesting the cognitive transition in the students’ relationship with the notion being discussed (Rowland, 1999, p.25). For further evidence of Sarah’s increased transition to the more connected state of understanding to the material she was writing about, I did a lexical search for the words ‘you’ and ‘I’ in the journal
entries I transcribed in the MaxQDA program in which I was doing my data analysis. The graph in Figure 4.2 is the number of occurrences of each pronoun for all twenty journal entries for the whole year.

![Graph of Pronoun Occurrences]

*Figure 4.2 Occurrences of First Person vs. Second Person Pronouns in Journal Entries*

There is a distinct decrease about half-way through the year in Sarah’s use of her “you” voice and an increase in the first person indicating an increase in her ownership and authorship of her understanding and connection to the material.

In the following I-Poem, Sarah is talking about her ability to direct her own learning and the way in which Ms. Brown shared her authority with the class in order to promote student agency and increase their empowerment.
Initially, Sarah’s “I” voice is speaking of internal feelings of external behaviors that she felt she could do in class – have more control, things she could say. The “you” she turns to represents Ms. Brown speaking to the class, including her, so it is both generalizing and individual here. In the first person plural, she is speaking for the class when she says that the group felt that they were able to respond to Ms. Brown’s questions of how they could be involved in the decision-making about assessment, homework and problem solving methods. They became more empowered by being given that authority to “fill in the holes” and do the “recap.” It is interesting that something as simple as being asked to summarize or give an opinion (part of the pedagogy of relation model) makes such an impact on student empowerment.

Sarah’s Transformation

Overall, my appreciation for what Sarah had been through in middle school went far beyond the listenings, but made an impact on me because of my own negative
experiences in the mathematics classroom. I was encouraged by how she found a place
for herself and made a connection with this classroom and Ms. Brown and saw that
mathematics could be seen with a different lens (‘I try to solve problems in different
ways’). Although she still struggles with the strength of her ability and being solid in her
confidence, she is moving forward with this idea which is certainly progress from where
she was.

Most telling about Sarah’s transformation to a student that enjoys mathematics is
her final statement in our time together. She is expressing appreciation for the
opportunity to see mathematics from a different point of view and at the same time share
her initial feelings of hesitation and residual fear from her previous experiences. I
appreciated that greatly.

Sarah: Well, I had a really great experience this year [laughs]

Ms. S.: You did?

Sarah: I wasn’t expecting it.
“It’s changed my identity and given me kind of like a voice in math. Whereas I didn’t really have one before. It was a silent voice.” – Leona, 15

CHAPTER 5
Leona’s Story: Finding Your Voice

I first sat down with Leona in a quiet, empty classroom in a building on the campus of our school in spring of the year she was taking M210, the mathematics course taught with RPBL. Leona was a sophomore in this course and she would characterize herself as having somewhat mid-level ability in mathematics and relatively low interest. She said she was really a humanities-type student who loves reading, debate and all things related to the stage. From a White, upper-middle class background, Leona was a boarding student who was known as a “legacy” at Greystone - her mother also had attended. Leona prided herself on the fact that she loved the school and has deep respect for learning and the independence the school fosters.

I believe that Leona was looking forward to sharing her ideas about learning and mathematics mostly because she had seemed to enjoy the class throughout the part of the year we had spent together so far. She was a student in my class so seemed very comfortable speaking with me but when we met for the first interview, she may have been somewhat nervous. However, it seemed more about the unexpected nature of the questions as opposed to any anxiety she might have been feeling. She started off, almost apologetically, explaining that she was “really a lecture-based student and had been warned by” students who had taken the course before that it would be hard to transition to learning in a problem-based environment. But in the same thought process she almost instantly became a huge fan of the course and claims that “what I’ve
experienced…opened me up to a new kind of style and I like it.” Her identity as a mathematics student is rather negative and self-effacing although she sees herself as an outspoken and independent learner (“I’m a little weird”, “[Math’s] never been my strongest subject”, “I consider myself an English or history person”, “I normally have low self-confidence when it comes to math, just because it’s not my strongest subject”, “It’s hard for me to fathom”, “I’m not that good at it”). She attempted to summarize what it is that stands out to her in comparing this type of learning to her previous experiences in more traditional mathematics classes.

It’s not the teacher sitting in front of the classroom being like, “Oh, do you remember when we did this? Well, this is like that.”…On my homework for example, using Pythagorean Theorem to find the length of the hypotenuse and then having to find a distance on a coordinate plane, and relating the concept back and applying it to that. Versus, “Oh you can do it this way” like a teacher sitting and saying “Here’s all the options of how to do it, now go and do it.” It kind of gives me a satisfaction of being like, “Oh I’m smart enough to connect that point and understand that.”

Leona’s comments here describe her overarching feeling of this course giving her a larger sense of ownership of and control over her own learning. The feeling and belief that she was “smart enough” to make the connections on her own or that she would be able to not need the teacher to tell her which way to do a problem. This was one of the many themes running through Leona’s interviews and documents.

**Independence as Empowerment**

As stated previously, empowerment in the classroom can be seen as the belief or feeling that a student has the power to direct and a sense of directing her own learning.
Leona was found expressing these feelings a number of times in the listenings. When I listened for the voice of the self, in her final interview at the end of the year, a clear theme of empowerment came out in this I-Poem. I had just asked her the question of how she felt the course may have affected her view of her mathematical identity, reminding her that she had said that she did not view herself as a “math person.” She begins the segment by saying that she feels that she’s a very independent person and she’s “been given that almost freedom, in a way, to express myself” and that this has helped change her identity.

<table>
<thead>
<tr>
<th>I</th>
<th>You</th>
<th>We</th>
</tr>
</thead>
<tbody>
<tr>
<td>As you grow</td>
<td>When you turn 18</td>
<td></td>
</tr>
<tr>
<td>You have the power</td>
<td>You get to express yourself</td>
<td></td>
</tr>
<tr>
<td>No matter what side you’re on</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

I feel like
I could be on
I like to solve it this way

We both get to express
One of us is wrong
If one of us is right
Or even if both of us is right

Changed my identity
Given me a voice
I didn’t really have one before
I think
I could always
I guess
I could go
I needed

We had each day

I could ask
I didn’t really feel
I could go
I couldn’t go
I could ask how
I could ask what
I couldn’t ask why before
I think
I mean
I hope so
In this passage, it is striking that Leona began with the “you” voice, or the second person, distancing herself from the idea of growing up, getting older, maturing and having power. She may see this as something that will happen in the future, perhaps when she is out of school that is when you get to express yourself. She then takes the “I” voice, or the first person narrative stance where she says that she “likes to solve it this way”, where you can distinctly hear her voice expressing her own opinion, something she said she didn’t think would happen, or should happen, until you are older. She then moves into the third person, into the “We” voice speaking as the class as a whole, or two classmates who disagree on their ideas in class, coming to the conclusions that even if they had both used different methods that disagree, “both of us is right.” This idea that there might be more than one “right” solution is actually the very essence of the freedom that Leona is looking forward to in the future. The idea that she can independently come to conclusions based on her own ideas is what is freeing, what has changed her identity and given her a voice (one she didn’t have before in mathematics class). It is clear in the last stanza of the I-Poem that Leona is still conflicted between what she can and cannot do (by the alternating “could” and “could not” lines), but in the end she is clear that she was deeply affected by the methods utilized in this class.

There was evidence of growth in Leona’s empowerment in her writing as well. In her 17th entry from the middle of the academic year writing about a problem where she was asked to find the slope of one of the midsegments of a triangle when given the
coordinates of the original triangle, Leona writes “This question only asks for one midsegment, but I decided to give all three, because it is a three sided figure.” In the drawing she showed how creating the three midsegments of a triangle creates another triangle interior to it, constructing some of her own knowledge and extending the problem she was working on. In another problem from almost six weeks later, she has moved on to choosing her own solution method in her writing. A problem where she was given three coordinates A(4,6), B(5,0) and C(9,9) asks for angle BAC. Leona chose to “form right triangles around the point A and to calculate the angles that when added to A makes it a straight line.” (see Figure 5.1).

She then went on to find the lengths of AC and AB with the Pythagorean Theorem after finding the horizontal and vertical sides of the right triangles she created “indicated by the dashed lines” she wrote. She then solved for the angles adjacent to A “that I have labeled X and Y” using the inverse sine function. Although were many other methods of find the angle BAC in this problem, Leona was empowered to not only use
the solution that she thought of, but to write about that solution in an empowered way using the first person (“I have labeled…”) as if she were the mathematician.

In speaking with her, Leona struggled with the concept of empowerment in the classroom (“I could definitely use that word…to describe how I felt…it’s a really strong word.”, “When I think about it, I think of a rally or revolution or something like that…”). She felt like this was something from her prior experiences that it was something that she felt she wasn’t supposed to have in the classroom. She could remember times when she was asked to present a homework problem when she really didn’t have a solution to give, “I do remember one problem that I was really struggling with…I just stood up there and I felt so defeated that I couldn’t understand it.” But this was quickly followed by a positive feeling of support from her peers as she stated, “And then, everyone, kind of chimed in or a few people started to chime in, and like ‘Oh, you could look at it this way. Oh, like just try it this way’” and this sense of camaraderie encouraged her to pursue an avenue at the board that she may not have attempted on her own. I clearly remember observing this interaction as her teacher and the transformation that seemed to occur from a student who was disempowered to one who was empowered.

There was a tension between Leona’s willingness to accept her own empowerment in the mathematics classroom which was when I listened for the contrapuntal listening. There seemed to be a clear tension between a voice of empowerment and independence and a voice of disempowerment or interdependence - almost as if the mathematics student of her prior class was telling her that her earlier learned habits were the ones that she needed to fall back on. For example, when I asked her about how the connections were made for her in her learning of the mathematics, she
had a difficult time articulating whether to give credit to the group or to take credit for those connections herself.

_Ms. S._: Well, if you put it in the context of making connections, like, what...how did those get made? Did somebody tell you what the connections were or did you decide what they were? Like I guess that’s the thing I wonder.

_Leona_: No, no, no, absolutely, I, I **made them**

_Ms. S._: Ok

_Leona_: Well, I mean, with, with, I mean… whooo (sighs)

_Ms. S._: [laughs]

_Leona_: I **don’t want to take all credit** for things because obviously…

_Ms. S._: Well, you’re allowed…

_Leona_: Other people [laughs] **other people are there to like help you** and support you and things like that, but no, I do think it was, it’s a very **independent process**.

In this short segment I could hear the voice of her own empowerment in the classroom stating the strength she has incurred from the work she has done over time and she had “absolutely” made those connections herself. But at the same time, she clearly knew that some of those connections were made because of the influence from her peers and her teacher and she had a responsibility to share the authority and credit with the others. Part of what creates Leona’s tension here is the perception that in our American culture it is of higher value to be able to accomplish feats on our own. The ability to work in a group or find value in its relational workings in order for a collaborative solution to exist is not as highly praised. We can take this assumption directly from
traditional modes of assessment which rarely took into consideration the work of a shared group.

You can also hear the conflict in the voices in the following I-Poem taken from a segment from her initial interview where I had just asked her to compare a prior math class experience to the RPBL class. The first stanza is very much disempowered – basically as she described what she did as she was going through the motions of the traditional class and how that worked well for her then. It moves towards a more active learning mode when she is talking about the RPBL class.

I recognize that
I did
I really like that
I consider
I’m doing
I mean
What am I doing?
I’m solving these answers
Here I feel like
I mean
I’m getting
I’m understanding
I’m doing
I’m doing it
I’m doing it
I can

What am I doing?
I’m solving these answers
Here I feel like
I mean
I’m getting
I’m understanding
I’m doing
I’m doing it
I’m doing it
I can

You get the “why”

You get that

You get that

I really like that
I consider
I’m doing
I mean
What am I doing?
I’m solving these answers
Here I feel like
I mean
I’m getting
I’m understanding
I’m doing
I’m doing it
I’m doing it
I can

You get the “why”

You get that

You get that

I recognize that
I did
I really like that
I consider
I’m doing
I mean
What am I doing?
I’m solving these answers
Here I feel like
I mean
I’m getting
I’m understanding
I’m doing
I’m doing it
I’m doing it
I can

What am I doing?
I’m solving these answers
Here I feel like
I mean
I’m getting
I’m understanding
I’m doing
I’m doing it
I’m doing it
I can

You get the “why”

You get that

You get that

I recognize that
I did
I really like that
I consider
I’m doing
I mean
What am I doing?
I’m solving these answers
Here I feel like
I mean
I’m getting
I’m understanding
I’m doing
I’m doing it
I’m doing it
I can

What strikes me as meaningful in this I-Poem is not only the movement from the more passive voice to using more active verbs when describing the RPBL classroom, but also the question (“What am I doing?”) and the statements (“I am getting”, “I am understanding”). Leona describes her experience in this classroom in terms of agency.
She is the one doing the solving, she is the one understanding – which is coming from a voice of empowerment.

**Relational Learning: Being in Another Person’s Mind**

When I considered this tension between Leona’s value and strength of her independence and her interest in and desire for interdependence with others, it made me wonder about her feelings about relational learning. This is consistent with what is known about girls, (Brown & Gilligan, 1992) but not necessarily utilized or focused on in mathematics classes in the US. Leona was very articulate about what it was about the relational aspect of this classroom that helped her learning. She said she liked how it “kind of put you through another person’s mind, in a way.” She even extends herself to say that “for me, when I have a better relationship with a person, I want to listen to them more.” She tries to explain that wanting to listen to them more and wanting to learn from them are inextricably tied together since “seeing the way another person thinks, [allows me to] develop a respect for them.” She follows that thought saying that “I just think it opens up a lot of discussion…which promotes learning inevitably…and creating new ideas and things like that.” At one point in our initial interview, I asked Leona what she thought about how the open discussion allowed students to share their own ideas.

It’s nice because we all do things differently, like as different people, everyone has a different personality and everyone thinks differently and it’s really nice to see how I think or look at something versus how someone else like in my class looks at something and being like “wow, that could work, I could use that”, or “I could use my way, whichever feels most comfortable.” But it’s nice to have that option presented by not only the teacher, but the student too because, I think, in a way, it develops like a relationship with your class that you don’t really have because you’re talking to them and you’re learning how they think.
This might be something she is used to in an English or history class, but is actually very novel in a mathematics class where she is used to there being “no other way to look at it” than the way the teacher showed you. This idea of bringing multiple perspectives on a problem to the discussion really worked for Leona mostly because of the relational aspect of learning. She had such a deep respect and appreciation for other people’s ideas that it was natural for her to learn this way. When asked for an anecdote from class, Leona gave an example from a class period that I remembered vividly. Here was the problem:

An airplane is flying at 36,000 feet directly above Lincoln, Nebraska. A little later the plane is flying at 28,000 feet directly above Des Moines, Iowa, which is 160 miles from Lincoln. Assuming a constant rate of descent, predict how far from Des Moines the airplane will be when it lands.

In class, another student had presented this problem by using slope as the rate of change (i.e. 8000ft/160miles) and used 28,000 as a y-intercept and wrote the equation of the line. She had then graphed the line and found the x-intercept to find how far from Des Moines the plane would be when it landed. This made no sense to about half the class, who were thinking geometrically, including Leona. So another student said they just did it by “counting” – they started at 36,000 and went down by 8000 and tried to see how many times they needed to do that to get to the ground (i.e. 36,000/8,000=4.5). So, they figured they needed to go over to the right 4.5 times 160 miles and that’s where the plane would land. That seemed to make more sense to a few more students, but then Leona got up and said, “Oh, so it’s like drawing a bunch of triangles with sides of 8,000 and 160 from 36,000 to the ground?” (see Figure 5.2)
It took a few minutes of discussion for her to show how what the other student said had inspired her geometric approach to this solution, but then a great connection was made between the other student’s algebraic approach and this one. The students realized that finding the x-intercept of the line was actually the same as finding the landing point the way Leona and the other student did. It was experiences and discussions like these that allowed Leona to grow in her appreciation of the multiple ways in which students viewed different problems. She learned a great deal from seeing these different perspectives and it only added to her learning experience. In our initial interview she made the statement that, “I really like that you get that ‘why’ in a few different ways – from your teacher, from your friends, well, I consider them my friends…” and because of the relational aspect of the learning, she really did consider the majority of the class her friends even if they were not close friends outside of class.
One part of the relational learning that pleases her the most is the fact that there is interaction and connection between the students in the class. This interconnectedness and responsibility for each other seems to give her some satisfaction not only in her own learning, but in the learning process in the classroom as a whole. She says

I feel accomplished that I get to…not influence, but in a way influence others and at the same time receive influence from others, because…then I feel accomplished like I’ve done something [that] not only affects myself as a learner, but others as well. And…it’s just a good feeling that I could hope to make others understand, if I’m correct with what I’m saying. And even if I’m not, I mean, everyone learns from mistakes so to present myself and kind of put myself out there, too, in front of people, it’s nice to have them accept what I’m saying, or choose not to. And so, I feel accomplished.

Later on in the same interview, she returns to this idea when I asked her how much of her own voice she felt was in asking questions or in giving input.

Ms. S.: I wonder how much of that was actually you sharing your actual ideas with the teacher or with the class or with…you know with me…a totally different experience…Your voice in that class might have been asking questions, and your voice in this class is actually giving input into the way people are learning which is really more powerful in a lot of ways, you know?

Leona: And it affects me and it affects other people…hopefully…I mean

Ms. S.: Oh I know it has…

Leona: I hope so, I mean anyone though…I mean I feel like I’m affected by other people and I hope to affect others or influence as well. Influence is a better word.

Ms. S.: Right, I think that’s great.
She seems to gain so much emotionally from this mutual influence that the classroom community shares and that emotional profit helps in her learning. She spends a great deal of time discussing her feelings about the time she spent in a traditional mathematics classroom and how the environment there did not foster this part of her. She tries to explain the difference here:

There’s a lot of things that my teacher, my old teacher, kind of said here’s what it is, but there was never that way, there was the how, there was the what, but that was it, and it’s nice to kind of understand now, what I’m doing, in a way, because I mean it was just like now here’s ten to twenty problems, go use what I’ve just taught you to solve the homework, well, what am I doing, I’m solving these answers, but for what? So here I feel like you get the why and it’s nice.

Here the “why” she is describing is the relationship with the material that was missing in her old classroom - the key link and part she played in her understanding of the material and in constructing that understanding. She was able to have a more conceptual understanding of procedures that were previously done by rote. Interestingly, she talked about the part that she “could” play in the classroom which was asking questions – she said “questions were always a possibility. Teachers never denied us of that, like, privilege; I guess you could call it.” As she said this I noted the sarcasm in her voice when she said “privilege” as if the students should be grateful for the ability to ask the questions that the teachers already knew the answers to, but the input in the construction of knowledge generally stopped there. This follows up with the fact that the types of questions she could ask and the participation in the construction of knowledge was basically limited to asking questions for clarification of the information that the teacher
had given them. It was not really any inquiry questions or thoughtful expansion of the
knowledge that perhaps she was curious about. She says

I could ask questions, but I couldn’t go and um…just understand it more
and it kind of goes with that “why” thing. Like I couldn’t go and ask…I
could go ask what, but I couldn’t ask why, which really affected me too.

I believe this speaks to Leona’s desire for a relationship with the material. The following
I-Poem is from my first interview with her. In it you can hear the voice of the relational
learner and her aversion for the disconnected classroom. It is from a segment where
Leona was talking about the needs she had that weren’t being met in a traditional
classroom and how it caused feelings of aversion towards the material.

<table>
<thead>
<tr>
<th>I</th>
<th>We</th>
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<tbody>
<tr>
<td>I kind of know</td>
<td></td>
</tr>
<tr>
<td>I have to do</td>
<td></td>
</tr>
<tr>
<td>I might as well</td>
<td></td>
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<tr>
<td>I want to do</td>
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<td>I want to be</td>
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<td>I know</td>
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<tr>
<td>I could</td>
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<tr>
<td>I’ll choose not to</td>
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<tr>
<td>I feel comfortable [this year]</td>
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<tr>
<td>I dread</td>
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<tr>
<td>I actually really enjoy</td>
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<tr>
<td>I don’t know</td>
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<td>I’m not 100% sure why</td>
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<td>I think</td>
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<td>I know</td>
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I felt like my teacher wasn’t 100%
I don’t know
I kind of base my classes
I think
If I have good relationships
I just didn’t really want to go

We gotta go

Affected how I felt about the material
I didn’t really mind
I really didn’t look forward to it

To listen to her talk to us
I’m looking forward to
I consider a pain
I’ll choose not

When I listened for the voice of the self, I heard her in the first person using the “I” voice feeling a sense of obligation to do math, to go to class, to do well. But there is also a movement back and forth between her feelings between this year and the past, moving between positive verbs and less positive ones and then finally saying “We gotta go” in the third person—an ultimately seeing it as a requirement and she just had to give in.

However, in the second stanza, she is talking about relationship—how the relationship with her teacher and the material ultimately affects her learning in mathematics. If these are working for her, she will look forward to it, but if not, she can choose to take the prior stance and give in once again.

The desire to want a connection to the material in the mathematics classroom—to want to know the reasoning behind solutions and ideas—is what is missing for her. Leona is feeling the central desire for a complete learning experience—the idea that no learning environment is complete without all three aspects of I-Thou-It (Hawkins, 1974) being the student, teacher and material intertwined inextricably. This can be further extended in the RPBL environment to the teacher as extended learner, subject matter knowledge as curriculum and relationship and respect as aids and impetus in the learning process (Rodgers & Raider-Roth, 2006). Her negative feelings about the lack of relationship in a prior classroom were so strong that Leona went so far as to state “I just didn’t really want to go and sit and have to listen to her kind of talk to us.” This made me feel that she would go into that classroom and felt as if she were giving up any sense of
agency that she may have had initially going in there (if she had any in the first place). It also gives a sense that the teacher “talking” to the class is really the only type of communication that is of importance in this type of classroom.

Mathematics Classroom as Agentic and Disagentic

In listening for the contrapuntal voices, I decided to try to name the dichotomous states of the attitudes that I am investigating in this study. I found it hardest to name the antithesis of the stance I was calling student agency, which was something that Leona spoke about a great deal. I spent some time first working with the autonomy part of the definition of agency referring back to the definition in my earlier writing:

Agency, with respect to learning, can be seen as the ability to act in a willed or voluntary way. It can also be seen as having the capacity to behave in an autonomous or independent way on one’s behalf while learning in the classroom (Biesta & Tedder, 2006).

In our first interview together, I left the idea of agency rather ambiguous and allowed Leona to simply discuss her feelings about her ability to be independent in her learning or to her capacity to act on her own behalf in the classroom. In the following I-Poem, I was struck by Leona’s ability to speak on her own behalf and about what she believed was her classmates’ behalf (in the “We” voice).

<table>
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<tr>
<th>I</th>
<th>You</th>
<th>We</th>
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<tbody>
<tr>
<td>I think</td>
<td>We are accustomed to</td>
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<td></td>
<td>We’ve realized</td>
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<td></td>
<td>We’re all different</td>
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</tr>
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<td></td>
<td>We’ve been exposed to it</td>
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</tr>
<tr>
<td></td>
<td>To see whether we’re right or wrong</td>
<td></td>
</tr>
<tr>
<td></td>
<td>What we’ve done</td>
<td></td>
</tr>
<tr>
<td></td>
<td>We’ve realized</td>
<td></td>
</tr>
<tr>
<td></td>
<td>We’ve realized</td>
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</tr>
</tbody>
</table>
The way we like

I know
I know I do

You’re given that
opportunity
Who you are as a person
How you learn

I independently learn
I know

Control over your life
I have control
How I perceive
Why wouldn’t I want that

This segment of her interview describes a sense of agency on her classmates’ behalf (the repeated “we’ve realized”, “to see whether we’re right or wrong”). She moves into a more personal voice describing her own agentic self where she has the control to see how she perceives her own learning and she definitely appreciates that opportunity in this classroom. When I asked her if she had always felt that math was certain, she responded by saying that it was always like the teacher “being like two plus two was four” but now she feels like she can be the one that actually does something. “I drew a number line or I counted on my fingers” relaying her appreciation for the fact that she has free will in her learning. She also feels like she has more power and control in how she does academically. When I asked her how her view of herself as a student changed over time, she replied, “Well, in the beginning of the year, I was like getting C’s on exams and things like that and I was like ‘this needs to change’…” Although math had always been challenging for her, her negative attitude had reversed because she was able to take control of it herself. This feeling of autonomy and being able to act on it perhaps was missing from a more traditional classroom for Leona. She said of her old classroom in her old middle school,
My teacher - basically she told us, “OK you plug this into your calculator and out pops the answer.” It’s just like I said, the way it is ‘you do this and you get that’, and when you’re looking at a triangle this is the place where your answer goes, but there was never that why and this [RPBL] is putting the pieces together for me and I really like that.

The enjoyment she is feeling from being the agent of her learning is allowing her to be a more active part of the process. She craved more control over what to do and how to do it, as well as connection with the reasoning.

In listening for Leona’s voice of agency, it is clear that she takes pride in her work and is grateful for the opportunity to find her own voice in mathematics. In considering student agency in learning, it seems clear that agency would need to coincide in a classroom with a student’s sense that they had a voice and free will to use it. But there was a clear contrapuntal voice talking about the mathematics classroom as disagentic as well. When I asked her to talk about how this course has possibly changed her as a mathematics learner or her identity as a mathematics learner she had this to say:

…I’ve been given the opportunity to express myself. It has made…my identity has changed as a person. Just kind of as you grow, as you turn 18, for example, like a simile, when you turn 18 and you have the power to vote now, you get to express yourself no matter what side you’re on. I feel like this course is kind of like that. I could be on the side where I like to solve it this way and someone else could be on the side where they like to solve it that way, and the fact that we both get to express our opinions and even if one of us is wrong and one of us is right, or even if both of us is right. It’s changed my identity and given me kind of like a voice in math. Whereas I didn’t really have one before. It was a silent voice.

Leona’s experience of having a “silent voice” in the mathematics classroom can be extended to many marginalized students in the U.S. today where the ‘silencing’ constitutes the process by which contradictory evidence, ideologies and experiences find themselves buried, camouflaged and discredited.” (Fine, 1987). Whether she was
actually silent by not talking at all, or she was silenced in this way where her ideas were buried or discredited by a learning environment that was not conducive or welcoming to them, is really irrelevant - what was important is that is how Leona felt. I have to admit that at the moment when she was sharing this story with me, I wanted very much to share with her my own story of my high school education, but kept the interview focused on hers. This very emotional moment for the both of us reminded me of a feeling of oppression - the way she was describing having a teacher that told students the way to learn a certain topic or held back her own ideas or thoughts. Leona spoke of not wanting to “go to listen to her talk to us” which I interpret as math class basically was all about being “talked at” instead of having interaction with others. This type of oppression on the part of the teacher reduces the students’ agency in learning in that it does not allow them to express their ideas or investigate their questions. In Marilyn Frye’s classic essay on oppression in *The Politics of Reality* (1983), she makes the analogy between oppression and a birdcage.

If you look very closely at just one wire in the cage, you cannot see the other wires…You could look one wire up and down the length of it, and be unable to see why a bird would not just fly around the wire any time it wanted go somewhere…There is no physical property of any one wire…that will reveal how a bird could be inhibited or harmed by it except in the most accidental way. It is only when you step back, stop looking at the wires one by one and take a macroscopic view of the whole cage, that you can see why the bird does not go anywhere; and then you will see it in a moment. It is perfectly obvious that the bird is surrounded by a network of systematically related barriers, no one of which would be the least hindrance to its flight but which by their relations to each other, are as confining as the solid walls of a dungeon. (p.5)
What were the “wires” that Leona described for me? A classroom that did not allow for multiple perspectives or value differences in learning styles or choices, a teacher that was directive in methods as opposed to questioning in ideas, a classroom community that was isolationist as opposed to creating a cooperative community of learners, a teacher who held all of the mathematical authority instead of dissolving the hierarchy of authority throughout the classroom – she could go on. It also seemed that there was a form of self-silencing that came from knowing that the type of questions that were acceptable were ones that kept things moving along and were not creative or interesting (“questions were always a possibility. Teachers never denied us of that, like, privilege; I guess you could call it”). I was impressed with the depth of understanding of the subtleties of the classroom that she was able to share with me and how articulately she verbalized her thoughts.

In her final interview with me, Leona was able to capture the essence of her thoughts on agency by sharing what it was about RPBL that worked best for her.

*Leona:* I remember more of the things that I get wrong than I do right – if I mess up and something’s wrong on a test

*Ms. S.:* Mm

*Leona:* I remember that more than if I had done something and got it right

*Ms. S.:* Oh wow

*Leona:* Because it sticks out in my mind for some reason, I don’t know.

Interestingly, she feels like she learns better when she makes mistakes, so being allowed to make the mistakes helps her, even on a test. She also appreciated being able to
volunteer to put problems up on the board on a daily basis (“Saying like oh I want to do
this one, or oh this one sounds good”). This gave a sense of being in control of the
classroom and hence in part of her learning.

There are many examples in Leona’s journal entries of articulations of her sense
of agency. In her 18th journal entry, which was sometime in early February of the
academic year, she was writing about a problem where the students are given the
equation of a line in standard form $3x + 2y = 12$ and they are asked to find the measure of
the angle formed by the line and the x-axis. Leona writes about her thought process by
saying that “the first thing I thought to do was to turn the equation into slope-intercept
form so it becomes easier to plot by isolating the y-variable.” In this sentence, she is
directing her own problem solving process by choosing what would make the problem
easier for her in order to visualize the line, graph it more easily and have it in a linear
form that makes sense to her. She goes on to say, “This decision gave me a y-intercept of
6 and a slope of $\frac{-3}{2}$.” I find it interesting that she did not just report that the y-intercept
is 6 and the slope is $\frac{-3}{2}$, but sees these two values as a result of a decision that she had
made in the problem-solving process. Further, she draws in her diagram the graph of the
line which has a y-intercept of 6, and using the slope finds that the x-intercept is 4, so she
states, “In the diagram to the right, I have labeled the angle x, and drawn a small arrow
for clarification. When you draw lines in from the y-intercept of 6 to the origin and (4,0)
(or the x-intercept) to the origin, a right triangle is formed.” She goes on to describe how
she can now use the inverse tangent function to find the angle requested. Leona’s
powerful language of how she drew the diagram (being clear to use arrows) and showing
where the right triangle comes from in order to solve for the angle shows her autonomy in the understanding of the process she is in this problem (see Figure 5.3).

Another example where her voice of agency comes through in her writing is in her 19th journal entry which was probably at the beginning of March that same year. Here, Leona was beginning her study of circles, reviewing circumference and area formulas from middle school. One question asked if four cup saucers, each with circumference of approximately 12.57 inches, could fit on a single layer in a square box with side of 9 inches. Leona moved through the arithmetic of solving for the diameter of each saucer easily \((12.57/4.00)\pi \approx 4.00\) and then goes on to give two different ways in which to view the solution to this problem.

You multiply the diameter by two, this means the diameters of the two plates is eight so it fits \((8<9)\). Another way to check is that there are four
plates so you know it’s a 9 x 9 box which is an area of 81 and the two plates are 8”, 8 x 8 is 64 which is also less than 81.

Although she is using the more distancing, generalized “you” voice here, I believe that in giving two different answers, she is promoting her own agency by voluntarily choosing these two options willingly. By giving herself options or writing the alternatives, she is being consistent with what she values in the classroom and thus making use of her autonomy in her learning.

Overall, both an agentic and dissagentic voice came through in the contrapuntal readings showing that tension exists in Leona’s mind about how she perceives herself as a mathematics learner. She seems to crave the agency and voice of independence in the mathematics classroom that has brought her satisfaction in this course, but her past experiences have taught her that this is not the case in most schools in the U.S. Perhaps she has come to believe that the exception that has brought her the most confidence and enjoyment, but which is not the norm, is worth fighting for.

**Positive Steps: Self-Confidence, Value & Enjoyment**

Leona was very clear from the beginning that mathematics was not her favorite subject (“I didn’t go in with a very positive attitude,” “I dread going,” “You have to drag me to go.”), and that she didn’t think she was very good at it (“I didn’t really understand,” “I normally have low self-confidence when it comes to math.”) However, she did seem to see a point to learning it – that there was value and even beauty in mathematics.

It’s so important to everyday life and, I think it’s wonderful, I mean it’s universal, in every language…I think it’s so important and so vital to
everything…but I love it as a whole…I, I think, I mean, I love it as something like, I mean I don’t love necessarily doing it.

Saying that she loves it as a whole, but she doesn’t necessarily love “doing it” implies that she loves the ideas and the concepts that were discussed and most likely the act of discussion and making those connections, but the reminder of “doing it” and getting the answer right or wrong may not be enjoyable to her. In reading for the voice of the self at this point, I believe I heard Leona talking about an appreciation for mathematics and problem solving that she sees now. Perhaps now that she realizes that she can have that relationship with the material that was missing before. When reading for the voice of the self, I believe this I-Poem sums up Leona’s sense of confidence gained through her experiences in this classroom.

I
I could
I’m smart enough
I think or look
I could use that
I could use my way
I think

You
You can do it

Relationship with your class
You don’t really have
You’re talking
You’re learning
You think

I guess
I like it
I have it here
I have a better relationship
I want to listen
I want to actually learn
I know
I could
In her own first person voice, Leona stakes her claim that she is “smart enough” and she can use her own way to do a problem. As she switches to the “you” voice in order to tell more of a story about relationship and what happens within the community of learners, it is easy to feel a sense of comfort and enjoyment, even freedom. She clearly likes what she realizes she has in this environment. Leona has what she wants from a learning environment – the better relationships, wanting to listen in class and actually learning mathematics possibly for the first time in her education. In her journal entries there are a few pieces of writings that could be interpreted as evidence of enjoyment where she described something as a “neat equation” or even drew a small emoticon “☺” to signify enjoyment of a specific process. Observing her in class, one would definitely get the impression at times that she was enjoying herself, jumping up to the board to give a classmate a boost of confidence or realizing herself something she forgot, but once again the enjoyment for Leona came from the process and not necessarily the product in the problem solving. And honestly, for her, that was just fine.
“It’s not as direct, like just between you and the teachers, like here it’s between like you and then like the person you’re sitting next to. If they don’t understand, and then like you ask a question and you know that other people benefit from it and it’s not just you not understanding.”

-Isabelle, 14

CHAPTER 6

Isabelle’s Story: “Seeing Through” Objectivity

 Sitting down and talking with Isabelle was such a pleasure mostly because, as one of the participants who was not one of my students, I was really looking forward to getting to know her on a deeper level. I had seen Isabelle around Greystone Academy and had interacted with her briefly on a freshman class “bonding” trip in the fall, but had little occasion to speak with her throughout the school year. We had arranged an afternoon for the initial interview when she was free and she came to my office across the grassy quad from her dormitory. The first thing that struck me about Isabelle was how she seemed much older than a freshman in many ways. I felt that she had a quiet confidence about herself, although she does not describe herself has confident. Isabelle is a boarding student from a suburban area whose parents care a great deal about education. She has big beautiful brown eyes, curly black hair and describes herself as rather shy and from mixed race ethnically - one of her parents is Sudanese and the other is white. She did not come across to me in our conversations as shy, but as very articulate and composed for a freshman in high school. Isabelle came to Greystone Academy from public schools where in sixth grade she was taken from the “regular” track and placed in the “accelerated” one in mathematics class which gave her an awareness of being a bit “behind” the other students who had been together in sixth grade. It seems that this move caused her to have a view of herself as an “average” mathematics student (“within the
accelerated math class, I was average”) since she was comparing herself to more able students since the switch. Interestingly, this may have also caused a change in her motivation as her teacher, Ms. Johnson also noted that Isabelle lacked passion and interest in mathematics in the classroom too (“I don’t see a lot of passion in Isabelle,” “She doesn’t always go deeper.”). As Isabelle and I got to know each other, and I observed her in the classroom, it was clear that there was a disconnect between what she wanted to get out of learning mathematics and her own personality and confidence in herself unrelated to learning mathematics. I found this to be an interesting tension for Isabelle and one that actually worked in her favor.

Self-Confidence Motivating Inquiry

When I observed Isabelle as a member of her classroom community, it was clear that Ms. Johnson depended on her as a regular contributor to the class discussion. On more than one occasion in both classes I observed, Ms. Johnson counted on Isabelle to confirm others’ ideas or share her own ideas (whether right or wrong) by writing her solutions on the board and explaining them in detail. Isabelle was clearly a valued member of the class in many ways. She confidently would ask questions of her classmates and give feedback to others. However, in our discussions together, Isabelle would regularly admit to not seeing the value in doing the mathematics. In many ways, in listening for contrapuntal voices, I could hear contrasting voices of value and worthlessness. I once asked her if she saw mathematics as something that has value. This was her response.
Isabelle: Yeah, um, well, I can see how it’s useful and stuff, but, like I wouldn’t choose to study this if it wasn’t relevant in like every single aspect of our life

Ms. S: Yeah?

Isabelle: Like if it was just math for fun or whatever, I wouldn’t take it.

Ms. S: So, you do it because it’s required?

Isabelle: Mm hmm

Ms. S: And it’s not like, you don’t see yourself kind of pursuing it?

Isabelle: No

So although Isabelle can see how mathematics is relevant to her life and probably can be applied to helping her solving problems, it is not something she would choose to study if given the choice. However, I can hear in her voice that she does have some understanding of why it is required. She says it is “relevant to every single aspect of our life” so she knows that there is meaning to be made in studying mathematics.

Interestingly, she also says in our final interview together that unless you are studying for a certain profession, she doesn’t think that it’s “absolutely necessary” to study mathematics, but later on in the same interview she says that

I think it’s definitely important, like everybody should know how to do certain things, like multiplication and addition and all that... It does help you with like your problem-solving abilities.

So although in one respect she can’t see that math is necessary for people to study, in another she can see the benefits of studying it. Also, although she freely admits that math is not her favorite subject, she does “like math” because she thinks “it’s really interesting when you can connect different ideas together” signifying something that she has realized
about mathematics in this course. All of this begs the question, what would make a student who does not see the value in a subject or think they are particularly able, enjoy studying it? What seems to have worked well for Isabelle in this situation was that she had an inherent sense of confidence in herself and what she was asked to do in her class. When I asked Ms. Johnson about Isabelle’s level of self-confidence, she commented that it is “pretty good” and in fact, it had allowed her to “get some answers wrong sometimes, because she’s just so confident about it, she gets them wrong” which is actually a positive aspect of this type of pedagogy. Ms. Johnson works with the ideas that are wrong in this pedagogical style and allows students to discuss at length the reasons why students have gone down the wrong paths and allows them to correct mistakes themselves. Isabelle’s lack of inhibition has probably allowed her to be able to be a leader in these discussions and helped others to understand more by the mere fact that she is inherently confident in herself.

While reading for voices of tension again, along with those of value, I could hear a voice of doubt in her ability in mathematics. Although Isabelle is a confident young woman, she has had experiences that have led her to doubt her abilities in mathematics. Seeing herself as “average” in the accelerated class and having her teacher choose to place her in those classes later than the other students in her grade have led her to believe that she may not really belong and perhaps may not be as able as the others. This shadow of doubt comes up when she talks about times when she is confused and how this classroom has helped her (“if I didn’t know something and I didn’t think it was right, I wouldn’t put it up on the board.”). However, the voice of confidence can also be heard
when she realizes how much she can accomplish on her own. I asked her how she felt about seeing non-routine problems on assessments throughout the year.

*Isabelle:* And I’m like OK, so I can just start doing a problem and then hopefully it develops into the right answer.

*Mrs. S:* [laughs]

*Isabelle:* And usually it does, so… Just using what you know to um, like solve a problem

*Mrs. S:* Yeah [long pause, writing] Yeah a lot of it is kind of like just remembering ideas…

*Isabelle:* Mm hmm

*Mrs. S:* And being like, “Oh I can use that here…”

*Isabelle:* Yeah

*Mrs. S:* And, “Oh, I can use that here.” Yeah, that’s true.

*Isabelle:* It’s more like a puzzle than a test.

It seems that although she may have times when she doubts her abilities, it is also true that she has times when she sees problem solving as fun, like doing a puzzle. She ends up feeling accomplished when she tries something on her own or with her classmates. In fact, Isabelle seemed to have a very strong voice of unity between her need for understanding and what she can do for the class, as an interdependent relationship. This I-poem shares her confidence in the mutuality of the relationship she has with her class.
In this segment, Isabelle is speaking only in the “I” and “we” voices, indicating that she is being totally inclusive in what she is saying. She is moving back and forth narrating her feelings about what she is doing, knowing and needing for herself, and what the class as a whole (including herself) is doing, knowing and needing. However, the different processes for problem solving (personal vs. group) somewhat parallel each other and she has a role that she plays in both. I believe that her own confidence has helped play a part in her ability to see that she can be a more active participant in mathematics in this classroom and part of a community of problem solvers.

*Is mathematics really all about procedures?*

Isabelle also identified herself as a student that “really like(s) algebra” because of how procedural it is. She believes that that’s how she learns best – just understanding the algorithmic procedures and doing it over again. In a conversation about what subjects she found most interesting, she was discussing how she was excited to take chemistry the following year because she knew it was a lot of memorization and formulas or “plug and chug.”

*Isabelle:* And I really like algebra, so… yeah…
Ms. S: So it’s like more procedural stuff?

Isabelle: Yeah

Ms. S: Like, if you, if there’s a way to do something…

Isabelle: Like yeah, I like steps [laughs].

I had her describe to me what she thought a person who was good at math was like (“button-up shirt, pants, tie, glasses, ruler, you know, really straight-forward and stuff”) and she definitely did not see herself like that. I then asked her if more math classes were taught the way her current math class was taught (with RPBL), did she think that that type of person would be good in that kind of math class. Instead of answering my question, she responded in the following way which I felt was much more enlightening. In the following lengthy segment, it is important to listen for the tension between Isabelle’s voices of a desire for structure in the mathematics class and a need for freedom.

Isabelle: Mm hmm. Like if more math classes were taught like this, I… I might like them a lot more.

Ms. S: You would like them a lot more?

Isabelle: Yeah.

Ms. S: Um, But do you think that other person would, like them a lot more?

Isabelle: Um… I don’t know. It really depends on the person.

Ms. S: Yeah?

Isabelle: Because if, they like really like steps, and like “OK, this, this, this, this, go for it” [simulating doing the steps of a problem]

Ms. S: Mmm…

Isabelle: That type of thing
Ms. S: Yeah

Isabelle: Then they probably wouldn’t like it, but if they’re more open to discussion, then

Ms. S: Yeah

Isabelle: They probably would

Ms. S: Hmm. Yeah. Well you just said you like steps though?

Isabelle: Yeah… It’s… [pause]

Ms. S: That’s weird?

Isabelle: It’s contra- I’m contradicting myself…

Ms. S: Yeah

Isabelle: You know, like… [pause]

Ms. S: So I wonder what it is about this that you like then?

Isabelle: It’s like steps, but it’s more relaxed. It’s… I can’t really explain it. It’s like more open but like, well, in geometry there’s not really necessarily always steps to do problems.

Ms. S: Yeah, right, sometimes you have to see how to do it…

Isabelle: Because there’s like twenty, yeah, like there’s twenty different ways to do a problem.

Ms. S: And yeah, right [laughs]

Isabelle: But like um, like I could see this happening in like an algebra class,

Ms. S: Mmm?

Isabelle: because that like requires steps, but you can also have discussions about it if you didn’t understand and that could help a lot more, so…

Ms. S: mm hmmm, yup, yeah, that’s true. So the discussion doesn’t necessarily have to be about like the alternatives,

Isabelle: yeah…
Ms. S: it could be about how to do it or…

Isabelle: mm hmm.

Ms. S: Like “I don’t understand how to do it, can you explain a different way how to do it?”

Isabelle: Yeah.

Ms. S: Or something like that, yeah. Alright. [pause writing] But… the heart of it has to be that openness to discussion?

Isabelle: Mm hmm

Ms. S: [long pause, writing] What do you think makes the discussion um, easier?

Isabelle: Um… That, like everybody else has questions and will participate and it won’t be just like asking a question and complete silence.

Ms. S: Mm

Isabelle: Like somebody will jump in and respond.

I feel that there are many important statements made in this segment. When Isabelle thought she was being contradictory by saying she would like math class more if they were run more like this class, she starts trying to figure out what it is about the class that she likes. She expected that what she liked about the class was talking about the procedures with her classmates - seeing how to do the problems and having someone explain them to her step by step. But then she saw it from a different perspective (“it’s like steps but it’s more relaxed”) and that perhaps it’s not the algorithmic conversation that she appreciated, she said that “in geometry there’s not really necessarily always steps to do problems.” What she realized at that moment was that classroom conversations centering on the problems were not always about how to do the problems, but might have
been about how the students related to the problems – their opinions about the problems, their concerns about the problems and their ideas about them (“you can also have discussions about it if you didn’t understand,” “there’s 20 different ways to do a problem”). These connections to the problems caused a different type of interaction with the material and in the end her larger realization is that her classroom experience wasn’t about procedures at all. It was about the community and the interhuman relations that existed. She observed that if students’ curiosity and inquisitiveness are fostered they will participate by asking questions because they will be invested in the process and the relations. In contrast to her prior experiences in a more traditional classroom, the teacher won’t just be “asking a question and complete silence” will follow. The people in the classroom had no relationship to each other; there was no reason for them to participate. Here there is a connection and she is convinced that someone will jump in and respond – in fact she knew it, it was a given.

This realization that mathematics can be more than just algorithmic procedures came up in a number of other statements that Isabelle made. It was clear from these statements that she was moving towards a new understanding of what learning mathematics meant to her. I asked her in our discussion at the end of the year what she thought helped her enjoy the class more than math in a more traditional classroom. She replied by saying that because there was so much note-taking and individual homework assigned it minimized the interaction in class with others, but in this class,

Well, you got to like speak your ideas out loud so it’s in the - it’s easier to think, at least for me, when I’m talking about it, because then I just have to make up random stuff but sometimes like it actually works.
When Isabelle compared “making up random stuff” to following an algorithmic procedure in a traditional class, the process of problem solving seems very arbitrary. She is amazed when it works for her although she admitted that it is easier for her to think in this type of process, as opposed to just following “steps” that someone else gives her. In class, when she would present a partially complete homework solution at the board, I observed how Isabelle synthesized her thoughts from the night before with the comments and ideas from the class in the moment. What she saw as “random”, I would describe as connective, or even catalytic, as a classmate would remind her of something they had discussed the previous day that might have been important to the solution of the problem. Once she realized that connection, she could then finish the problem on her own. Time and again, she organically synthesizes her ideas and this forms a mathematical process for herself while she is constructing her own processes as opposed to taking on someone else’s framework. However, this method helped her enjoy mathematics more and allow her to understand the material better in her eyes.

There is still the voice of longing for the direct, procedural instruction (“she jumped in and explained more,” “if you didn’t understand, she’d just explain it more.”) but she also explained how much she enjoys being a part of student construction of knowledge (“it’s really easy to understand certain concepts because of everybody else, like giving in their feedback and like the teacher giving their feedback,” “It helps me like remember what I learned by restating it to the class and like teaching it to somebody.”) This tension between her new vision of learning mathematics and her old habits of the direct instruction mathematics classroom echo Alanna’s habits of mind that were cited in Anderson’s research on students in feminist classrooms (2005) and their resistance to the
level of independence required. Although the tension exists, Isabelle is clear that this newfound potential in learning mathematics has created more opportunity for enjoyment for her time in the classroom.

Enter Subjectivity

As Isabelle starts describing more attributes that seem to be adding up to her enjoying the class more, I am trying to paint a picture of what it is that produced her enjoyment. The interesting thing is that it is not the mathematics she is enjoying, but the class – the interaction between the people in the class, and should they be solving some interesting problems that pertain to mathematics, that’s OK too. What Isabelle described enjoying about the class is the way in which she saw mathematics as no longer black and white, with only the teacher’s information as what counts. I asked her to describe for me what it’s like in class with Ms. Johnson.

*Isabelle:* Like it’s, if you have a question you can just ask it and then that can lead into like some conversation or she can ask a question and then kind of leaves it out there for us, the kids to answer it, so…

*Ms. S:* OK, and why do, why do you like that better?

*Isabelle:* Um, because it’s not so uptight and *laughs*, like it’s not like focused, “memorize all of this stuff”,

*Ms. S:* Hmm

*Isabelle:* It’s more relaxed, and that helps me learn better I think.

Isabelle’s more traditional view of the mathematics classroom with its “uptight” and rigid nature reminds her of memorizing facts and formulas and she stated that she responds better to a classroom that, in her eyes, is more “relaxed” and interactive allowing her
views and responses to matter. This is consistent with not only Maher’s (2001) view of the feminist classroom’s responsibility to “deliberately position students as academic authorities” in order to allow them the input for the feeling that their responses matter, but so that they do not “dismiss their own emerging sense of themselves.” Also, Isabelle’s feelings are consistent with what Keller (1985) once called “dynamic objectivity” which she defined in terms of how we might be inclined to think about the idea of integrating student input with factual mathematical knowledge.

*Dynamic objectivity* is a form of knowledge that grants to the world around us its independent integrity but does so in a way that remains cognizant of, indeed relied on, our connectivity with that world. In this, dynamic objectivity is not unlike empathy, a form of knowledge of other persons that draw explicitly on the commonality of feelings and experience in order to enrich ones' understanding of another in his or her own right (p.117).

We can view this more flexible way of viewing knowledge as necessary for including students like Isabelle who find the more rigid mathematics classroom not conducive to learning. She would rather remain connected to the material and the persons in the classroom with her in order to facilitate learning for herself. Isabelle truly enjoys the fact that students are the contributors to the knowledge and share in the presence of authority in the classroom. Because of the openness to the dynamic objectivity of the knowledge, the students (and she) are able to accept that their input is valuable. When I asked her why she thought the students felt so compelled to participate in the classroom, she had this to say.

*Ms. S:* Yeah, there’s almost a guarantee that people will… I wonder why? I wonder what guarantees that everyone will have something to say.
Isabelle: Well [both laugh] it’s probably just because geometry has like twen... like a lot of different ways to do certain problems so there’s a lot of variations in the way that people do them, so…

Ms. S: Hmm.

Isabelle: That might be it, or it might just be that people feel comfortable in the situation they’re in to participate and it’s not like, “OK nobody ask questions so we can leave now.”

Ms. S: [laughs] Yeah. Ok. So there’s a certain amount of like motivation to want to talk about it?

Isabelle: Yeah.

Ms. S: because it’s like interesting to hear what other people did? [pause] Um, yeah, I can’t figure that out.

Isabelle: I think everybody like shares the same curiosity level and like when somebody… like I know in our physics class he never tells us the answer to questions and it drives everybody crazy…

Ms. S: Huh…

Isabelle: And then we all start talking about it to try and figure out if like we can find out the answer ourselves so and the same thing happens in my math class so…

Ms. S: Yeah?

Isabelle: I think it’s just the motivation to find the right answer and like, because I know everybody in my class wants to understand.

Isabelle describes a tacit understanding in her class of the dynamic objectivity of the knowledge and the part the students play in its formation. When presented with a problem when the solution is unknown and the teacher presumes a certain level of authority in the students, the students take on a level of responsibility and curiosity in finding solutions and methods for those solutions. Not only does Isabelle describe herself as “more comfortable” in this classroom, but her teacher reiterates this as well (“she’s a
lot more easy-going [than other students], “she’s just so chill”). Isabelle clearly feels at home in this type of learning environment.

The Dance of Agency

Boaler (2003) utilized the work of sociologist, Andrew Pickering (1995), who described the “interplay of knowledge and agency in the production of new conceptual systems” as the “dance of agency.” Boaler wrote that through mathematics classroom practice, students can develop the skills needed to complement the rules of mathematics (or the ‘agency of the discipline’ [2003]) with the student’s own individual and collective agency in the classroom. It is just this “interweaving of standard methods and procedures with their own thoughts” that allows the students to gradually allow them to build knowledge and draw strength from their own agency in the process. Isabelle mentions on more than one occasion the cooperation that occurs between students in building knowledge (“if I didn’t understand a problem for math homework, I would go up to somebody who did and ask them to do that problem with me on the board,” Well, my class was really nice and so I wasn’t like afraid to go up and present problems even if I wasn’t sure what the answer was to them.”), and I observed her ability to work well with others and the teacher. It is interesting to me that Ms. Johnson did not feel that Isabelle was much of an advocate for herself as she did not seek much extra help outside of class when she seemed confused about material (“She hasn’t ever reached out to me for a meeting”). When I asked Isabelle about this in our conversations together, she seemed confused about going to see Ms. Johnson for extra help. Isabelle really saw her time in class as the time where she would clear up confusion.
Ms. S: Were there days when you would leave class confused or dissatisfied with the material, like not understanding what happened?

Isabelle: No, because like everything... if you had questions and you didn’t understand something, um, you either asked it or Miss Johnson could always tell...

Ms. S: [laughs]

Isabelle: If you didn’t understand something so then she’d just explain it more.

Ms. S: Ok

Isabelle: Because if you had this confused look on your face then she’d like actually take that into consideration and explain the problem more

Ms. S: Wow

Isabelle: as opposed to the more traditional classes where the teacher would probably be just like “Oh, whatever”.

What Isabelle appreciated most about the group’s social interaction seemed to be the reciprocated empathy to be sure that everyone understood what was going on. She felt that Ms. Johnson was giving individual attention in class as well as taking care of the learning needs of the whole class. Although her teacher felt that Isabelle still needed further clarification outside of class (in order to be doing better, as a teacher should) Isabelle was feeling confident and secure in her learning because of the interactions in the classroom. What Ms. Johnson was interpreting as a lack of agency on Isabelle’s part outside the classroom was Isabelle’s interpretation of herself actually being agentic in the classroom. This is an interesting question about the understanding of expectations and agency for further study.
Although Isabelle claimed to be an average mathematics student, she did show evidence of agency in her own learning in many ways. Other than in class observations and in our interviews, I found in her writing that she often had a voice of agency. In a journal entry from the fall of the academic year in which she was taking the RPBL algebraic geometry course, she wrote about this problem:

In a dream, Blair is confined to a coordinate plane, moving along a line with a constant speed. Blair’s position at 4 am is (2,5) and at 6 am it is (6,3). What is Blair’s position at 8:15 am when the alarm goes off?

Isabelle begins her journal entry with a diagram (see Figure 6.1) of discrete points that are labeled with coordinates and times as in the following diagram. She writes:

I wanted to find out where 5 am would be plotted so I would have a unit ratio, or so I could have a slope that would have one hour increments. To do this, you can find the midpoint of 4 am and 6 am or (2,5) and (6,3).

She calculates the midpoint of (2,5) and (6,3) with the midpoint formula that they learned in class, but also explains that it is basically just find the average x value and the average y value.

She then realizes that every hour, Blair is moving vertically down one unit and horizontally across to the right two units. So she’s created her “unit ratio” as she called it in a one hour increment. She writes, “After plotting the endpoint I know that the slope between the hours is \(-\frac{1(down)}{2(right)}\).” Her current information only gets her Blair’s locations on the hour and the question asks for her position at 8:15 am. However, Isabelle is aware that her midpoint method worked when given a two hour increment and decides to follow this method further.
Then you find the midpoint of 8 am (10,1) and 9 am (12,0) to get to 8:30 am (average 8 and 9) [shows the arithmetic work for this answer], then you find the midpoint of 8 am and 8:30 am to get 8:15 am (10,1) and (11, .5). [shows arithmetic for these averages as well]. So Blair’s position at 8:15 is (10.5, .75). This problem involved midpoints and slope.

This problem was actually supposed to be introducing students to the idea of scaling vectors to get new coordinates in the plane. Isabelle used her own agency and took ownership of the problem and made it about midpoints, which is a way of scaling. She took what she saw a unit of measure (the down 1, right 2 movement) and halved it until she had the correct unit she needed to get to the position for 8:15. Although this was not the solution that was discussed in class where Ms. Johnson and the class came to an understanding of using vectors and scaling them by multiplying by a constant (in this

![Figure 6.1 Isabelle’s Diagram 1](image-url)
case \( \frac{1}{4} \)), she showed her ability to control her own learning by writing about the way that she originally understood this problem, which ultimately led her to understanding the concept of scaling vectors which she wrote about later on in another journal entry as well. Ms. Johnson commented on how nicely Isabelle explained this solution method in her journal and also wrote that it would’ve been great for her to connect her solution with the “one [they] did in class” to make the entry fully complete.

When reading for the voice of the self, I found this I-poem to describe Isabelle’s feelings while grappling with the ideas of the dance of agency. She moves back and forth between understanding and not understanding, being proud and excited about discussion and remaining confused and questioning.

<table>
<thead>
<tr>
<th>I</th>
<th>You</th>
<th>We</th>
</tr>
</thead>
<tbody>
<tr>
<td>I think</td>
<td>You can have discussion</td>
<td>We all start talking</td>
</tr>
<tr>
<td>I know</td>
<td>You didn’t understand</td>
<td>We can find out</td>
</tr>
<tr>
<td>I think</td>
<td></td>
<td></td>
</tr>
<tr>
<td>I like</td>
<td></td>
<td></td>
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<tr>
<td>I know</td>
<td></td>
<td></td>
</tr>
<tr>
<td>I think</td>
<td></td>
<td></td>
</tr>
<tr>
<td>I didn’t understand</td>
<td></td>
<td></td>
</tr>
<tr>
<td>I would go up</td>
<td></td>
<td></td>
</tr>
<tr>
<td>I could understand</td>
<td></td>
<td></td>
</tr>
<tr>
<td>I expected everybody</td>
<td></td>
<td></td>
</tr>
<tr>
<td>I have a question</td>
<td></td>
<td></td>
</tr>
<tr>
<td>I ask it</td>
<td></td>
<td></td>
</tr>
<tr>
<td>I know</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>You’re more in control</td>
<td></td>
</tr>
<tr>
<td></td>
<td>You can</td>
<td></td>
</tr>
<tr>
<td></td>
<td>You have to participate</td>
<td></td>
</tr>
<tr>
<td></td>
<td>You don’t have to</td>
<td></td>
</tr>
<tr>
<td></td>
<td>You choose to participate</td>
<td></td>
</tr>
<tr>
<td></td>
<td>You want to help</td>
<td></td>
</tr>
<tr>
<td></td>
<td>You want to understand</td>
<td></td>
</tr>
<tr>
<td></td>
<td>I didn’t understand</td>
<td></td>
</tr>
<tr>
<td></td>
<td>You should not do it</td>
<td></td>
</tr>
</tbody>
</table>
You find out the reason why
Then you’re like “OK”

Isabelle moves between the “I” voice and the “you” voice here in an attempt to own her agency (“I would go up,” “I would understand”) and also disassociate from the agency, knowing that some of the agency is the group’s or the discipline’s (“You want to help,”, “You want to understand”). She is aware of the interplay between her own ability to move toward directing her learning and what she can do about it and how part of that is directed by others and the material itself. This is just part of her transformational experience in this course.

**Connection to Others – Seeing the “Elephant in the Room”**

Another way that Isabelle learned to navigate this idea of her own agency was in internalizing the input from her peers and teacher. In hearing others’ ideas, she had to learn to critically listen to alternative methods and decide for herself which helped her make decisions in problem solving, as opposed to direct instruction. Although direct instruction was clearer, it did not help with these critical thinking skills and form the relationships with others. Here’s what Isabelle had to say about these key relations.

*Isabelle:* To hear other people’s um, point of view… it’s like a beneficial way to find a better way to do the problem. Or if it’s not, to see why you should not do it that way, like explain more, so like “don’t do this.” It’s not like “don’t do this”, it’s like “don’t do this because… it will lead to like a difficult situation” or something like that, so…

*Ms. S:* Do you have a, a sense… do you appreciate that kind of “because” part now [laughs]?

*Isabelle:* Mm hmm, yeah
Ms. S: And you didn’t have that before, like when you…

Isabelle: Yeah, it was basically like direct instruction, “Do not do this”, but there was no, “it’s like why?”, “because!” [laughter] It’s more like you find out the reason why and then you’re like “ok, I won’t do that because I don’t want to end up in that type of like 2000 decimal point situation” or something like that so…

In helping others with their reasoning skills, Isabelle sees that she is also improving her own reasoning. She also has picked up on the fact that in the direct instruction classroom she was not satisfied with the justifications for many of the methods. What she’s said here is that the majority of the time the justifications sounded as if they were not important (“I don’t want to end up in that type of like 2000 decimal point situation”) or non-existent (“because”). Being a part of a classroom where there are interhuman relations at play is more satisfying to her in so many ways. Not only is it more satisfying, but it has become more important in her learning and she realizes it is something she can do. Further, she is empowered to make the decision about following a specific solution method because of the fact that she knows why it works or doesn’t work.

Isabelle commented on the value of working with her peers in class many times in speaking with me (“If I didn’t understand it, I can…while we’re putting it up, I can learn what I didn’t do right from the other person,” “It makes me feel good because I’m helping somebody”). I also observed her freely questioning others at the board when they were presenting and when she was working on problems in groups during class (“It's like a circle and the radius is 80, right?”). This aspect of the RPBL pedagogy was clearly something that was part of Isabelle’s experience that she valued.

When I listened for contrapuntal voices and heard the tensions between the voices of longing for direction instruction and the appreciation for the freedom of the classroom
discussion and dynamic objectivity, I also detected a subvoice to the voice of appreciation. Isabelle made many comments where I could hear a voice speaking about the transparency of knowledge in the RPBL classroom. It’s as if in a more traditional classroom, the lack of discussion keeps the amount of knowledge that students have an “elephant in the room” – an issue or problem that everyone knows about but will not discuss - and there exists a presumed understanding of the material by everyone in the room. Because of the lack of connection and relationship between the members of the community, there is isolation and almost competition between those who understand and those who do not. Here is what Isabelle first had to say about this.

*Isabelle:* But, like, the teaching style makes me more relaxed because I know that some of the people don’t understand the problems because like I see them talk about it to other people and stuff like that

*Ms. S:* Hmm

*Isabelle:* And when they learn something they share it to the whole class. And yeah, that helps me understand more.

*Ms. S:* [long pause, writing] Hmm. So like the, the openness of people’s ideas like makes you more relaxed too because there’s no like hidden agenda of like

*Isabelle:* Mm hmm

*Ms. S:* Who knows it all and who doesn’t know it all and

*Isabelle:* Yeah

*Ms. S:* That kind of thing. Oh, that’s interesting. That’s kind of cool, I never thought about that. Huh. Um… everything’s kind of transparent [laughs]

*Isabelle:* Yeah!
So, in other words, one of the aspects of the RPBL classroom that makes Isabelle more relaxed is the way that she is aware of the other students’ questions by their discussion. Everyone’s confusion or understanding is right there, out in the open, so there is no question of “who gets it” or “who doesn’t” and the majority of the time they are “all in it together” grappling with the understanding of a problem. In comparing it to a direct instruction class where most students ask individual question directly to the teacher to clear up their own questions, Isabelle stated:

It’s not as direct, like just between you and the teachers, like here it’s between like you and then like the person you’re sitting next to if they don’t understand, and then like you ask a question and you know that other people benefit from it and it’s not just you not understanding.

At one point, Isabelle said to me, “There’s no debate in my mind, like saying, ‘Oh, are other people already going to know this?’ because I know other people are curious.” She has experienced the curiosity in their questions and interest in the problems and by watching them present their partial solutions at the board. There is a reflexivity and mutual influence that is shared in the relationships that have been built by the common classroom experiences. Isabelle describes this reflexivity by saying:

Yeah, because I know not everybody in my, um, like I don’t think everybody in my class always understands every problem. So, if I understand something I like to share with other people because I’ve been in their position.

It is not only the transparency of the other students’ understanding that Isabelle appreciates, but the fact that everyone shares the different positions of presenting a solution and being correct, presenting a solution and being wrong, watching a
presentation and being able to help, or watching a presentation and learning something new. It is the reciprocity of the positioning in learning that she said motivates her to be a part of the shared experience. This is in fact, at the heart of Foucault’s definition of knowledge, wherein, knowledge is produced within a social field of relations where the power is dispersed and shifting (Pinkus, 1996). Isabelle and her class seem to be benefitting from the goals of the RPBL classroom of the relational trust and dissolution of hierarchy in that they are creating the relationships that improve the dynamic for learning for her.
“I notice things for myself so I have a better grasp and understanding of it in my own eyes. Whereas, I’m just learning from what you’re teaching me it’s like your opinion not mine.” — Alanna, 14

CHAPTER 7

Alanna’s Story: Realizing Your Own Power

Alanna was one of the very first students that I interviewed in this study and I was so grateful for her interest and flexibility. I would consider her an extremely outgoing, well-spoken and high-achieving ninth grader who is hard-working and has done well for herself. Alanna is one of those students who is truly inspiring considering she has found her way to Greystone Academy by a rather circuitous route. With her status as a day student at Greystone, Alanna and I were able to meet while school was not in session, so we decided to talk to each other one afternoon over our spring break vacation. The school grounds were rather empty and the building was quiet, except for some outside crew working on some construction. I told her how much I appreciated that she would take the time out of her vacation to meet with me. The generosity of giving her time on her vacation told me a lot about her as a person.

Alanna is an African-American ninth grader, growing up in low-income circumstances with a single mother and moving from school to school. She often found herself unchallenged in many of the public schools she attended and because of different circumstances, there were many. When asked, she described herself as “lazy” and “distracting to others” in math class mostly because she really didn’t see any value in it. In reality, her ability was much higher than the care that her teachers could provide for her and although she received good grades, she never really enjoyed mathematics. Alanna described her past experiences in mathematics classes where she would already
understand what the teacher was presenting to the rest of the class and she would give
Alanna other work to do.

Ms. S.: Were you always, like, in the accelerated math classes or the, I
don’t know…?

Alanna: There really weren’t any of those. The thing about me is that I
get bored very easily. So if I learn something, I just want to learn it and
go onto the next subject.

Ms. S.: Oh.

Alanna: And that always happened to me in all my schools, that’s why I
would just get A’s without any effort. I wouldn’t study for anything. I
just got it.

[few minutes later…]

Alanna: Well, for me while sitting in the classroom, I’ll always be the
distraction. I would always be the one always talking ‘cause I understood
it. I got it. But I would always distract students who didn’t get it. It
would always be me and a couple people who got it. And it was just like
talking, but it was never really like me paying attention.

Ms. S.: Wow that must be really hard. I mean, did you crave more
interesting material?

Alanna: Not really because I got lazy. Like I wouldn’t do my work and
then talk.

Ms. S.: Wow, that’s too bad. I mean think about what your teacher
could’ve been giving you to think about.

Alanna: Yeah, she used to give me extra work but what would happen is
she would have to pay attention to other students and I was just like, “Well
I’m done.”

Her experiences in math class were to get through what the teacher gave her and be done
with it. There was no appreciation for the material and it was an easy A. “It was just like
talking” but there was no interaction or actual communication of concepts or ideas going
on in the classroom. It almost seems like most of what was happening in terms of
learning was coming naturally to her and she almost didn’t even need the teacher to be
there, she may have learned it all with the same quality of learning whether the teacher
was there or not. It appeared to me that her prior mathematics class experiences were
unfulfilling and dissatisfying in terms of giving her any reason for feeling like
mathematics was something worth pursuing or having value in learning.

As one of my own students, Alanna and I had been through a lot together by this
time in the year. Coming from public schools where she had been mostly in traditional
classes, she had a difficult time transitioning to the RPBL classroom. When we talked
about this in her initial interview she said, “In the fall, I was just down right against it…”
but then she learned to adapt to it. When I asked her what she felt she had “adapted” she
said,

The ability to connect other things…’cause before they would teach you
something and you’d go home and practice it. But in this class you have
to like be able to bring back other information and then do the problem
so...I think I’ve learned that skill.

So, to Alanna, the skill of making connections between skills, content and the act of
problem-solving was actually not only new to her, but an important part of mathematics
that was missing for her. In fact, she felt so strongly about this idea, that she believes that
she transferred this skill to other disciplines. She says that vocabulary has been coming
more easily in this class and in others (“It also helps with history and stuff because
everything flows together now.”). I asked her for an anecdote or story that might
describe how learning how to make connections has helped her.
Alanna: There’s like nothing always specific. It’s just like, I learn better like through self-discovery. I learned that about myself. ‘Cause like, I would have to like teach myself …not really teach myself, like I would look at something and then I will learn it through trying to figure out how to do it. So it’s like through self-discovery. Or like in class when somebody’s doing a problem and it connects with something and I understand it better that way. Instead of having somebody teach it to me and do it their way.

Ms. S.: Do you think you learned that this year or you always knew that about yourself?

Alanna: I think I learned that this year. Because it became evident in other subjects…Well ‘cause, I’m taking piano lessons, and what I’m playing now it’s just… like when I’m learning theory, it just makes sense when I teach it to myself. For example, like in the circle of fifths, I understood it better, when I looked at it, better than when my teacher explained it to me.

In my study of Alanna, it became evident that like Leona and other girls, Alanna was craving connection – to the material (a purpose in her learning), to other learners (by a desire to communicate in the classroom) and to her overall education (in seeing connection to her other endeavors). However, the way she was learning in mathematics in the past was not facilitating these connections for her nor was it offering opportunities for relationships.

Relational Learning (or lack thereof)

In my discussions with Alanna she spent a great deal of time describing to me how she didn’t understand the reasoning behind mathematics class. When I listened to the voice of the self, there was a clear sense of frustration, even sadness, when she spoke about this lack of understanding. The following I-Poem came from a passage from my
initial interview with Alanna where she and I were discussing her memories of her past mathematics classes. She tried to summarize what those experiences meant to her.

<table>
<thead>
<tr>
<th>I</th>
<th>You</th>
<th>We</th>
</tr>
</thead>
<tbody>
<tr>
<td>they would teach you</td>
<td>you’d go home and practice</td>
<td>we’ll learn something</td>
</tr>
<tr>
<td>something</td>
<td></td>
<td>we’ll investigate something</td>
</tr>
<tr>
<td>you have to be able</td>
<td></td>
<td>teaching us something</td>
</tr>
<tr>
<td>I think I’ve learned</td>
<td></td>
<td>we go home and practice it</td>
</tr>
<tr>
<td>I was pretty much screw</td>
<td></td>
<td>we didn’t have midterms</td>
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<td>ed</td>
<td></td>
<td>we’d take a test</td>
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<tr>
<td>I’d forget it</td>
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<td></td>
</tr>
<tr>
<td>I remember stuff</td>
<td></td>
<td></td>
</tr>
<tr>
<td>I take a test</td>
<td></td>
<td></td>
</tr>
<tr>
<td>I feel</td>
<td></td>
<td></td>
</tr>
<tr>
<td>I have, I have to retain</td>
<td></td>
<td></td>
</tr>
<tr>
<td>I don’t know yet</td>
<td></td>
<td></td>
</tr>
<tr>
<td>I would have to know</td>
<td></td>
<td></td>
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<tr>
<td>I want to do</td>
<td></td>
<td></td>
</tr>
<tr>
<td>I’m not really sure</td>
<td></td>
<td></td>
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<tr>
<td>I remember them</td>
<td></td>
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<tr>
<td>we’d just learn words</td>
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<tr>
<td>we never went back</td>
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<tr>
<td>we’d just learn words</td>
<td></td>
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<tr>
<td>we never went back</td>
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</tbody>
</table>

What struck me as most meaningful about this I-Poem is the initial use of the “you” voice to describe her experience of the lecture-practice method which is very standard and assumes a set of objective factors. It would be natural for Alanna to disassociate herself from that process if she does not feel it was the way she should be learning or that it did not work for her. She then speaks in the “we” voice as the students in the class are talking about “learning”, “investigating” and “practicing” the skills that are taught in this RPBL class, but somehow it all sounds very passive and disassociated from herself in the first person plural voice. She claims in frustration that she was “screwed” on the test since she never really fully constructed any knowledge nor did she have any opportunity
to do so. Once she gets to her “I” voice in this poem, she is extremely active in her frustration with the expectations of knowledge that she has never gained from the processes of the class. She’s not even sure she can remember something that she was supposed to have learned at all. Most touching is the fact that “we just learned words” not concepts that they would go back to and have them actually make meaning in context of something else once again. I believe that Alanna’s voice in this I-poem is clearly expressing her frustration with the lack of relationship she had with the material in her past class – it was what was missing for her and perhaps what would have answered the question of what the “point” was in being in the mathematics classroom.

Alanna also found value in the relationships with her classmates and teacher, which contrasted with her past experiences. The interdependence that she felt between her classmates, perhaps for the first time while studying mathematics, affected her deeply (“everybody is so dependent on each other”, “obviously I learned from other people doing problems”). In many ways, it was a relief to her because as one of the brightest kids in all of her classes, students relied on her to always have the answer, but she was never interested in engaging with them. Conversely, this classroom was more anxiety-filled because never before had she actually had peers in her classes who were at her level so the relational trust needed for this type of learning was not presumed to be part of the classroom community in the beginning. When I asked Alanna about whether or not this was difficult for her she answered, “Yeah, ‘cause I never really had to depend on anybody for like learning and stuff. It’s good because it doesn’t leave all the pressure on me and it gives me a different perspective.” Having other students’ perspectives on mathematical concepts actually interested her and she felt that it added richness to her
The diversity of the perspectives of the people in the classroom actually contributed to the level of involvement and “fun” of learning for her (“It’s fun, it’s only fun depending on like your surroundings – for me.”). This part of her voice kept resonating with me as the part of her that knew mathematics class as boring from before and that it had to be “fun” to be important or engaging to her. In order for it to remain purposeful and interesting it had to be “fun”, I believe, because that was the language that she had for important and interesting. She continued with this voice when I asked her to elaborate on what part of her surroundings made mathematics class fun for her. Alanna replied that “if she was just sitting down in a college class learning math, [she] probably wouldn’t be able to focus” because she would need something to “distract” her from how boring the class would be. I asked her to be more specific about what elements of the class would make it “fun” for her.

**Alanna:** Like people that keep it entertaining.

**Ms. S.:** Ah, OK… And what keeps it entertaining for you now?

**Alanna:** Um, like, Annie and Lisa and Ariana and Toni, like all those personalities altogether just talking about stuff. Like you as a teacher, it’s not like you’re just boring and just talking at us. You ask us questions we respond and interact.

**Ms. S.:** Oh, OK, so there’s like a relationship between the people in the class who are talking to each other and you’re asked what your opinion is and, you know, and they care about what you know and what you want to say.

**Alanna:** Yeah, like that. Yeah, that makes a lot of sense.

Alanna had a difficult time putting into words that it was the relationships between the people that were integral to her engagement, but she was able to list the people and the interactions between the people that made the relationships important. Expressing herself
in relation to the others in the classroom community allowed her to be more comfortable and find purpose in learning. Like many African-American young women from urban culture, Alanna considers herself very loud and outspoken and she may see her cultural and social personality in conflict with what is acceptable in the traditional mathematics classroom. Lim (2008b) said this of the internal conflict that young African-American students grapple with in a traditional mathematics classroom:

Black students’ communication style (e.g., free verbal expression and talking aloud) and learning preference (e.g., holistic, relational, and field-dependent) were rarely respected in the classroom space; rather they were considered disruptive behaviors or, at best, an attitude non-conducive to mathematics learning.

Alanna found that her personality and outspoken attitude was valued in the RPBL classroom because sharing her thoughts and creating relationships were encouraged. It actually worked in her favor. I could hear her voice telling the story of her participation in a collaborative process that allowed her to partake in a partnership of learning. In our final interview together, she said this of the classroom practice of student presentation and class discussion of problems that weren’t necessarily correct or complete:

*Ms. S.*: Did you mind putting stuff up on the board when it was wrong, or when you didn’t know if it was right… I guess that’s a better question.

*Alanna:* Well, I think I wouldn’t mind

*Ms. S.*: Yeah?

*Alanna:* Well, it’s just a way for me to learn from my mistakes I guess

*Ms. S.*: Ok.

*Alanna:* But I would make every, like I would make it known that I don’t know what I was doing, I didn’t know what I was doing…

*Ms. S.*: Yeah
Alanna: So everybody just would just go with the flow and copy it down.

Alanna is so willing to be open and honest about what she knows and what she doesn’t know because she feels a connection with her classmates. In this type of environment where there is less of a hierarchical authority and more a shared authority among the community members, she is able to be more herself and allow herself to make mistakes and learn from them. There is evidence that, especially for students of color and low-SES, a more cooperative learning environment and processes that relate to their everyday life (like authentic problem solving scenarios) foster deeper appreciation and higher achievement (Boaler, 2008, Lim 2008a). Alanna sums up her appreciation for this pedagogical style when she says:

‘Cause that’s like basically the essence of the class - just working together. So to do some good stuff, we ended up skipping a lot of problems but in the end we were able to come back and like, she saw something I didn’t see, and then we just went off that - just bounce ideas off each other, right?...like do somebody else…incorporating what they say into what I say and just making something out of it.

It is just this relational aspect of the RPBL that Alanna seems most grateful for. In class, I would observe Alanna truly enjoying putting problems on the board and sharing her solutions with the class, but as the year went on, I watched as she learned to sit back and allow her classmates to present their solutions because she knew that they learned just as much from making their own mistakes at the board and not necessarily always watching her present. This was part of Alanna’s realization in her growth, which there was much of throughout the year.
Empowerment and Agency as Independence

An epiphany for Alanna in this RPBL mathematics class was how being empowered changed her view of independence in the mathematics classroom. She was always confident in mathematics and knew that she was a capable student (“I was never not self-confident,” “If I needed help, I would ask. I wouldn’t be ashamed.”). In fact, as noted previously, her past teachers would have to give her extra work to do to keep her occupied in order for her not to disrupt other students during class since she was more capable than other students in her classes. Alanna was used to working independently and perhaps even feeling as if she were extremely capable of doing so. So what was missing? Why was mathematics class so unfulfilling if she could work independently at her own pace and feel confident doing so? When listening to the contrapuntal listening, I heard a tension between her pride and enjoyment of that independent aspect of the traditional classroom (which may be more isolating) and a voice of enjoying the agency and empowerment that came from the RPBL classroom. In the final interview I had with Alanna, she was describing how her other “independent work” in middle school worked for her.

*Alanna:* In my old school what would happen is, in math class for like 6th and 7th grade, no 7th and 8th grade, my teacher would always give me other work to do

*Ms. S.:* Ohhh

*Alanna:* So, it was just, I, if I understood something, then I would tell other people, like this is how you do it and I would always get other work to do, so in terms of this class, I think it was just strengthening of that

*Ms. S.:* I see. Ok. So you’re kind of used to that

*Alanna:* Yeah
Ms. S.: Being your own like learning agent. Because if you, if they hadn’t given you that extra work um, you would have just kind of not really done anything because it was so easy for you.

Alanna: Yeah [laughs], just be bored in class.

I do believe that Alanna’s voice of pride in her own work was being heard here. The source of this voice being viewing herself as her own learning agent in her past class and perhaps seeing learning “on her own” as what her teacher wanted her to learn. The idea that she would choose to not “just be bored in class” doesn’t strike me as actually being her own learning agent, although she was working independently. It is clear that in the more traditional classroom, Alanna was extremely proud of the fact that she helped others (“I would tell other people, like this is how you do it”) and that she would be given independent work by the teacher in a traditional classroom because she was more advanced than most of her classmates (“I would always get other work to do”). However, I also could hear a voice of pride in her agency when she could actually “learn” or make connections on her own in the RPBL classroom.

Cause in that classroom it’s not as hands-on as in this classroom. And like, you notice things for yourself. That’s how I like to see it. I notice things for myself so I have a better grasp and understanding of it in my own eyes. Whereas, I’m just learning from what you’re teaching me it’s like your opinion not mine.

She gives a lot of credit to herself for the understanding that comes from her own ideas and her own opinions, which is justified. The self-validation that comes from learning from forming your own opinion as opposed to being given an idea from the teacher seems to mean so much more to her than being given independent work to do by the
teacher. Alanna sees much more value in the agency that she has formed from the RPBL classroom. The following I-Poem describes different aspects of the pride that she feels and her process of solving problems and what she can accomplish now.

<table>
<thead>
<tr>
<th>I</th>
<th>You</th>
</tr>
</thead>
<tbody>
<tr>
<td>more opportunity for you to affect your own learning</td>
<td>you get another idea</td>
</tr>
<tr>
<td>you just do it</td>
<td>you see something</td>
</tr>
<tr>
<td>you notice things for yourself</td>
<td></td>
</tr>
</tbody>
</table>

I like to see it
I notice things for myself
I have a better grasp
I’m just learning
I’m just that person
I think
I think I’m that independent
I want to learn it for myself

In this I-Poem, Alanna’s first stanza is completely in the second person, disassociative, “you” voice as she is speaking generally about the workings of the RPBL class. She seems to be describing students who are in the class as being able to get ideas and “do it” for themselves, perhaps as opposed to in a traditional classroom. She then switches to the “I” voice describing not only her preferences about what she likes and wants, but making grand statements about who she is and what she is capable of as a mathematics student. It is very powerful that she is “just that person” and she’s “that independent” as if she knew it all along and is finally in a place where she can value that about herself mathematically.

In a journal entry towards the end of the school year, Alanna wrote about an interesting phenomenon that many students do not attempt to grapple with. She entitled the journal entry “Proportions” but little did she know that she was dealing with much
more than that. She drew a diagram of two sets of axes and lines right next to each other as in the diagram below. The problem she was commenting on asked her to draw two lines with slopes that were in proportion to each other and to see if the angles they formed with the x-axis were also in the same proportion. The example of a proportion of slopes she choose was 2:1 as her lines were $y = x$ (see 7.1a) with a slope of 1 and $y = 2x$ (see Figure 7.1b) with a slope of 2.

This is what she wrote in response:

In graph 1, the slope of the line is 1, while the angle created with the x-axis is $45^\circ$. If slope and angles were in proportion, the line in graph 2, which has a slope of 2, would create a $90^\circ$ angle, the axis, but this is not the case. To determine the relationship between angles and slopes we use a function called tangent: $\tan(\text{angle}) = \text{slope}$ OR $\tan^{-1}(\text{slope}) = \text{angle}$.

With that being said, the angle created in graph 2 is $63.4^\circ$.

This was a wonderful description of not only a reason why the slopes and angles are not in proportion (which so many students want to be true), but the correct relationship between slopes and angles and how you would find the angle that goes with a slope of
two. However, Alanna did not stop there. We had just started discussing circles and the proportions that *are* true in circles, so she thought that it would be interesting to continue this journal entry with a counterexample of when angles and “lengths” actually are in proportion.

On the other hand when it comes to circles, everything is in proportion. This is because all circles are dilations of each other. In all the circles here [see diagram] the arc angle is the same, but the arc lengths are dilated. The ratio of similarity is the radii of the circles. With that being said, I can know [sic] come to the conclusion that the arc angle and the rest of the circle are in proportion to the arc length and the circumference of the circle.

\[
\frac{\text{arc angle}}{360^\circ} = \frac{\text{arclength}}{\text{circumference}}
\]

Alanna continued the journal entry by actually doing an example from class where they were asked to find the length of an arc of a circle with radius ten inches and a central angle that was 106°. Many students in a traditional classroom might simply be taught a formula for arclength that they memorize, but Alanna had made some important connections about why the arcs and central angles were in proportion for circles and why angles made by lines were not. She felt her own agency and was empowered to write about it on her own (she was not prompted in any way to make these connections). This journal entry is illustrative of Alanna’s agency in her learning and how much power she felt in seeing the connections between concepts that were moving her forward in her understanding. This ability to be “independent” yet rely on the material and her class shows the interdependence that Alanna has come to
value and appreciate as part of mathematics learning in the classroom. As a learner, the aspect of agency has deepened her experience of understanding in the mathematics classroom, although her ability has played a part in that as well.

**Education as Growth**

When I think of the overall story that Alanna was telling in both of the discussions I had with her, it is evident that this course and the experiences that she had was the impetus for a great deal of growth on her part. Of course, for adolescents it is difficult to say what is responsible for the growth that develops at this pivotal time in their life. However, for Alanna the experience that she has in mathematics class is so inherently different and it shows her such a different part of herself that she has not been able to see before. This may have been what was able to instigate growth in this area for her. In fact, John Dewey said that the educational process was one with no end “beyond itself” but one of “continual reorganizing, reconstructing, transforming” (1916, p.50) which can be seen as growth. To look harder at some of the transformations that Alanna went through in her experiences in the RPBL classroom, I listened to the contrapuntal voices of self-confidence and self-doubt. This was dominant in both of the discussions we had together and I was often confused by the tensions between them. For example, when I asked in our first interview whether it was confusing to hear other students’ perspectives on a specific problem, this led into her berating herself for her horrible algebra skills describing to me what she sees as a big difference between her process skills and her thinking skills.

*Ms. S.:* And that’s not confusing - to see other people’s perspectives?
Alanna: Sometimes it is, **if it’s simple.** [She laughs]. I can get it. If it’s algebra, like if they do it, I **wouldn’t really comprehend it really well,** but if they show it to me **like to physically show it me, then I probably would.**

Ms. S.: Then you feel like you’re a more visual or geometric learner?

Alanna: Yeah, when it comes to math.

Ms. S.: Do you think that seeing other people’s algebraic solutions helps you understand algebra more, or would it be easier to not even have the algebra part?

Alanna: **I want algebra to die…I don’t like algebra.** I don’t. It’s just that…I don’t know. Maybe it’s just that I don’t remember or get it. But…

Ms. S.: Huh…what …how do you think you would respond if algebra was taught this way?

Alanna: Oh no…no…**you can’t do that to me.**

Ms. S.: Why?

Alanna: Because it’s so abstract. It’s like x could be anything and **I would have to do the right thing to get x.** It’s not like there’s a lot of different things I could really do.

Ms. S.: Hmm…so and you don’t think that geometry is abstract?

Alanna: Not really, like with vectors and the slope thing, well, up four over three. That just makes sense to me. **It just makes a triangle so just do the Pythagorean Theorem to find the length of it.**

What Alanna says here is interesting in so many different ways. There is one voice that is telling a story of how algebra is too “abstract” for her, but it seems like algebra has a set of given algorithms that she can learn to follow and in that way, it is not too abstract. You have to “do the right thing to get x” and she does not believe there are a lot of different ways that x can be found. As long as someone shows her the algorithmic process, she will be able to understand. However, she also sees that geometry is “not abstract” but it helps that she is a visual learner (implying abstractness or the ability to
abstract understanding from a diagram), but then describes solving a geometry problem with vectors by an algorithmic approach (“up four, over three,” and “just do the Pythagorean Theorem to find the length.”). It is difficult to tell which she finds abstract and which she find procedural in these descriptions. I think it is extremely important that in the sixth line of this segment where Alanna is proclaiming her hatred of algebra, she stops for a moment and almost changes voices and says “Maybe it’s just that I don’t remember or get it.” For a moment here, we are hearing a voice of frustration, regret and perhaps sadness that she has this anger (“I want algebra to die.”) towards the subject matter that she now knows she has an interest in and it could be because in that more traditional classroom, she was not given a chance to understand or retain the information in a way that worked for her. In a way, this is the contrapuntal voice of self-confidence trying to get out. She is now more confident that she could have been more interested and engaged in class, had she been given the chance. When reading for the voice of the self, I heard in this particular I-Poem a great deal of tension between her confident self and her doubting self.

<table>
<thead>
<tr>
<th><strong>Self-Confidence</strong></th>
<th><strong>Self-Doubt</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>I can get it</td>
<td>I wouldn’t comprehend it</td>
</tr>
<tr>
<td>I probably would</td>
<td>I want algebra to die</td>
</tr>
<tr>
<td></td>
<td>I don’t…</td>
</tr>
<tr>
<td></td>
<td>I don’t know</td>
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<tr>
<td></td>
<td>I don’t remember</td>
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<tr>
<td></td>
<td>I would have to do</td>
</tr>
<tr>
<td>I could really do</td>
<td>I can’t do those</td>
</tr>
<tr>
<td>I can draw</td>
<td>I wouldn’t know</td>
</tr>
<tr>
<td>I have those</td>
<td>I wouldn’t know</td>
</tr>
<tr>
<td>I just combine</td>
<td></td>
</tr>
<tr>
<td>I think</td>
<td></td>
</tr>
</tbody>
</table>
I know the way
I just had to plug
I’ve had practice
I was taught that
I got it
I know the formula
I just put it together
I try not to get confused

In reading this I-Poem, I was struck by how I could almost eliminate the lines that were
the voice of self-doubt and I would have a complete I-Poem from the voice of self-
confidence. It has truly contrapuntal, interwoven voices showing how conflicted Alanna
is about her feelings towards mathematics and her ability. Although she has proclaimed
that she was “never not self confident”, it is clear that she still struggles with realizing her
true ability (“I’ve always been good at math” vs. “I can’t do those”).

There is also a distinction between different meanings of confidence in her voices
as well. At one point, Alanna uses the word “confidence” but I feel like she really means
security. When I asked her at one point in our initial interview if she feels that her belief
in her mathematical ability had changed at all, she talked about her feelings about the
instructional methods of the two types of classes.

Well, I guess with the other class it’s you’re more confident because you
know that this is what you have to do, you know for sure, but with this
class there’s just so many different possibilities that yes, I’m confident
when I know this might be the only way. Or I know for sure this is a 30-
60-90 so I have to do this, or this is a median so I have to do this. I guess
it depends on what the problem really is…Or how solid I am in that
subject…

In this moment, the voice that I hear is not really a voice of self-doubt or a lack of
confidence. It is more a voice of discomfort with the lack of objectivity in the RPBL
classroom (“you know for sure.”). This voice is heard in other places as well (“I was just
taught that and I got it.”). Many students are uncomfortable with being given the freedom that comes with this type of problem solving and the habits of mind that have been formed over years of being in a direct instruction classroom often take over.

Anderson (2005) found that many students have grown “accustomed to learning in a classroom that required little from [them] in terms of engagement with mathematics” and that even with collaborative group work, microcosms of a traditional classroom can be formed because of these unconscious habits of mind. This insecurity can be confused with self-doubt, and externally can be articulated that way. It is clear though that Alanna is moving forward on a journey of embracing the subjectivity of problem solving since she enjoys it so much. On a number of occasions in our time together, she explains the satisfaction of solving a problem or the pride she feels in her accomplishments (“You’ve had that sense of accomplishment like you actually did something,” “I feel accomplished.”). When reading for the voice of self in the following I-Poem, I was struck by Alanna’s feelings on her appreciation for what this class has done for her intellectual growth.

<table>
<thead>
<tr>
<th>I</th>
<th>You</th>
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<tbody>
<tr>
<td>I think</td>
<td>you just add that together</td>
</tr>
<tr>
<td>I taught myself</td>
<td></td>
</tr>
<tr>
<td>I never really understood</td>
<td></td>
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<td>I got it right</td>
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<td>you did it on your own</td>
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<td>without anybody having told you what to do</td>
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<td>you’re putting it together on your own by yourself</td>
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you’ve had that sense of accomplishment
you actually did something

By the last stanza she is speaking completely in the “you” voice, she is disassociating herself, but in a positive way. I see her stepping back and reflecting on how far she has come and describing herself as “you.” Almost looking at where she started and where she has ended up and describing herself in the second person in order to see herself as another. “You did it on your own” and “you’ve had that sense of accomplishment” as opposed to before when “you” weren’t able to or never felt that before. This I-poem encompasses the feelings of her first describing how she feels as an individual describing her experience in the first person (“I taught myself,” “I never really understood,” “I ended up diving in,” “I was happy”) to her looking at herself from an external perspective, almost as an evaluator, with pride in her accomplishments and in how far she has come.

Another area where she realizes she has improved is in her communication of mathematics. She sees how the interdependence of learning with her peers has allowed her, almost forced her, to necessitate talking mathematically with others. Alanna has come to an appreciation not only for the inquiry in mathematics but for the moments of deep understanding (“There’s always that Ohhh…moment, I get it. So it’s just always spontaneous in a way ‘cause you can just do so many things that you just never thought of”). It was moments like these that kept her engaged and interested in the conversation about problems. I asked her how she was part of the discussion in her past mathematics classes.

Ms. S.: How much would you talk in a regular class?
Alanna: I really wouldn’t say anything. I would know the answer, I just wouldn’t say anything.

Ms. S: That’s so sad. Why not?

Alanna: I felt like I should give other people a chance to get it instead of always saying the answers.

Ms. S: So the opportunities that you had to ask like extension questions…

Alanna: They were like non-existent.

Because of the fact that she did not have experience or practice discussing mathematics at this inquiry level, Alanna initially had difficulty talking productively about problems and concepts and working through her ideas.

Ms. S.: How did you feel, um… when you had to explain stuff to other students? Because that probably didn’t happen in your other classes, right? Or did you?

Alanna: Not exactly

Ms. S.: Yeah

Alanna: Well, I don’t, I’m not really good with words, so I’d always have to show them what I’m doing. Not… I have it in my head, I just can never get it out properly

Ms. S.: Yeah?

Alanna: My words just get mumbled.

Ms. S.: Well, that’s funny, I thought you were pretty good at explaining stuff to people [laughs]

Alanna: Well, I don’t… [laughs]

Ms. S.: Yeah? Huh…

Alanna: I don’t know, but I would just have to show it, like draw pictures and do numbers in front of them.

Ms. S.: So do you feel like you improved in that area, that you’re…
Alanna: Well, yeah, because everybody is so dependent on each other, so...

Ms. S.: Hmm [long pause, writing] what do you… is there anything that you feel like you didn’t improve on?

Alanna: I don’t know

Ms. S.: No… [both laugh] I’m not saying there is, I’m just saying…

Alanna: Well, I don’t know, I don’t – I’m good at math, I got better I guess [laughs] Uh, I guess more outspoken, more than I was before.

It is clear from this segment that Alanna is proud of not only her mathematical ability, but that she has changed because of what she has been asked to do in this class. Because of the relational nature of the classroom practice, Alanna had to change her own classroom practice. Forcing herself out of her comfort zone (drawing pictures and “doing numbers”) by having to describe her process and even writing in her journal (“Like with the journals, you have to focus on one problem or one or two problems and actually dissect it and stuff like that”), she began to realize there was more to doing mathematics than just procedural algorithms. In our final interview together Alanna tried to articulate her newfound appreciation for the depth of mathematics by saying

It’s just the whole process of like seeing how you can go from one thing to another thing and seeing how that thing affects a whole bunch of other things.

To which I replied, “Could you be more vague?” and we had a good laugh. But somehow I knew what she was talking about. She went on to explain that the process of self-discovery and discussion helped her retain more information in this class and helped her appreciate mathematics more. We talked a little more about how this particular type of classroom engaged her as a learner, as opposed to the more traditional method.
Alanna: ‘Cause in that classroom it’s not as hands-on as in this classroom. And like, you notice things for yourself. That’s how I like to see it. I notice things for myself so I have a better grasp and understanding of it in my own eyes. Whereas, I’m just learning from what you’re teaching me it’s like your opinion not mine.

Ms. S.: Why does that mean more?

Alanna: ‘Cause I’m just that person. I think it has to do with personality and stuff. I’m just that independent person that I want to like learn it for myself – get it myself – instead of having you just teach it to me.

Ms. S.: So you think it’s your personality?

Alanna: Yeah [laughs].

Ms. S.: Um…but being able to notice it for yourself, makes you feel more in control…I don’t want…

Alanna: Yeah…

Ms. S.: To say like…well, you don’t have total control of what you’re learning…I’m the one that gives you the problems and says this is what we’re learning today…that type of thing. But you have more say because you can figure out what you get out of that problem to some extent?

Alanna: Yeah, cause like if there’s something in a problem that I didn’t see when I was doing it directly that might be what I look for in that I have to do tonight. Like, if there was a median thing and that had to do with 30-60-90 to find out the angle, I might just look for that in another problem tonight instead of just like completely disregarding it.

In this part of our conversation, I can hear Alanna’s voice of independence and pride in seeing and understanding on her own. Being able to own an idea is extremely important to her and this comes from a classroom where dialogue is “constructive, active and reflexive” (Chow, et al, 2003) so Alanna could feel that she was learning from her own opinions and not solely the teacher’s authority. She has become a student that reflects on the classroom discussion when looking at her homework problems in the evening.
attempting to integrate her own ideas and possible solution methods with the ideas of her classroom community members. Ultimately, this is a more valuable and much more rewarding experience for her.

Finally, Alanna has seen her mathematical identity transform from one of a lazy student with very little responsibility in the mathematics classroom to one of a student who has a great deal of agency and power. This transformation has been very fulfilling and liberating for her. As Alanna spoke about her experiences in her more traditional classroom in middle school, although she knew she was a mathematics student with high ability, there was seldom a voice of pride that I could hear. She would describe herself as “lazy,” or the “distraction” in class because she knew that she would talk to students because she was bored. Here she describes a charter school in an urban area in large city where she once lived.

*Alanna:* I was in a charter school. And the thing about it was they basically did everything for you. Like I really didn’t have any responsibility.

*Ms. S.:* Oh no.

*Alanna:* Yeah, …

*Ms. S.:* Even like the homework responsibilities?

*Alanna:* They checked our homework like every single day. Like even if it wasn’t due the next day they checked it for us meaning that like we had to …we didn’t have time to plan our time to say that I’m gonna do this tonight and do this tomorrow. It just all had to be done the next day.

*Ms. S.:* Hmmmm…so because they had these strict um guidelines you didn’t have to be the one to say, I better do my homework tonight,

*Alanna:* Yeah,

*Ms. S.:* Gotcha, so that sense of responsibility wasn’t there.
Alanna: They even gave us pencils. [laughs]

Ms. S.: Wow.

Alanna: I had one binder for every single subject. Like everything just fit in that one binder and we had packets like they gave us packets and we just had to fill in the blanks for our notes... But for the most part, we really didn’t have any responsibilities.

The system at this school may have worked for some students who needed the structure of teaching them discipline and study habits, but for Alanna what it did was allow her to rely on others to get her through her academic exercises. It belittled the potential of intellectual practice for her and sadly shaped her identity as a student as one that had low expectations of herself. Once she was in a place with higher expectations she needed to relearn what she was capable of in terms of intellectual practice. Her view of herself in mathematics class has totally changed and she herself is more agentic – with so much more ability to do mathematics.

Ms. S.: Do you have…what do you think about when you…like now, when you think about math as a subject, do you think it’s enjoyable? I mean ….or do you think it’s kind of boring still?

Alanna: Oh it’s not boring…

Ms. S.: You can say it is…I mean…

Alanna: For me…well right now it’s not…there’s always that Ohhh…moment, I get it. So it’s just always spontaneous in a way ‘cause you can just do so many things that you just never thought of.

The idea that she sees mathematics as “spontaneous” and not contrived and planned out by the teacher anymore is extremely liberating. She has transformed from a student that has been isolated by the learning of mathematics to one that is connected and engaging. The fact that she can now articulate her own impact on her learning (the fact that she can
do “so many things” she “just never thought of”) shows that she has realized that she can be more of an agent in her learning than what she was in a traditional classroom. This newfound authority can be exemplified in a wonderful journal entry that Alanna wrote at the very end of the year. We were studying the relationships between similar solids, a topic often found in a traditional geometry class. However, students were asked to conjecture on the ratios of the areas of similar solids in many problems that we had discussed. Alanna started writing a journal entry attempting to summarize some rules about similar solids before we had completely finalized the topic in class. She began with some background information on circles:

When looking at the ratio of similarity between two figures we are looking at the figures in 1-D. For example, all circles are dilations of each other. The ratio of similarity is the radii of the circle.

She then continues her writing with a more complex example, moving in two dimensions. To get from 1-D to 2-D, you must square the 1-D variables, as seen on the previous page. Since it is established that the ratio of similarity is a 1-D variable, I can conclude that to find the ratio of the areas, I can square the ratio of similarity. For example:
In order for me to get to 3-D, I must go back to the 1-D variables and cube them. The reason Why I cannot jump from 2-D to 3-D is this: If I squared the 2-D variables my answer would be in the fourth dimension $->$
units^2 \times \text{units}^2 = \text{units}^4$. So in terms of ratio of similarity and volume, in order for me to get the ratio of the volumes, I must cube the ratio of similarity. If I am given the ratio of areas, I have to first find the square root of that to get the ratio of similarity.

Although this isn’t the greatest journal entry because there are a lot of “loose ends” that she doesn’t discuss, I believe it is a great example of how far she has come and her own authority in mathematics. The key component here is her attempt at explaining why she needs to “go back to the 1-D” values in order to get the ratio of the volumes in three dimensions. She explains this by counterexample by saying that if she were to simply square the ratio of the areas, she would be dealing with units that were already squared. By squaring the units that were “square units” you would not end up with cubed units, which is what you are supposed to get with volume. This makes so much sense to her that it seems obvious that you need to go back to the units for a one-dimensional measurement and cube it. Probably the most amazing thing to me (as her teacher) is that she makes the leap to saying that if she squared the two-dimensional measurements her answer would then be in the fourth dimension, which we hadn’t even discussed in class at
all! It was truly amazing and fulfilling to hear this coming from a girl who once considered herself lazy and unable to ask extension questions in class.

**Ultimate Enjoyment**

At times in the beginning of the year, I would observe Alanna being rather competitive about putting problems up on the board. She would show great pleasure in being able to solve a problem that other students could not, not in a self-aggrandizing way, but in a very self-gratifying way. It was as if each time she was able to share some knowledge with another classmate, she was really proving to herself what her true ability was. I believe that she has come to a different understanding of enjoyment in learning mathematics. At one point in our initial interview, I asked Alanna for a specific story that might help me understand what inspired her learning the most in this class. What I ended up hearing was a voice of pride and enjoyment for the work that she has done and wonder in herself as a mathematician. There was a question that asked “If the Greystone Academy Freshman class has about 112 students and the day:boarder ratio is approximately 3:5, how many students in a freshman English class of fifteen would you expect to be day students?”

*Alanna:* Well, I taught myself proportions the other night on the homework. *[she laughs]* ‘Cause I never really understood ratios…the one that Kathy did.

*Ms. S.:* The day student and boarder one? Oh OK.

*Alanna:* Yeah, I never really understood that. I was able to teach myself it in a way. Like I ended up like dividing it by three and then 8, or multiplying it by 3. Cause in my mind I was like if there was 8 boarders and 3 students I just took it in parts, so there’s 8 students and 3 boarders, so you just add that together that’s one group and then…stuff like that.
That’s how I put it in my head, I just wrote out steps for it and I was like I’ll just do this for other problems.

Ms. S.: So was that before we went over it in class? Or was it after?

Alanna: Yeah, before...[she laughs]

Ms. S.: So just from reading the problem and thinking about it as 8 parts like 5 boarders and 3 day students or something like that...

Alanna: Yeah..

Ms. S.: Wow, that’s great. And then when she talked about the way she had done it...

Alanna: It was similar.

Ms. S.: Yeah, I think she did it that way too. Yeah that’s right. How did that make you feel to hear her kind of discussing what you had thought about in that problem?

Alanna:: I was like oh OK. [she smiles]

Ms. S.: Were you satisfied?

Alanna: I was happy. Yay, I actually got it right...

What I was hearing her discuss in this segment was the mathematical process of trying different methods on her own and before coming to class it seems she came up with a solution that she wasn’t sure was right, but decided to accept what she had done. After coming to class and hearing another student who struggled in the same way and settled on the same answer in a similar way, Alanna felt a kinship with this other student - like they had been through the same experience of grappling and had come out satisfied in the end. She felt the satisfaction of working it through and doing it correctly. This enjoyment that she was experiencing was not isolated and also was a different sort of happiness than what she experienced in her prior mathematics classes. When I asked her if she ever felt this type of “happiness” in her other classes this is what she had to say:
So it’s like in this class you figured it out on your own, like you did it on your own, without anybody having told you what to do. Like it’s a matter of you putting it together on your own by yourself so it’s like you’ve had that sense of accomplishment like you actually did something. That other happiness is like yeah I got it right, but nobody else did “ha, ha, ha.”

There was distinction in the reason for working independently for Alanna. Before it seemed that she would be “happy” because of the competition she would win by being the one that would “get it right” and there was pride in her ability and perhaps speed in understanding. She’s come to a realization about what she values in her independence in the mathematics classroom and she can be the one to “put it together on her own by herself.” It is the sense of accomplishment that is making her happy now and not another’s lack of a sense of accomplishment.

In the following I-Poem, I believe Alanna’s voice of describing her journey to learning about herself as a learner in relation to the subject of mathematics is clear and proud. I believe this I-Poem begins by calling on her past mathematical identity and then ends with who she now knows she has become.

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<th>I</th>
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<tr>
<td>I didn’t see myself going anywhere</td>
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<td>I didn’t have that chance</td>
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<td>I can</td>
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<td>I can do so much now</td>
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<td>I’m not capable of everything</td>
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<td>I’m a little bit more confident</td>
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<td>I have ambitions and goals</td>
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<td>I’m not there doing nothing</td>
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<td>I would be able</td>
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<td>I think</td>
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<td>If I come home</td>
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<td>I freak out</td>
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<tr>
<td>I’m not satisfied</td>
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<td>I will show</td>
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“You’ll be fine”
I push myself
I am
I push myself
I’m not going to let myself sink below
I wouldn’t be scared
I would still want to be outspoken
Just be who I am
“Math makes sense when you understand it.” – Kacey, 17

CHAPTER 8

Kacey’s Story: From Independent Learner to Free Agent on a Team

When I interviewed Kacey, it was a Friday afternoon and classes had all finished. We met in a quiet classroom at the end of a hallway in the mathematics building at Greystone Academy which we both agreed would be fine. Kacey had been a student in my Algebraic Geometry course since September so she felt very comfortable with me during both interviews. She was a new student to the school, as well as what was called a “repeat” tenth grader – a student who chose to come to Greystone and repeat the same academic grade that they had just completed at their prior school. So as a student, Kacey was rather articulate and outgoing as someone who was older (17 years old), Caucasian and a boarding student from a Mid-Atlantic state. She was a top athlete at Greystone on both the varsity tennis and track teams, known for her prowess with the shot put. What also made Kacey significant was that she was an openly gay student and well-respected by her peers for her political stances and ability to speak her mind in all areas. Although this was a point of contention with her parents, Kacey felt that being a boarding student allowed her to form her own identity and be herself in all ways, including in her learning community at Greystone.

Kacey’s past experiences with mathematics and mathematical learning were extremely diverse. She spent her sixth and seventh grade years being homeschooled using the Calvert Math program materials. She went to her nearby public school for eighth grade which followed a rather traditional pre-algebra curriculum and pedagogy. Kacey found that just going for extra help would get her 100’s on tests so her parents
took her out again for ninth grade where they bought her textbooks and she just worked out of the textbooks with online support. This did not work out very well, so for tenth grade she went back to public high school. Kacey shared with me the story of what happened next.

I went to public high school for one month, and it was weird because I wasn’t in any math classes. They didn’t have math until halfway through the year, ‘cause there was block scheduling which I didn’t like. But I went there for one month and I came home one Friday and I was like, I’m not going back to school on Monday I’m gonna do my own thing because this place sucks, so I told my parents that. So they were like OK, write out what you want to do, so I wrote it out that weekend, so I didn’t go to school Monday.

Kacey’s parents seemed to have a difficult time deciding what to do about her education which was unfortunate for her mathematics learning. She had such a positive attitude towards learning though that by the time she came to Greystone Academy, although she was older, she was ready and willing to try anything. She was extremely open-minded and had a love of learning that many younger students have not had the time to foster yet. Kacey had been exposed to many different types of learning environments and perhaps that contributed to her ability to be open to a problem-based classroom the year I met her.

When reflecting on her different experiences in the classroom Kacey saw the advantages and disadvantages of all of the different types of learning environments in which she participated. She clearly appreciated the autonomy and self-direction of her homeschooling experience (“they were like ‘write out what you want to do’, so I wrote it out that weekend”, “I begged my mom to buy me these CDs where this lady would talk and I would watch it on a Smartboard”). However, she knows that she missed basic skills
that were not taught to her by someone who knew how to communicate the true nature of mathematics.

I feel that part of it is missing like…yeah, that foundation. Not having anyone there to teach you how to do something, and then you kind of learn how to do things on your own, like I guess and check all the time, just because I didn’t learn how to do things the right way.

She’s clearly aware of her strengths and weaknesses based on her background. She is also aware of what she appreciates about the classroom and how she learns best too. From her time learning alone, she definitely appreciates the ability to ask questions or even to have someone of whom to ask questions. When being homeschooled, she could remember times when she craved interaction with other people (“All I want to do is talk to somebody or do a math problem. I would try to go upstairs and talk to my parents.” “I think everybody has a need to talk about it.”). When she was in a more traditional public school classroom, she was frustrated with the way students would silence themselves.

I feel like everyone has questions but they don’t always ask them. And I think it’s good to ask questions because that’s the only way you’ll get something out of it and if you don’t understand something, say you don’t understand it otherwise…that’s not good.

Since she had spent so much time alone, she could not understand why students would spend so much time not asking the questions that they had. At the same time, she couldn’t blame the students because the teachers set the tone for the interaction by implicitly telling the students what they wanted them to do (“You’re not talking, you’re listening. You’re not contributing, you’re having someone talk to you and lecture you.”). She had a very cynical view of teachers from her time in public school and her attempts
at questioning. I asked her if there was any sharing of thoughts like she was craving while she was homeschooled. She responded by saying not at all and that there was no real thought process discussed (“there was just ‘this is how the layout of how your problem should look’, you know, if you get the numbers right then you’re good.”), as if all that the teacher cared about was making sure the manipulation of the numbers was clear and demonstrated and then they would move on. Kacey went on to explain the following:

But it wouldn’t have done anything because the teacher just moves on. She just goes on and as soon as you finish it you move on and you say “Oh well I liked this problem. Can we talk about it?” “Uh, no…cause we’re going on to the next problem.” Oh and another thing, 30 problems a night, something like that, we had these things called problem sets, like PS1, PS2, PS3, sometimes she’d assign just the evens or the odds so it’d be 15 or 20 or 35 problems a night … I think that’s really important not to just have mindless…the same ideas, concepts over and over again.

She had come to expect that even if she did utilize her voice and express some interest in a problem, there would not be the time for or interest in the class exploring that extension question or an alternative solution (which in the teacher’s defense may not have been in her control). Through homeschooling, she came to appreciate her own ability to see connections between materials and the repetition in material seemed “mindless” and not thoughtful. At one point I asked her how she would view the traditional classroom now after experiencing an RPBL classroom this past year. Her response describes Kacey’s need for independence and agency in her own learning as well as a relationship of mutual respect with the teacher.

If I didn’t have discussion, I don’t think I would have class. It just wouldn’t make sense to me anymore. Like going to a public school; I think would be a complete slap in the face. It would be totally different.
Not being able to sit around and say something without raising my hand and waiting forever for the teacher to get done writing the problem and she then ignores you with your hand up.

Clearly, Kacey seemed to have found value in this type of learning. She described the RPBL in many ways as liberating and helping her to gain a sense of ownership for her own learning; which has a sense of irony considering one would think that homeschooling would allow for that. It might beg the question of why her homeschooling experience did not allow for that, but perhaps it may be that learning for Kacey is much more of a social and relational experience than her parents thought. I asked her if she thought there was something about being with other people that was better than being homeschooled.

Yeah, I missed hearing new ideas. I really missed hearing that. Because I’m so used to just hearing myself and my ideas. I wanted to know what other people thought. I like it when someone…I’ll solve a problem this way, and then someone in my class will come up and say I solved it this way and I like hearing all the different perspectives. I’d just gone for so long without hearing any of it so yeah, I like being in a group, it helps because I felt so deprived for so long, I’ve been so lonely.

She knew that in order for her to learn and move forward in her knowledge it was better for her to hear alternative methods. She even goes so far as to describe her learning as “lonely” when she is not able to discuss ideas with others, although when given the choice, she most likely would not have wanted to be in a direct-instruction classroom, as evidenced in her tenth-grade decision to come home after a month. Even in class, Kacey exhibited a true love of learning by not being able to just sit back and be passive in class. She was actively engaged in class every day asking questions. This is entirely consistent with Mau and Leitze’s assertion that a problem based curriculum offers girls that space to
become self-assertive and also to fulfill their need to have a deeper connection to a problem and hence the material in general (2001, p. 38). Mau and Leitze quote Belenky et. al by saying

Girls need to know that they are integral parts of the classroom and that their thinking [is] of interest to the teacher. Their ‘connected teacher’ starts with what the students know and what new knowledge they gain through the act of learning. They suggest that “connected teachers try to discern the truth inside the students (2001, p. 39).

This may also be true of some boys, but was found to be especially true in girls’ ways of learning and especially in mathematics. Kacey was a particularly fine example of this.

Relational Learning with the desire for a “Connected Community”

In describing the differences between her two classroom experiences – public school vs. the RPBL classroom, Kacey was very clear that in her public school environment there was very little interaction between the students, or even between the students and the teacher (“I mean sometimes students would raise their hands, and ask ‘Oh, well I don’t get how you did that’ and then the teacher would work the problem again.”) and it was that interpersonal connection that she had missed being homeschooled. Although early on Kacey was enjoying our class, her grades were not necessarily what she had hoped for. There were holes in her background that were not easily accounted for because of her diverse mathematical education (which I was not aware of at the time) so it was difficult to tell on an assessment what the reason was for her incorrectly answering a question. Kacey had a hard time finding the balance between asking questions and being independent. It was a tough transition at first.
During this transitional time, the relational nature of the classroom allowed Kacey to come to new understandings about mathematics and the mathematics classroom. When I listened for the voice of the self in the interviews, I heard a voice of appreciation for a newfound understanding of the value of the subjective nature of mathematics. The following I-poem speaks to the understanding of this potential for mathematics to be subjective and the possibility that there is more to her work and learning than just right and wrong. It also has a sense of the struggle with this new concept since she is grappling with accepting the idea that mathematics could possibly be different from the way she has viewed it for so long.

I

<table>
<thead>
<tr>
<th>I thought you either get it you don’t get it</th>
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<tbody>
<tr>
<td>I used to think you didn’t get the right answer you understand the concept</td>
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<tr>
<td>I’m kind of understanding I will get partial credit</td>
</tr>
<tr>
<td>I would get the wrong answer I don’t understand it I hate it I understand it I’m close to getting there I feel I can get it I just need to make something click I usually think I think I think I feel like geometry I think you approach something</td>
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The increased use of the first person in this I-poem is quite telling as Kacey begins to take more ownership of the idea of mathematics as being subjective or of allowing there
to be value in her attempt at a problem. She seems to be giving *herself* more credit as she earns more “partial credit” on problems for the risks she takes and methods she tries on problems. She can go from totally understanding it to be “close to getting there” and that’s OK because she feels that she “can get it” eventually. In the end, it’s really how “you approach something” that matters anyway, not necessary having to be there.

As Kacey had said before, the aspect of the RPBL classroom that she appreciated the most was probably the discussion (“If I didn’t have discussion, I don’t think I would have class”) which stems from her lack of having individuals to interact with in her homeschooling experiences. Kacey’s desire for connection with others and the material demanded more justification for her questions than her traditional public school experience would allow for (“I’ve actually come to expect people to answer the questions of why and how and go into great detail of explaining to me”) and her appreciation for those justifications is immense. Through discussion of mathematics she has grown to see mathematics in a new way and her abilities through a new process (“Interaction with other people, hearing different ideas, choosing the right idea for myself, and contributing, questioning, always question it even if it’s right.”)

I was intrigued by this obsession she had with the discussion in the classroom. I observed at times in the classroom that there was rarely a class period that Kacey would not speak at all. In fact, other students sometimes often grew tired of her intensity and intellectual curiosity, with adolescent eye-rolling and impatience. However, I would allow Kacey her interesting questions while at the same time moving the class ahead as time permitted. It was a delicate balance since I could tell how much she was gaining from the experience. In my discussions with Kacey, I decided to probe a little further
into what she felt she received from the discussion in terms of her learning. I heard a very clear voice that kept repeating messages about Kacey’s appreciation for her classmates’ ideas and their influence on her understanding. She seemed to realize how much what another student said could change and shape her understanding.

You think you say “Oh, I’m stumped, I don’t know what to do,” but then someone says something and someone else says something and maybe the group doesn’t get it as a whole but somehow what they said makes a connection in your head and you know how to do the problem.

She often attempts to describe her learning process as the organic and dynamic system that is rather chaotic and then somehow becomes organized at some unknown point (“I won’t even necessarily know that I’m explaining this to myself but just by saying what I know and someone says something and it all just combines into this big mass”) but there is inevitably an outside force that moves her toward understanding. There was a class period that I can recall where students were doing an exploratory lab regarding the Inscribed Angle Theorem which states that the inscribed angle of a circle is equal in measure to half of the arc that it intercepts (See Figure 8.1a). During the class period, one student, Camille, had projected her diagram on the whiteboard and discussion ensued. Kacey was, of course, right in the thick of it. This was a moment where the
students did not understand why the inscribed angle was half of the arc it intercepted but one student noticed that the two radii of the circle were the same, and then another student realized that that made the triangle isosceles. This made someone else realize that the central angle would be 120 and the exterior angle would be 60 and then finally Camille realized that the arc that was intercepted would have to be double the original inscribed angle. (see Figure 8.1b). Here is Kacey’s account of that time:

*Kacey:* I remember one part, one unit uh when we were doing the triangles and like the intersection of like what is it? The median to the vertex

*Ms. S.*: Mm hmm

*Kacey:* Those intersections and the triangles, like I saw it and then I kind of saw the exterior angle theorem like worked into something – I forget what it was – but, I remember I noticed that and we were like it was that one really good day in class when Ms. Johnson was there and someone said something and I said something and Camille said something

*Ms. S.*: Oh right, yeah, yeah, yeah…

*Kacey:* And we were like, “Wow, that was perfect.”

Ms. Johnson, another teacher, was observing the class for professional development and all the students were so excited that another person was there to have witnessed their mathematical prowess and cooperation. It was “perfection” in relational learning and I could see not only the pride but the satisfaction on Kacey’s face that day and in her remembering it during the interview. That proof could’ve been easily demonstrated on the board by the teacher and in less time. However, in RPBL the pride and satisfaction generally come from “perfection” is working together, valuing everyone’s ideas and creating that sense of connection with the material and peers.
The discussion and the relations that she appreciated did not only exist between student and student, but between teacher and student. This may have been Kacey’s first experience where her teacher truly added to her learning experience in a positive way. In our discussions, I did not directly question Kacey about what she liked or did not like about my teaching style, but we were discussing how the class is run in order to be able to see different perspectives and how that helps learning for her. Kacey found a voice at that moment of describing the way that I question.

One thing I liked about “my teacher’s” teaching style is that she will ask questions that are like answers at the same time. Because someone will have a problem and they won’t understand how to do it and then you’ll ask a question like well “what do you think about this?” It’s not like you’re giving a specific answer like “This is how you do the problem.” Because, I don’t know… I feel like it’s just adding to the process of making the connection on your own and having people participate in the discussion.

Unlike other students who would get frustrated by a teacher who was evasive or coaching in their teaching style, Kacey fed off of the encouragement with her curiosity, motivation and eagerness to problem solve. She found that my continued prompts or “questions” actually helped her answer her own questions which gave her a further sense of accomplishment and fulfillment. This worked much better for Kacey’s learning than her experience in a direct instruction traditional classroom where the teacher “would teach [her] from the chalkboard ‘this is how you do the problem’” and Kacey’s voice had little input into the direction of the learning in the classroom.

I won’t know how much of Kacey’s need for discussion stems from her homeschooling experience and being denied the ability to discuss her ideas with others for some key years in her middle school learning experience, but for whatever reason, it
is a key component in her learning. She sums up how important discussion is to her learning of mathematics in this way:

> If students aren’t talking, I don’t feel like I learn as much. Even though they’re just students and not teachers, and they don’t necessarily know what they’re talking about. Just some people know specific parts and then this person will know this part of the problem and that person will know that part of the problem and then it comes together, but if there’s no discussion, I don’t think there’s really effective learning.

As I read this, I heard a voice, not only of relational learning but one that appreciated the collaboration – what each individual brings to the discussion and how it all comes together. Kacey’s ability to value each person’s perspective even though “they’re just students and not teachers” is an important one because I feel that in my class she shared in the important task of helping students feel as if they were the authors and owners of the material – that their voices mattered. I felt that this was a very important role that she played in the classroom dynamic.

**Group and Individual Ownership of Learning**

There is not a great deal of research currently available concerning ownership in learning and mathematics, but the concept is often thought of in relation to autonomy, choice and control when a student or group of students ultimately “develop[s] an idea towards a deepened understanding” (Enghag, Gustafsson, & Jonsson, 2009). One research study showed that through discussion of problems groups of students can actually increase group ownership, and in certain cases individual ownership, based on the “student’s own personal questions posed in order to increase understanding and not only to produce a fast answer to the task” (Enghag, Gustafsson, & Jonsson, 2009, p. 470). In Kacey’s case, her questions most definitely were posed to increase her understanding...
regularly in class and hence, I would conclude that any ownership that she was involved in was both group and individual. While observing her in class, I would witness Kacey jump up to the board and become almost possessive of a concept as if once she understood it, it was her responsibility to explain her newfound understanding to others.

Kacey would take ownership of the material in her journal writing as well. In a journal entry from January of the year of the study, Kacey was writing about a problem in which she first had to construct the correct diagram from the description in the problem and then find a missing angle.

In triangle ABC, it is given that \( \angle A = 59^\circ \) and \( \angle B = 53^\circ \). The altitude from B to line AC is extended until it intersects the line through A that is parallel to segment BC; they meet at K. Calculate the size of \( \angle AKB \).

Kacey drew a wonderful diagram and labeled unknown values that she was able to calculate (see Figure 8.2).

![Figure 8.2 Kacey’s Diagram 1](image)

In describing her process of finding the measure of \( \angle AKB \), Kacey does something interesting. The language she uses is not just procedural. She actually stops and tells the reader to take note of the method she uses and then how she moves on. She uses possessive verbs like “found” and “have” to describe her problem solving and she is writing in the first person.
I must first identify that the point at which the altitude of B intersects side AC forms a 90 degree angle. I know [sic] have one side of my triangle figured out. Notice how I am using a smaller triangle to prove the angle of AKB. Next I look at angle C which I found from subtracting the sum of angles B and A from 180. Angle C corresponds with (and she drew an arrow to KAC in her diagram) so they are congruent because the lines are parallel.

Kacey continued with great journal writing throughout the year and found through authorship, another outlet for her individual ownership of her learning.

However, she also took great pride in the group ownership that was evident through the discussion of the problems. At one point in our conversations I asked Kacey explicitly how she thought this type of teaching method gave students a sense of ownership or how students got invested in the learning process through problem-based learning. Her response was one of the most articulate, descriptive and moving moments of our interview time together.

People identify it like this is our class. If we don’t talk there’s not gonna be a class, if you don’t share your ideas there’s not gonna be a class, because Ms. Schettino’s not gonna tell you “get up there” and tell you how to do every single problem on the homework, you’re gonna have to say something. You’re gonna have to get up on the board, and you’re gonna have to write it down, and you’re gonna have to get up and actually explain to the class. That process of explaining how you did your problem, not only helps everyone, but I think it helps you. So that helps explain your own work and just the fact that this is your problem. I like that part about the day. You always start out with your homework, this is my problem, this is how I did it and everyone talks about your problem.

Kacey’s emotional emphasis of the possessive pronouns in this segment shows how seriously she takes the idea of ownership of not only her understanding, but the classroom community as a whole (‘this is our class’, ‘how you did your problem’, ‘this is my problem’, ‘everyone talks about your problem’). She also has totally invested in the
belief of her classmates having ownership of their understanding, too (‘You’re gonna have to … explain to the class’, ‘process of explaining… not only helps everyone, but… helps you’). She is fully aware of how much that process of discussion helps not only the class’ understanding but her own and is extremely grateful for it. In fact it is the ownership itself that she really enjoys – the pride she feels talking about her problems and sharing her ideas with everyone (‘I like that part of the day’).

Eventually, I was so curious about this aspect of Kacey’s vision of learning in the RPBL classroom that I asked her how she thought this sense of ‘community ownership’ per se, gave students a deeper sense of agency in their learning than a direct instruction classroom.

Ms. S: How do you think that aspect of the class gives students a little bit more, not power, a sense of control that they don’t have like in a traditional math class?

Kacey: I think… uh, that’s a hard question. How would you answer that? Like, I know for me, like, I feel like it’s my geometry class and I feel like compared to a public school, I couldn’t say something. I couldn’t come in and put in my two cents on the problem even if it’s just a little part of the problem. I feel like no one would raise their hand in public school unless they knew the entire problem, so you could kind of add in your perspective and it kind of gives this sense like “Ooh, I helped with this problem” and then another person comes in and they helped with that problem and by the end no one knows who solved the problem. It was everyone that solved the problem. Like, everyone contributed their ideas to this problem and you can look at this problem on the board and you can maybe only see one person’s handwriting, but behind their handwriting is everyone’s ideas. So yeah, it’s a sense of “our problem” it’s not just Karen’s problem, it’s not just whoever’s problem, it’s “our problem”.

She begins by claiming her ownership of the class and then comparing it to a more traditional class where that ownership was not felt and why. Her voice was not valued in
terms of ownership of the material and neither was the voice of others students. In fact, most students’ voices were only used to confirm the teacher’s statements (‘no one would raise their hand…unless they knew the entire problem’) which only perpetuated the idea that their individual ideas were not important. It was her input into the problems that helped increase the idea of the value of her ideas even “if it’s just a little part of the problem.” Kacey so beautifully uses the description of the “ideas behind the handwriting” which helps create the community problem and lack of individual possession of the problem in the end. I appreciate her description of that process as someone who had observed students moving through that process.

The following short I-poem captures Kacey’s voice describing both the individual ownership of the material and what the class does as a whole.

<table>
<thead>
<tr>
<th>I</th>
<th>You</th>
<th>We</th>
</tr>
</thead>
<tbody>
<tr>
<td>I feel like once</td>
<td>you understand the connection</td>
<td>we learned</td>
</tr>
<tr>
<td></td>
<td>you actually become smarter</td>
<td></td>
</tr>
<tr>
<td>I think that’s the</td>
<td>you can make connections</td>
<td></td>
</tr>
<tr>
<td>beauty</td>
<td></td>
<td></td>
</tr>
<tr>
<td>I love the problems</td>
<td></td>
<td></td>
</tr>
<tr>
<td>you can have one problem</td>
<td></td>
<td></td>
</tr>
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<td></td>
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</tbody>
</table>

This is one of a few I-poems that I found in Kacey’s transcripts that actually included the use of all three pronoun voices (I, you and we). It signifies that she is attempting to speak for everyone in the class, herself, a general student, and the class as a whole with herself included in that group. There is ownership and belonging described on behalf of all three of those groups here; ownership, appreciation and learning.
Independence vs. Interdependence as Agentic and Empowering

Kacey loved to talk about the relationship she had with her classmates and how it affected her learning. However, in listening to the contrapuntal listenings it was evident that there was tension between how much credit she gave to herself for her learning and how often she recognized the group for her understanding. She continually would say that she would make connections “on her own” with no one telling her the information that she needed to know, but then she would attribute her understanding to something that someone else said in class and oftentimes, in the same sentence (“I think because you’re overcoming the problem on your own, like you know, you have help in class with other people”). In the following excerpt Kacey holds many tensions between the two ideas of being independent in her learning and how much the interdependence is important.

*You do your homework and you come back and discuss the homework and you work it out together as a class, and then the teacher is there to kind of guide the discussion and when people don’t get it the teacher steps in and says “well what about this, did you ever think about this?” but you don’t ever get the direct answer it’s more like helping everyone to kind of realize it on your own I feel and once you do that it’s …it helps because you make the connection by yourself with guidance.*

It’s almost as if the idea of doing it alone-with-help-from-others is one and the same for Kacey, not ideas that are juxtaposed against each other or would have to happen separately from one another. In Kacey’s mind it is possible for her to be an independent learner yet completely interdependent on the other students and the teacher for her learning. I believe this mindset is based on the relationality of her learning and how much her knowledge is truly contextually based on the relations she has created between the individuals in the class and the relationship she has with the material. These
relationships are somehow exaggerated in Kacey’s learning perhaps because of lack of accessibility of them in her “lonely” homeschooling and impersonal public school experiences in her past and her innate need for relational learning. I could hear these contrapuntal voices in these two I-poems as well.

- Independence

  you don’t ever get the direct answer
d  kind of realize it on your own

I feel

  once you do that
  it helps because you…
you make the connection by yourself
  but once you get it
  you could get it
  you could
  you could have someone tell you
  you could say “oh yeah, I get it”
you told me it

I figured it out
I think

  you figure things out
  for yourself it sticks

- Interdependence

I feel like everyone has questions
I think it’s good to ask questions

  the only way you’ll get something out of it
  if you don’t understand something,
you don’t understand it otherwise

I think it’s this one
“I don’t understand this.”
“Oh I understand this”

  they’re just like you
  they help you

This complex way of describing her learning may be how others viewed their learning as well. In the first I-Poem, Kacey clearly feels as if she is speaking for other students with
a disconnected “you” voice representing the general student in the class. Although she is speaking of her own experience, she is projecting how other students may be having that experience as well. She takes a short break and reminisces about how learning in a direct instruction classroom was less than fulfilling (“you could have someone tell you’) but then the learning is not deep because it was not something that was experienced (“oh yeah I get it, you told me it’). The “you” here quickly shifts from student to teacher. However, she has now learned that the real learning happens when she figures it out on her own. At the same time, the second I-poem proudly claims her appreciation for the contributions of her peers. Speaking in her “I” voice she is encouraging others to ask questions – speaking to them, giving them advice in order for them to increase their understanding. Then she begins speaking as them in the first person in an “I” voice, sharing their voices and how she heard their voices in class. Finally, she shares how she sees them as “just like you” – where “you” is any student in the class, even herself. Everyone is on even footing and they all share the authority and right to ask questions and learn.

But what is it about the RPBL classroom that gave Kacey that sense of agency, or the feeling that she was directing her own learning? It may be that she had had that feeling before, during the time when she was homeschooled and so it was familiar and comfortable, but she had also been in a classroom where she hadn’t felt it as well. In reading for the story, I could vividly remember asking her about gaining a sense of agency throughout the year and she responded by saying that every student would leave this class feeling agentic (“I can’t imagine someone who wouldn’t have that because… unless like, you just weren’t trying at all”). I asked her to elaborate on that comment.
You come to, like, a certain point where you want to solve the answers yourselves, because, not because, well, I mean you’re given the homework, but **you want to find the answers**. Because *you* want to and not because like…you’re being instructed and taught everything, but **you’re actually looking for something that you don’t know that’s not taught to you**. Ah, like that versus like a teacher telling you this is how it’s done you’re actually searching for how it’s done and it, and it **makes you want to do it and it makes you reach out and look for other things just because it’s you, personally, that wants to do it because you have to do it for the course because it’s the way everything is set up**.

What Kacey is describing here is her reaction to expectations of the course. Because the course is set up in such a way that there are no answers given, just discussion by the students on the process, there is a focus on what each student brings to the class each day. Each students’ contribution is what is important, not the answer that (potentially) the teacher might have brought to class. Additionally, because that’s what’s important (“you’re actually looking for something that you don’t know’, ‘you’re actually searching for how it’s done”) students eventually shift their view of where the agency in learning comes from and realize that they bring the direction of learning with them to class (“because it’s you personally’, ‘it makes you reach out and look for other things’) or ask more directive questions while in class as well. In this segment above, Kacey clearly outlines how her agency is connected to the classroom culture created by the RPBL environment.

When I asked Kacey about how she felt this course had empowered her in her own learning she actually said that she felt that “this was probably one of the most empowering classes [she’d] taken” and it was amazing when “half way through the year, like three-quarters,” when she came to realize, “Oh I can, I can do this myself.” This next segment from our dialogue shows the joy that she felt when she realized her own
empowerment. She speaks from the heart and with a voice of empowerment by creating an analogy between her and another person who has made an empowering discovery.

Kacey: You know, and even though I was still part of a group, I, I could sit in my, at my desk and I could make a connection myself and then, there was one point in time where I made this, I forget what it was or what problem, but like I made a connection, I was like [gasp]

Ms. S.: [laughs]

Kacey: I didn’t have to go to Ms. Schettino to figure this out! And then I felt like that guy, the Roman or Greek guy who was sitting in his bathtub and then he discovered water displacement, you know, by himself. Do you know that guy?

Ms. S.: I don’t know that guy.

Kacey: You know, when you drop something in water and it like goes up?

Ms. S.: Oh right, right, right…

Kacey: Like water displacement, he’s the one who figured it out, and…

Ms. S.: Ohhh [laughs]

Kacey: It’s like how you discover if like gold is pure… or someone’s cheating you.

Ms. S.: Right…

Kacey: You can like put it in, so…

Ms. S.: Well, I’ll have to look that up.

Kacey: Yeah, like making that kind of connection, like he ran out into the street naked and said Eureka! [both laugh]

Ms. S.: Right.

Kacey: That guy. [laughs]

Ms. S.: Oh my god…that’s funny. [laughs]

Kacey: So, so yeah. Making a connection in geometry is maybe on a smaller scale than that.
Ms. S.: Ok, I get it.

Kacey: I’m not exactly going to walk out in the hallway naked and say Eureka [laughs].

She was convinced that she was responsible for making things “stick” in her learning and it was just that individual responsibility that gave her self-satisfaction and in turn, empowerment (“When you make a connection and you know that you were the one who made the connection, that’s very empowering’). Inevitably, whether Kacey attributed her learning to herself or to a combination of the group and herself, didn’t seem to matter because she truly ended up enjoying mathematics and her mathematical learning a great deal.

Enjoyment without High Performance

Through my time working with Kacey, she always contended that she was not a “math” person. Her identity centered on sports, writing, and the arts as she was an amazing tennis and track athlete and also, prolific writer and deep thinker. However, as she developed in this course it became clear to me that she came to value mathematics in a way that many students who do not have high ability in mathematics generally do not in a traditional mathematics class. One way that Kacey did this was in seeing how RPBL helped her to understand mathematics better. Kacey knew that in a direct instruction classroom she could have a teacher just “tell you how to do something” but she really never had a deep understanding of why the mathematics fit together and it didn’t “stick” for her. In other words, there rarely was a lasting understanding. I asked her to elaborate
on why she thought something that someone else tells her would leave her understanding as opposed to something she figured out on her own.

I like to think about compared to me throwing the shot put without the technique. Because like you could just do it with just brute strength but it…and you could do it faster. But you would have to backtrack and go through the steps through the technique and learn it like down from…like start from the bottom, and build yourself back up and it goes slower, but in the end you’ll like throw it so much farther. Just like when you do geometry, it takes like…it’s a slower process I feel like, but it’s so much better in the end. And you feel smarter, like, I didn’t really understand what is the point of math, I didn’t want to go in any career involving math, but I feel like once you understand the connection, you actually become smarter and you can make connections in other things as well. And it just helps everything.

What Kacey is describing is a phenomenon that many educators have realized through experience and many researchers have confirmed through experimentation – that retention of knowledge and development of learning comes with experience and deliberate thought processes. The educational researcher and teacher Caleb Gattegno once said of learning, "We are retaining systems and do not need to stress memorization as much as most teachers do. We hold better in our minds what we meet with awareness" (1976, p. vii). Because RPBL stresses the process of problem solving and the collaborative relationships between those involved in the process, the learning is enhanced by making students aware of processes by their own realizations and discoveries. That awareness is often more meaningful or creates more sense-making in terms of the mathematics for them in the long-run. As was stated by the National Research Council report on student learning in mathematics, “Metacognition and adaptive reasoning both describe the phenomenon of ongoing sense making, reflection, and explanation to oneself and others” (Donovan & Bransford, 2005, p. 218). Deeper
understanding and more active participation definitely increased Kacey’s enjoyment of studying mathematics.

Kacey claimed on many occasions that she was a more visual learner and enjoyed the geometric aspects of learning rather than doing algebra (“The more algebra stuff, I did not like at all, but the ones where I could see pictures, I was like, ‘Oh, this is so great’”). It was true that she did seem more engaged with the geometric concepts throughout the year and I can even recall the time when she seemed to make a breakthrough with her understanding, which happened with more visual problems. However, I think it might have to do more with holes in her background algebra knowledge than with the type of learner she was. Unfortunately, Kacey saw her mathematical limitations as innate inadequacies in her ability as opposed to problems with her foundational preparation. (“I think there comes a time when you realize there’s like a block that some students put up against math and science and say, ‘Oh, I’m an English person’ and I still don’t like to say I’m an English person.”)

It was at this point that I began to hear contrapuntal voices in Kacey’s description of her enjoyment of mathematics. There was a voice of surrendering to the idea that she just didn’t have strong mathematical ability, but at the same time there was a voice of excitement and enthusiasm for the beauty of mathematics. It was hard for her to come to terms the possibility that she might have more ability than she thought, but that her background doesn’t allow her to achieve at the level she would like (“I still don’t even get As, but I love the feeling you get”, “:) I love vectors (in journal)”, “I really like it”, “I really liked geometry and I’m sad that it’s over”).

And yeah, to me math is awful when I don’t understand it I hate it, it’s awful. But when I understand it, it’s like “this is fantastic!” But then
there’s that in between part where you’re like I’m close to getting there, I feel like I can get it, I just need to make something click, but when it clicks it’s great. I feel like that sometimes, it’s frustrating but it doesn’t make the class any less fun, even though it doesn’t necessarily show in my grades. I wish that colleges would take into consideration – they’re like, I got a C in this class, but I really enjoyed it and talked a lot.

Even within this short excerpt, Kacey’s voice of surrendering to her perceived lack of ability and how the external measures of the college process and grades judge her in very shallow and incomplete ways. However, I can still hear the voice of her appreciation for the satisfaction of finding a solution on her own and the value of problem solving and logical thinking. In this I-poem, Kacey describes her ability in the humanities which was reciprocated in terms of her grades, but the wonderful feeling that she gets from mathematics.

<table>
<thead>
<tr>
<th>I</th>
<th>You</th>
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</thead>
<tbody>
<tr>
<td>I think</td>
<td></td>
</tr>
<tr>
<td>I’ve taken</td>
<td></td>
</tr>
<tr>
<td>I can learn history</td>
<td></td>
</tr>
<tr>
<td>I can get an A</td>
<td></td>
</tr>
<tr>
<td>I have to study</td>
<td></td>
</tr>
<tr>
<td>I still don’t get As</td>
<td></td>
</tr>
<tr>
<td>I love</td>
<td></td>
</tr>
<tr>
<td>The feeling you get</td>
<td></td>
</tr>
<tr>
<td>You know</td>
<td></td>
</tr>
<tr>
<td>You make a connection</td>
<td></td>
</tr>
<tr>
<td>You know</td>
<td></td>
</tr>
<tr>
<td>You were the one</td>
<td></td>
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</table>

In her “I” voice she is identifying with her strengths and weaknesses academically. She expresses her personal frustration with studying in math and not getting the grades to show her hard work. Parallelling those feelings, are the feelings of pride she gets from making the connections in mathematics, but it is interesting that here she speaks in the
second person “you” voice, as if to distance herself from the possibility of having a mathematical part of her own self. She has claimed that she is not a “math” person, so it would make sense for her to not speak about herself as a mathematical self in the first person. She even says “you were the one” to make the connection, in the second person voice.

It is a dilemma for a student like Kacey who knows that the dominant culture will eventually be measuring her in one way, but what she values and sees as her strengths may not be measured as easily. Although she admits to her weaknesses in skills and algebra, she has a goal to take mathematics in college someday.

I love Geometry. The other day I was talking to a calculus student, and I looked at calculus, graphing all this stuff and I thought “Wow, that looks really cool.” And I want to take calculus now, it looks really fun, even if I have to take it in college, it looks really fun. I think I get excited about math like, I’m really enthusiastic.

So although she considers herself not a “math person” she freely admits that she loves math and it excites her intellectually. She attributes her enthusiasm and interest in mathematics, at least partially, to her work in this course because it has allowed her to grow in her confidence, to inquire and her ability to answer her own questions. We talked about the development of her confidence and how it affected her enjoyment of mathematics.

*Kacey:* So like, I developed the confidence to speak up in class and ask questions.

*Ms. S.:* Right
Kacey: And then after asking the questions and finding out what it was that I really needed to know, then I could answer the question.

Ms. S.: Yeah?

Kacey: And that’s where…

Ms. S.: That’s a really good point, it’s…

Kacey: So there’s like two parts to the confidence -

Ms. S.: Right

Kacey: The confidence to want to learn and then the confidence like that you get from actually succeeding.

Kacey had found that she was confident enough in her abilities to want to question and learn in a group setting, but even more that she could be successful when trying to answer those questions. Her inquiry was sometimes success enough, but in time her inquiry got her to correct conclusions and that confirmed in her that her curiosity had value. She eventually got excited and enjoyed the actual mathematics as well (“I love the problems we do where you can have one problem that involves everything we learned from the entire year and it all comes in”) and appreciated the connections in the material.

In the end, what Kacey left me with was the fact that a student who had a rather low ability level in mathematics could learn to love a subject that most students with low ability run in fear from. Not only did she learn to love it, but on a daily basis, she would dive in, head first with eagerness and excitement to see what a problem had in store for her. She learned to love the process of being a mathematician and being in a community of problem solvers. At the very end of our last interview, Kacey got me very emotional and thanked me for this experience that she felt had changed the way she looked at mathematics as a whole. Since then, I have thanked her as well. In this last excerpt from that interview, I can distinctly hear the voice of Kacey, the mathematician.
Kacey: And I really like… and I think I never would have learned that I liked that unless I had done this course, so….

Ms. S.: Wow

Kacey: So that’s… thank you for that…

Ms. S.: That’s really cool.

Kacey: So, I think I took it kind of as a learning experience.

Ms. S.: Hmm

Kacey: And found out just what I like about math.

Ms. S.: Ok.

Kacey: Maybe I’ll be like a philosophical mathematician or something…

Ms. S.: Hmm

Kacey: Write a book…

Ms. S.: Yeah

Kacey: [laughs]

Ms. S.: That would be cool. That would be really cool.
CHAPTER 9
Comparisons of the Girls’ Narratives

I will turn now to looking at the interesting intersections and disconnections between and among the analyses of the five girls in the study. Because of the ability to so closely listen to their multiple voices, a truly complex web of stories were told and layer upon layer of commonality and difference have been folded and unfolded. Recall from Chapter 3, the multiple theory Continuum of Girls’ Learning blended from theories of Belenky, Perry, Baxter Magolda and Brew (Figure 9.1).

**Figure 9.1 Continuum of Girls’ Learning**
It strives to describe girls’ learning not solely in a gendered way (as some of the individual theories did), but in how they are empowered by their sense of their voice in their learning (Do they have the right to be heard? Can they be heard? Is what they say valued?), their sense of knowledge as absolute truth (Is there only right or wrong? Can knowledge be subjective?) and from where does that knowledge come (Is there an absolute truth source from where it comes? Can some knowledge come from within themselves or from their experience?).

The relevance of this continuum to the five participants in my study is twofold. First, it considers the gender learning theory (Belenky, 1986), but it also looks at development learning of adolescents from a dualistic perspective of truth which is too often related to mathematics education. It would not be difficult to argue that for the majority of people, not only girls and women, traditional mathematics classes tend to draw people towards the left-hand side of this continuum. A teacher-centered, direct-instruction classroom focuses on a single authority – that of the discipline of mathematics – where the absolute truth lies and there is one method of solution. For most of the girls in this study, this was the sole mathematics experience they described to me in their elementary and middle schooling experience (excluding Kacey). However, by the end of a year in an RPBL classroom, I would place them at very different locations on the learning continuum as seen in Figure 9.2. Arguably, each of these girls started and ended in different places, dependent on their own past experiences. However, from their stories, it is easy to ascertain that the place they came to from their mathematical and relational experiences mathematical and classroom journey is a very different one with respect to their view of voice, knowledge and authority. This suggests that the pedagogy
of Feminist Relation which was used in the RPBL classroom played a part in the movement of these girls along the continuum. Moving from a place of Absolute Dualism to Contextual Commitment, it seems like the experiences in their mathematics classes and the way they viewed themselves as a learner were affected by the methods of RPBL.

Both Sara and Leona were students who truly saw themselves as passive mathematics students coming from experiences in extremely traditional classrooms. They had little internal agency and saw mathematics as an absolute truth – sitting and waiting for the teacher tell them what was right or wrong. Isabelle and Alanna had more of a cynical view towards the mathematics classroom as they both were more capable math students, but had prior experiences that had poisoned their view of their own agency. They knew they were capable, but still viewed mathematics as a rather dichotomous and objective
subject matter which fostered the idea that the authority was totally external. Only Kacey
was truly the student who began the year at the Transitional Multiplicity level, which
might have been attributed to her homeschooled experiences. She was aware that she had
an internal voice of authority, which had relied on because of that experience and had
rejected that authoritarian model of public school. However, because of her
independence in that forum, she was not aware of the power of multiple perspectives and
the connectivity of mathematics.

By the end of their time in the RPBL classroom, all girls clearly moved forward
on the learning continuum. All of them discovered their internal mathematical voice of
authority and discussed their view of connection and relation in mathematics. Although
they did not find the complexities of the interconnected of the knowledge at the same
levels, they did at least appreciate their own ability to integrate their internal voice with
the group’s voice – either through their own writing or presentation skills with the class
as a whole.

The Heart of Relational Problem-Based Learning

This information about growth on the Continuum of Girls’ Learning begs the
question of what aspects of the RPBL pedagogy were most influential to the girls’
experiences. Also, is this type of growth dependent on the utilization of RPBL? They
are questions worth asking since perhaps it is merely the idea of interesting, scaffolded
problems that the students were responding to. I would say not. In considering all of the
evidence that the girls gave which affected their enjoyment, self-confidence, value, sense
of empowerment and agency, I would conclude that it is the combination of the pedagogy
of relation and the PBL curriculum that fosters the outcomes they are describing in their stories. Figure 9.3 attempts to illustrate the relationship between the recurring themes in

![Figure 9.3: Emergent Themes of RPBL Framework from Narrative Analysis](image)

the girls’ stories and how the RPBL classroom attributes support those themes. The themes of Ownership of Knowledge, Justification – not Prescription, the Connected Curriculum and Shared Authority emerged from a code map of the girls’ descriptions of their experiences of the RPBL classroom. It is clear that each of these aspects of their experiences made an impact on their learning or one of the attitudes and senses in some
form or another. Because of this, it would be wise for the mathematics community to consider the advantages of learning with this type of pedagogy.

Ownership of Knowledge

In 2011, education researcher Grant Wiggins did an exhaustive national survey of approximately 7300 middle and high school students in order to obtain the learner voice in the bigger picture of American education. About 38% of students responded that mathematics was their least favorite subject (largest percentage out of ten choices) and of those students, 53% were female (Wiggins, 2011). The two most-often cited reasons for stating that mathematics was their least favorite subject were that “I am not interested in it” and “I am bad at it” both at an approximate response rate of 60% (Wiggins, 2011). It has been widely established that mathematics as a subject gets a bad rap of being boring and uninteresting to students and many do not enjoy the class. However, this was not the case with these five girls with respect to the RPBL classroom. One reason they gave for this class improving their enjoyment of and self-confidence in mathematics was their ability to take ownership for the material that was being discussed.

Student voice is one aspect of the ownership of the knowledge. If students can make their voices heard and realize all of the opportunities for this, a sense of ownership follows. There were numerous times where the girls shared stories of their appreciation and revelation that their voice mattered in the RPBL classroom. Sarah’s realization that her deep interest in mathematics would allow her to delve deeper into the extension questions that she always wanted to ask but “definitely” never able to in a more traditional classroom. Leona’s powerful statement that this class had “given [her] a
voice, whereas [she] didn’t really have one before. It was a silent voice.” This speaks volumes to her growth from a place on the continuum from Absolute Dualism to a more right-hand location where she knows that there is a time for listening and a time for speaking. Maybe most importantly, student voice in many cases replaces teacher voice so allows the students to experience the fact that the part that they are playing in the construction of the knowledge is valued and important. Kacey said, “It’s not the teacher explaining what the student did; it’s the student explaining what they did on their problem.” This very subtle difference in discourse moves actually makes a big difference in the message that is sent to the students, as evidenced in their attitudes toward student voice and ownership.

I was struck by Kacey’s impassioned description of the class as “our class” and “your problem” with a real sense of ownership and propriety for each of these items that are not even physical. In all of these statements and many more, there was not only an emerging acceptance of multiple perspectives, but an expectance of hearing multiple perspectives along with deep appreciation for them. These students knew that the different perspectives that others brought with them to a certain topic or problem would enrich and often clarify their understanding of a concept. The complexity of seeing through others’ lenses helped them to view mathematics as subjective and less as an absolute truth, bringing these students more into the realm of independent relativism.

If I extend the idea of rejecting the absolute truth of the knowledge of mathematics all the way to the end of the continuum, students arrive at the concept that the authority for meaning-making emanates from within (thus furthering ownership). Coming to this point on the continuum takes a great deal of confidence, intellectual
maturity and trust in one’s own judgment and risk-taking. It would be exceptional for a student to move through all of these stages in one academic year, therefore it is not surprising that many of these girls have not fully entrusted their meaning-making to themselves. However, the sense of ownership of the material grows as they experience this transition. Through her time “at the board,” Sarah was able to gain confidence speaking and presenting, which she considered most pivotal in her growth. Sarah admits that working with her peers and figuring something out “means more than just a teacher telling you how to do the problem.” In fact this was a common theme that all of the girls could agree upon, even Isabelle. They stated time and again that they knew they would learn it better and remember it longer if they “figured it out for themselves” but oftentimes that meant frustration and hard work. However, working with each other gave a “sense of accomplishment” and satisfaction that did not come when the teacher “just told you how to do it” that they all shared. In other words, making your own meaning and being your own authority was not only possible, but much more enjoyable and rewarding.

In my theoretical framework, I cited a definition of social empowerment that claims that it is a “social process…that fosters power in people (that is, the capacity to implement [a plan or idea]) in people, for use in their own lives…by acting on issues that they define as important” (Page & Czuba, 1999). In order to think about this in the classroom for adolescent girls, I tried to describe to them the sense that they were not powerless in the act of learning and that they had some sense of control in how they learned. The Ownership of the Knowledge seemed to make the difference for many of
these girls and it affected their sense of empowerment, their self-confidence and the value they placed in the mathematics classroom.

**Connected Curriculum**

All of the girls commented on the way the curriculum has topics running in parallel and revisits concepts regularly. Leona commented on this aspect of the curriculum as well as her learning in our first interview together.

I think what I really like …the concepts reoccur or it kind of reiterates a point. And I like that because it’s not like…it helps my learning because it reiterates, it repeats what we’ve learned in places ahead, a few pages behind and it’s not like we come across one thing and it just ends right there. It’s not like, “OK chapter 5 is all about like polynomials”,…but like the circumference of a circle like it’s not like that, and then you move onto like triangles, it’s all kind of incorporated and we’ll learn something and then you’ll see it later on, and you’ll say, “Oh my gosh, I know how to do that, I know what that is!” and that’s what helps my learning. It’s kind of that repetition.

Isabelle also made statements about how the topics kept coming back. She said, “I like how it’s like the spiral learning thing so you don’t forget anything. So, like we’re still doing the stuff, like we’re still using centroids and stuff – we did that in January. It helps me remember everything.” The piece that these girls are not articulating here is what the “repetition” is actually asking them to do. All of these problems are not “repetition” in the sense that they are the same type of problem, over and over again. It is merely the fact that the topic is part of another question that is actually different. So they are being asked to remember a concept and connect that concept – hence the “repetition” – to a new concept. It’s more than reinforcement or practice of the same topic; it is seeing how the concept actually affects and moves forward a deeper understanding.
Alanna described this appreciation for the connected curriculum when learning dilations for the first time by saying, “Like with the vectors and dilation thing. I saw vectors in that, I don’t know why, I just did.” She made the connection between her prior knowledge of vectors with the knowledge of dilations. Alanna said:

The ability to connect other things…’cause before they would teach you something and you’d go home and practice it. But in this class you have to like be able to bring back other information and then do the problem, so…I think I’ve learned that skill. Because it became evident in other subjects.

Kacey described her appreciation for the accumulation of the mathematical knowledge by stating that mathematics had beauty in the sense that “everything we learned now is kind of starting to come all together and I love the problems we do where you can have one problem that involves everything we learned from the entire year and it all comes in.”

These girls have learned that mathematical knowledge does not need to come in the form of chapters and facts from a published textbook. It is there to be discovered, as it first once was by mathematicians.

The acquisition of knowledge is seen from a different perspective as well and helps the girls move along the continuum. Since there is no textbook with pre-ordained, compartmentalized topics, the students are experiencing a curriculum that appears to have no higher-order authority – beginning or end – and allows them to take ownership of the development of the material more easily. It is not simply the sense of ownership of the knowledge being built, but the greater sense that feeling and belief that they are helping to construct it themselves because of the connections that they have made together and individually.
The curriculum that is evident to the girls is not necessarily the curriculum dealing with the subject of mathematics, nor how mathematics is used in everyday life. What it seems to be are the so-called “21st Century Skills” of critical thinking, problem solving and communication that have been proposed as critically important for the future of our job market in the U.S. (Beers, 2011). Through experiencing mathematics in a way that has allowed them to be active and engaged and immersed in the depth of mathematics on a regular basis, they have practiced these skills and they have done so relationally. Not every student will become a mathematics major, nor will they all be able to manipulate algebraic equations or visualize abstract geometric concepts – however these girls learned and valued skills that they will use in many other disciplines in their lives. As Alanna said to me:

Ms. S.: What do you, what do you think the value is?

Alanna: Like, it’s just the whole process of like seeing how you can go from one thing to another thing and seeing how that thing affects a whole bunch of other things?

Ms. S.: [both laugh] can you be more vague? No… just… [both laugh] So, the process…., so like learning how to work something out?

Alanna: Yeah

Alanna is expressing her initial thought on the curriculum as her newfound ability to make connections between concepts and use those connections to solve problems independently. However, after thinking about it, she also sees that these connections help her to remember the material because the RPBL curriculum regularly uses prior knowledge again and again. So by forcing the connections, the retention is fostered. The process of synthesizing and critically thinking about problem solving is actually, in
effect, the use of current and prior knowledge. This use of knowledge is then internalized and retained for future use.

Shared Authority

In most mathematics classrooms, the answer to the question, “From where does the knowledge come?” is clearly from the teacher – the knower of mathematics. In an RPBL classroom the answer comes convincingly from the classroom community members and their resources since there is no textbook with concrete answers or declarations of absolute mathematical truths. Initially, this was extremely uncomfortable for most of the girls, but they came to appreciate the freedom and camaraderie of the relational pedagogy. Many of the girls made reference to this moment where, although not one individual student might have known how to do the problem, once they started discussing their ideas the integration of the new ideas with their own helped move their thinking forward.

*Kacey:* You think you say, “Oh, I’m stumped, I don’t know what to do”, but then someone says something and someone else says something and maybe the group doesn’t get it as a whole but somehow what they said makes a connection in your head and you know how to do the problem.

*Alanna:* …like somebody else incorporating what they say into what I say and just making something out of it.

*Sarah:* I mean it’s kind of like a team thing almost…you get to work off of like each other’s knowledge.

*Leona:* And they kept giving me little hints here and tips or suggestions, and kind of felt like, “Oh, that’s how that works”

*Isabelle:* “[There’s] like a lot of different ways to do certain problems so there’s a lot of variations in the way that people do them”
The common theme here being that they recognize regularly in the classroom that the knowledge they are building is coming from each other and the teacher.

Sarah saw the RPBL classroom as fostering her agency in having to be at the board presenting, having the freedom to ask extension questions and also the major difference of the students’ ideas being the impetus for the problem discussion.

My old classroom, I mean, it was just the teacher at the board and she would like give us notes to write down, we’d fill in notes together, and then we’d do homework, come back and go over the homework. I mean we still go over the homework in geometry but it’s like a totally different format.

She saw the very different classroom structure and shared authority as encouraging her agency since she was the one directing the discussion instead of the teacher. The students would share their ideas on problems and make conjectures to prove, in place of their usual role of writing down what the teacher said or following the teacher’s notes and merely filling in the blanks.

The girls made comments not only about how the teacher was able to share her authority but how the students came to share individually in the authority of the class. This shared sense of responsibility and inter-relatedness in their learning helped them trust each other. Alanna’s sense of empowerment stemmed mostly from the revelation that her innate mathematical ability could be shared with others and she was encouraged to use her intuition.

It would be like in terms of - I can say this and this would happen and I would learn because it’s an open discussion class – it’s not just one person dominating the whole thing and the fact that a lot of other students come up to the board and like teach.
She used her power to affect others’ learning and she used that same power to make the choice to listen to others as well. She no longer was the “distraction” in class when she was finished with assignments first because she was actively engaged with doing mathematics and listening to her peers.

Leona appreciated the shared authority in the way it allowed her to have more of a say in the problem-solving process. She knew that her opinion mattered. The concept of multiple perspectives changed Leona’s view of learning mathematics.

We’ve been exposed to it …that kind of fuels that drive to see whether we’re right or wrong in what we’ve done. We’ve realized that the way the teacher might be presenting it on the board isn’t really the way we like. She valued the fact that her opinion meant something in the classroom and her idea, although different from the teacher and her peers, was important and could be right. She could choose the method she wanted and it was not prescribed by the authority of a textbook. Her natural curiosity and enjoyment of debate also fueled her drive for the truth and creating open, productive discussion.

*Justification*-* not Prescription*

Having the focus of a mathematics class be to justify the process was very different for many of the students (“I like the why”, “Here, we find out the why”). It is clear that the modus operandi for a traditional mathematics teacher is to merely prescribe a solution method and have students practice that method. There is little discussion on alternative solutions or options.

Because the course was focused on justification, rather than prescribed solution methods, all girls found ways to appreciate the consequences of that new focus. Kacey’s
confidence, which is such an integral part of who she is, allowed her to make the most of the relational aspect of this course. She questioned freely, conjectured regularly and took risks openly in almost every class. It was natural for her and she showed others how to “do mathematics” in many ways. Leona’s confidence lay in her experience in the theater and public speaking. This class offered her the opportunity to use those strengths in presentation while allowing her to explore her curiosity for math and appreciation for multiple perspectives. She always loved hearing others’ opinions in politics and history class and now she could hear them in mathematics too. This gave her confidence to feel more comfortable to ask questions and relate to the conversation as if it were a different class and to be a more active part of discussion.

Sarah, Leona and Alanna had all not had experiences where they had been asked to inquire in mathematics. However, when exposed to that type of thinking they did enjoy it, especially in a relational way. Leona’s innate talents of communication and debate made her a natural for discussion and when she realized she could apply this to mathematics class, it became much more fun and comfortable. Once Sarah was encouraged to follow her natural sense of inquiry and extension questions, she enjoyed being valued for those skills. Alanna needed to know that her high intelligence was of use in a problem-based classroom by allowing her independence to work for the class instead of being a distraction in a direct-instruction classroom.

The more the internal process of learning was made external by discussion, problem presentations, verbal questioning and answering by students, the more they were able to “see” that process unfolding and being constructed and feel more confidence that everyone was on the same page, or least that it was out in the open. Isabelle’s description
of the “Elephant in the Room” that was in the traditional classroom, where the amount of understanding that students have at any given moment was no longer a secret because of the open discourse. Not only that, but people were willing to help each other instead of competing for who knew the most. Because the amount of competition was reduced and there was less stress of prescribed methods, those students who, in a traditional classroom were not considered “good at math” because they were not quickly able to mimic prescribed methods, were all of a sudden “good at math” because they were creative and had interesting insights.

Alanna saw in a direct instruction classroom, not only the teacher’s opinion of how to do a problem, but a prescribed process that she must follow – a clear authority in the mathematical realm. Because she was so bright, she knew that she had the intuition to make those choices for herself. She craved a classroom where she was able to own the learning herself. I asked her to elaborate on the idea that she had control over her learning in the RPBL classroom and distinguish that concept between the teacher having a topic in the syllabus that needed to be taught and actually spelling it out for students.

Yeah, cause like if there’s something in a problem that I didn’t see when I was doing it directly, that might be what I look for in what I have to do tonight. Like, if there was a median thing and that had to do with 30-60-90 to find out the angle, I might just look for that in another problem tonight instead of just like completely disregarding it.

Alanna is talking about how she would make the connections between problems and concepts instead of needing a teacher to tell her the connections or what type of problem it was. This is a truly important part of Alanna’s growth in her agency and something it took her some experience to realize. Although there is a syllabus and topics are discussed
to some extent (or “covered” as a more traditional classroom would state), the RPBL allows the students direct their learning of those topics.

Because her teacher did not focus on only covering prescribed solution methods, Sarah was able to follow many of her own ideas and they were valued. One of Sarah’s new experiences in RPBL classroom was the opportunity to follow her inquiring mind to new places.

And then it kind of helped the [next] class because on the next few homework nights there were questions that were really similar to that and if we hadn’t done that extension question in class it would have been harder for us to solve the other ones. So it’s kind of... it helped the whole class, not just a random question.

The problems in the curriculum kept her moving to new places that her past teachers once stifled. Her newfound empowerment stemmed from the realization that those trains of thought were not just musings of a distracted mind, but interesting deep thoughts about complex problems. She was able to help the class in their understanding and even move them forward with new problems when they pursued her line of thinking. Sarah’s empowerment was fostered by a teacher that allowed her to think creatively and enthusiastically about the possibilities of mathematics and the world.

Leona’s appreciation of the teacher’s focus on justification was mainly because of the multiple perspectives that the teacher would entertain. It fostered Leona’s sense of agency the most. By seeing how other students viewed a problem from a graphical, algebraic or geometric viewpoint, it helped Leona choose the standpoint that helped her learn best. It definitely didn’t hurt that Leona was a very independent and outspoken young woman who felt almost restricted in a direct instruction classroom, not being able to share her opinions.
Cross Participant Comparisons

In the large scheme of the RPBL Framework, these four themes most consistently emerged. However, often the evidence cited by a participant (like a discourse move used by the teacher or the use of multiple perspectives) was attributed to a different part of the framework. This is the purpose of the arrows that go from each of the boxes that describe the characteristics of each of the themes. It seems that much of the girls’ experiences come from the combination of all of these characteristics of a RPBL classroom. Further research needs to be done to tell which are necessary and which of others might have been neglected to be included in the framework.

Of course, the five girls’ experiences were very different in many ways and while one girl might have consistently cited valuing multiple perspectives as a teacher behavior that helped her realize the course-valued justification of methods, there may have been another student that stated that using multiple perspectives allowed her a greater sense of ownership of the material. Each of these characteristics of a RPBL classroom may affect each girl differently based on their background experiences, level of self-confidence, place on the Continuum of Girls’ Learning and many other variables. Again, further research may help parse these experiences in more detail.

To return to the girls’ experiences for a final summary, I decided to compare the three attitudes and two senses that were part of my original research question – enjoyment, self-confidence, value, empowerment and agency. When comparing codes for these different attitudes and senses, some common themes emerged which can be
<table>
<thead>
<tr>
<th>Student</th>
<th>Relationship with Enjoyment</th>
<th>Concept of Self-Confidence</th>
<th>Vision of Value</th>
<th>Realization of Empowerment</th>
<th>Journey towards Agency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sarah</td>
<td>With support and praise, surprisingly enjoyable when allowed to follow her own inquiry</td>
<td>Still developing, mending the damage of comparison, competition and silence</td>
<td>Math is more of a tool for science, but problem solving is a life skill that is valuable</td>
<td>Realized strong voice of inquiry for extension questions, leadership skills and ability to take risks</td>
<td>Still dependent on teacher for validation, but strong increase in independence and interdependence in directing learning</td>
</tr>
<tr>
<td>Leona</td>
<td>Enjoyment comes from deeper understanding, reasoning – knowing the “why” and communicating</td>
<td>Improvement because the class played on her strengths of communication and relational skills</td>
<td>Math is ‘something you must do’, but the problem solving skills help in other disciplines</td>
<td>Strong newfound sense of her own voice in the classroom, but grappling with her power in her learning</td>
<td>Relational nature helped bring out her own agency in learning, appreciated multiple perspectives immensely</td>
</tr>
<tr>
<td>Isabelle</td>
<td>Views problem solving as fun, like doing a puzzle, enjoys relating others by helping and being helped</td>
<td>Sense of confidence not necessarily related to mathematics, but gave her an advantage in discussion and risk-taking</td>
<td>Sees the relevance of big picture problem solving or calculation skills, but does not see its necessity</td>
<td>Realized the possibility of the subjective nature of mathematics and how voicing her own ideas and/or listening to others added to the learning experience</td>
<td>Students can be more agentic because their understanding is “out in the open” - no one “keeping up appearances of understanding since they are constantly questioning</td>
</tr>
<tr>
<td>Alanna</td>
<td>Pride, satisfaction and fulfillment from accomplishing learning independently, in her way and above her expectations</td>
<td>Confidence in her ability remained constant, but concept of self as a student changed drastically (see empowerment and agency)</td>
<td>Mathematics is a worthwhile subject to study, but the class was always boring until RPBL</td>
<td>Being an independent learner doesn’t mean working alone, it means collaborating with your peers and building knowledge together.</td>
<td>Seeing things with “her own eyes” is more meaningful than “being told” by a teacher, found appreciation for agency in her learning</td>
</tr>
<tr>
<td>Kacey</td>
<td>Enjoyment flourished even though her achievement did not</td>
<td>Initial strength did not wane and allowed for risk-taking and sustained inquiry</td>
<td>Mathematics value is in its “beauty” not necessarily in its everyday use or applications</td>
<td>Grows from a self and group sense of ownership of knowledge that is co-constructed, stems from connections</td>
<td>Juggled balance between independence and interdependence– but the agency and direction in learning came from her and her peers</td>
</tr>
</tbody>
</table>
viewed in the comparison chart in Table 9.3. It was clear that across all five girls the one category in which the RPBL framework failed to make an impact on their experience was in their value of the content material that is taught in mathematics class. All five girls shared their lack of appreciation for what the subject of mathematics and the skills that the class taught them would bring to their lives (specific mathematical skills like factoring, writing equations of lines, etc.). However, they did grow to have an appreciation for the skills they acquired in communication, collaboration, problem-solving and synthesizing the material. They also expressed appreciation for the value in making the connections they made in the recognition of material that returned and how they were able to reuse knowledge again and again.

As you can see in the comparison table, all girls found that the RPBL classroom experience positively affected the other areas in question, which helped move them toward the higher end of the Continuum for Girls’ Learning. The relational aspect of the RPBL Curriculum enhanced the discussion, helped to bring agency to them individually (and as a whole) and allowed for increased self-confidence, according to each of them. In one way or another all of the girls realized that working independently relied on working interdependently, which in turn helped them become better agents of their own learning. Further, Kacey went so far as to say, “I think this is probably one of the most empowering classes I’ve taken” and the other girls realized more of their own power in numerous ways. All of this came to fruition because of the extraordinary fusion of the curriculum, the students and the teacher in the very special RPBL classroom.
CHAPTER 10

Conclusions

There is very little argument from professionals in the world of mathematics education today that it is in need of some positive change. According to the 2009 PISA (Programme for International Student Assessment) Mathematics Literacy test, 18 countries had higher average scores than the U.S. (NCES, 2009). Because of the lack of girls’ interest in math and science and the number of women leaving STEM fields, the AAUW published the report *Why So Few?* in 2010. In that report, it was found that girls’ achievements and interest in math and science are shaped by the environment around them (AAUW, 2010, p.xiv). Almost 20 years ago, in 1994, Fennema and Hart published an analysis of all of the research about gender and mathematics that had been published in the Journal for Research in Mathematics Education (E Fennema & Hart, 1994). Two of the major conclusions that were published in numerous articles were that “Teachers tend to structure their classrooms in ways that favor male learning” and “Interventions can achieve equity in mathematics.” Part of what this study has shown is that the experiences of girls in the some mathematics classrooms in the U.S. have not really changed in the past 20 years. The call of past research for equity in the mathematics learning community is failing with the current traditional mathematics pedagogy in place. It is time for us to move forward with changing the way students learn mathematics in the U.S. starting with the environment and classroom in which they are enculturated. In order to create more equity in the world of higher education and STEM field careers, more young women need to feel as if they belong in the classrooms of those courses.
Stepping Back from the Birdcage

Further argument for looking at a move to a Pedagogy of Feminist Relation in Mathematics could be made by examining the excerpt from Marilyn Frye’s essay from 1983, Oppression once more. I referenced this when speaking about Leona’s experience in her past classroom, but after reflecting on all five girls, I believe the analogy is relevant to many girls’ (and possibly other underrepresented students’) experiences in mathematics classrooms all over the U.S. Referring to Marilyn Frye’s emotional and articulate analogy of oppression to a birdcage, I claim that it is time that mathematics educators and researchers in the U.S. take this analogy to heart and step back to view the wires. They need the perspective that the girls in this study have given which allow a much broader view of the realistic picture. It shows that together all wires continue to keep girls (and boys) of all underrepresented and underserved groups from reaching their full potential because of these obstacles that they cannot overcome - obstacles that continue to keep them from not only appreciating the beauty of mathematics, but often from the awareness that they are capable of that appreciation.

Ms. Frye wrote, “If you look very closely at just one wire in the cage, you cannot see the other wires.” For years, researchers looked “one wire up and down” trying to explain the reasons behind why girls were not achieving in mathematics – there was a gender gap, they said. Once that was dispelled, they moved onto other wires of the cage of oppression in the mathematics classroom – standardized testing bias, test anxiety, self-efficacy issues, gender bias, cultural expectations, mathematical identity – the list goes on. Unfortunately, there is no “physical property” to any one wire that will explain for us all what the reasons are for the existence of the obstacles. The reasons are different for
many girls and women, people of color or students with learning differences. Ms. Frye
concludes her meaningful quote by saying:

It is perfectly obvious that the bird is surrounded by a network of systematically related barriers, no one of which would be the least hindrance to its flight but which by their relations to each other, are as confining as the solid walls of a dungeon.

What seems to be the case is that there is a “network of systematic barriers” in place and sadly it is the traditional, dichotomous, gendered and authoritarian method in which mathematics is viewed and taught in our society. It is this very system that serves as the “walls of the dungeon” for underrepresented students in mathematics and what hinders them from fully experiencing mathematics as I would argue it should be – a sharing of ideas and discovering of patterns in open creativity with intellectual peers. Allowing the current system to transition to embracing of a pedagogy of feminist relation using a problem-based curriculum has the potential to create a greater likelihood of enjoyment, self-confidence, value of the broader skills studying mathematics has to offer and a larger sense of student empowerment and agency in the classroom.

Implications for Future Research

While doing this study, it was clear that there were many limitations and there were often times when I stopped myself and thought, “Wouldn’t it have been wonderful if I could have done…?” I had originally collected data on eight participants which included a broader range of age, more students from the other teachers’ classes, and a greater diversity in race and SES. However, time limitations did not permit me to utilize the data for this study and my hope is to eventually make use of all of the data for a fuller investigation of these attitudes and senses in RPBL. Also, clearly, the fact that these
classrooms were all single-sex may have impacted any or all of the attitudes that these students had about their learning, so it will be important for follow-up studies to be conducted in a co-educational setting that uses RPBL.

Disaggregating populations to be sure that there is consistency between SES, race and gender would be my next consideration to see if girls are not the only underrepresented group in the field of mathematics for which RPBL makes some difference in their experience in the classroom. I would like to take on a larger mixed methods study at some point that would consider any correlation between the use of RPBL and some of these attitudes and sense that might be measurable. Having an experimental design with pre- and post-testing to allow for possible conclusions to be drawn would also be my hope in future research.

I would like to increase my work with teachers on professional development in this pedagogical theory and research how different methods of work with teachers might enhance student learning in different ways. My hope is to refine my methods of curriculum development and student discourse analysis in order to help improve the future of RPBL. Future research with teachers could include using the professional development and training framework I utilized with the Greystone Academy teachers (see Appendix F) to see if similar student outcomes were experienced with both single-sex and coeducational settings.

Considering the Framework for the RPBL Classroom, I believe it would be interesting to consider another qualitative study to try to understand more about the four aspects that I found to be effective (namely Justification – not Prescription, Connected
Curriculum, Shared Authority and Ownership of Knowledge) and see whether there is more to this framework that would give the same affects. I am open to the possibility that similar outcomes may come from a different framework and am interested in how this framework could be changed and to what extent.

**Moving Forward**

In a study of two schools with different pedagogical methods, Boaler once wrote “The Amber Hill girls [at the traditional school] found that they were unable to improve their situation, not because they were disillusioned by their own inadequacies, but because they were powerless to change the pedagogical traditions of their institution” (1997, p.302). In short, her advice was to “change the system, not the girls.” Still 16 years later, schools in the U.S. have not learned how best to teach our underrepresented students so that they feel empowered to learn in the ways that meet their needs. Personally, I have spent my career attempting to reach out to not only students, but teachers that are interested in this type of change in the hope of making a difference in mathematics education. I have been encouraged by how many individual teachers are looking for a change in their pedagogical approach to mathematics in order to have some semblance of equity, communication and sense-making actively occurring in the classroom.

At its lowest levels, what this study has done for me is confirmed my beliefs about how RPBL is valued in the experiences of young women studying mathematics. Their journeys, as told in their stories, touched me deeply and moved me as an educator. At the highest levels, my hope is for this research is to inspire further study and a movement in the education community to look for alternative and powerful ways in
which all students can have experiences in the mathematical classroom that are valuable and meaningful to enrich their lives and affect their futures with enough depth to see some of the beauty in mathematics. Because, as I said to Kacey, “that would be cool, that would be really cool.”
References


Appendix A: RPBL Teacher Facilitation Behaviors that connect to Tenets of a Pedagogy of Feminist Relation (Schettino, 2011)
Appendix C: PBL Course Student Rubrics for Assessment

Grading Rubric for Class Participation and Discussion – This rubric is considered on a continuum and +/- is included when improvement and/or growth throughout the year has occurred.

<table>
<thead>
<tr>
<th>Grade</th>
<th>Description of Work</th>
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<tr>
<td>A (90-100)</td>
<td>You take a strong, well-defined and well-communicated position in your solutions. Your presentations are well-organized, persuasive arguments with accurate supporting evidence. Your solutions end with clear, concise justified conclusions demonstrating logical thought processes. You question not only your own work, but others' as well. Not only do you listen attentively to others' comments, but you give others the respect they deserve in their understanding process. Your preparation for class includes a sincere effort consistently on all homework problems. You take total responsibility for your own understanding and oftentimes for your classmates'</td>
</tr>
<tr>
<td>B (80-89)</td>
<td>You take a clear position in your solution presentation and process. Your argument is well-organized with solid supporting evidence. You include much of your work, however unsure you are of the process. Your questions are usually in regards to clarifying your own understanding of a concept, while at the same time helping others to do the same. When confused, you might lose the ability to listen to others, but you obviously try to understand, and eventually find a resource for clarification. You sincerely attempt all homework problems, but there is not always evidence in the pages shown in class.</td>
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<tr>
<td>C (70-79)</td>
<td>Your presentations of solutions are poorly organized and general. Since you have little grasp of the material, your argument is unfocused and often confuses classmates. You have a difficult time connecting new ideas with prior knowledge even if they have been discussed recently. There is little evidence that you are listening to or engaged in discussion by your classmates or the teacher. There is also little evidence of effort on your part to complete the homework problems or to come for extra help and take responsibility for your learning.</td>
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<tr>
<td>D (60-69)</td>
<td>You rarely volunteer at all to share your thoughts on homework, since little was even attempted. When you do, your position is vague and unclear. You are rarely engaged in class discussion and only add irrelevant, critical or thoughtless comments. You do not question any topics or problems and do not take responsibility for your own understanding of the material</td>
</tr>
<tr>
<td>No Credit Given (&lt;60)</td>
<td>You seldom speak and when you do it adds no value to the discussion. Your presence in class is either negative or not felt at all. It is unclear if you have ever opened your book or attempted any homework problems and you do not take responsibility for your understanding of the material in any way.</td>
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Grading Rubric for Journal Entries – This rubric is considered on a continuum and +/- is included when improvement and/or growth throughout the year has occurred.

<table>
<thead>
<tr>
<th>Grade</th>
<th>Description of Work</th>
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<tbody>
<tr>
<td>A (90-100)</td>
<td>Your entries include well-written commentary on problems, formulas and class discussions and you select relevant and appropriate problems that incorporate multiple concepts and often integrate complex processes. Your writing is organized with persuasive arguments that use relevant formulas and terms. You write in complete sentences and diagrams are neatly drawn. You justify each statement and often make connections between concepts and from problem to problem.</td>
</tr>
<tr>
<td>B (80-89)</td>
<td>Your entries include written commentary on problems, formulas and class discussions. When writing about a formula, you provide an explanation or proof, and an example. You write down solutions, sometimes procedurally, without consistently justifying your work. You write in complete sentences and provide diagrams. You select problems that are relevant but often only on a single concept or process.</td>
</tr>
<tr>
<td>C (70-79)</td>
<td>Your entries include solution to problems, formulas discussed in class and other topics from class discussions. You do not justify your steps with persuasive arguments or mathematical reasoning. You are inconsistent about using complete sentences or providing diagrams with your entries. You make statements that are not valid and your solutions is often difficult to comprehend</td>
</tr>
<tr>
<td>D (60-69)</td>
<td>Your entries look like class notes or homework. You provide little justification for your work or show no work at all. Your have little to no commentary written on your work for each problem.</td>
</tr>
<tr>
<td>No Credit Given (&lt;60)</td>
<td>You do not complete the assignment in a timely manner or in an acceptable way at all.</td>
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Appendix D: Course Expectations for PBL course

Required Materials:
· TI-84 Any Edition
· Three ring binder with hole-punched graph paper or notebook with graph paper
· A composition notebook (with graph paper) to serve as journal
· Internet Access
· Laptop to be brought to class

How your work in class is valued and assessed:

1. Individual Student Contributions to Class (oral assessment) - being a regular part of class discussion, coming well prepared for discussion in class, showing enthusiasm for the learning experience, volunteering solutions with excitement, displaying a sincere interest in the mathematics of the problems, thinking analytically and critically on all in-class activities including computer lab work.

How do you view your homework? As a chore to get over as quickly as possible? As an aid in your understanding? As something collaborative you can work on with your friends? As a way to work with your teacher to help your understanding? Not sure how to improve your class participation grade? Check out Ms. Schettino's Rubric for Class Participation on our class’ Moodle page.

2. Excellent Preparation – this includes having all course materials each time class meets, exhibiting obvious perseverance on all homework problems, hand-in homework problems done well and on time, solid evidence of effort in all aspects of class, taking responsibility for misunderstandings and incorrect solutions, seeking extra help when needed from classmates, peers, and especially the teacher (including all resources at your disposal.)

3. Written assessments - Improved work on all types of problem sets, be it partner, solo, group, or hand-in, with notes or without notes. Well-written solutions, solid methods, and interesting ideas, evidence of the ability to write mathematics with clarity, precision and accuracy. Your written assessments will include a variety of types including problem sets, Quick Quizzes and journal entries. Want to see how Ms. Schettino will grade your journal entries? Check out my Rubric for Journal Entries posted on our class’ Moodle page.

4. All 6 (or 7) grades will be averaged together at the end of the trimester and weigh equally. At end of the fall and spring trimesters there will be a common trimester exam.

Classroom Community Members’ Bill of Rights

1. Right to Enough Class Time - Since we have a limited amount of class time that we would like to optimize, everyone is expected to get to class in a timely fashion (including the teacher!) If being late to class becomes a habit for a student
however, the teacher may report APs to the Dean of Students office. Be respectful of the class’ time and interest and be on time for class.

2. **Right to a Classroom Conducive to Learning** - Since Deerfield is a rigorous academic institution, class dress denotes your intention to come to class with intellectual curiosity and a serious attitude towards learning. Please be in dress code for every class. Also, it is important that every member of our classroom community feel comfortable speaking and listening with respect. We will talk more about this together.

3. **Teacher’s Right to Receive work in a Timely Manner** - In all cases of excused absences, students are expected to hand-in any assignments that were due during a missed class the next class day. Should you be feeling overwhelmed the night before a homework assignment, quiz or problem set, I expect to be contacted well before 10:00 pm via email or by phone. Please let me know what your workload is like and why you are feeling overwhelmed. Oftentimes, arrangements can be made, but sometimes they cannot. Daily Homework will be checked visually every day. If there is no evidence of sufficient time spent on an assignment, it will be noted and affects your class contribution grade. Missed hand-ins homework assignments cannot be made up, you must give them to me by the end of the day or you do not get credit for that assignment. Should you miss an assignment, quiz or problem set due to an unexcused absence it can be made up, but the grade is automatically a full letter grade lower than what is earned.

4. **Right to Understand When & What Assignments Exist** – Homework will be posted at least two weeks in advance on our Moodle page, as well as quizzes and problem sets and due dates for journal entries. If I am getting behind in posting assignments, please remind me and let me know that you need to know.

5. **REGARDING COLLABORATION** – (or **Students’ Right to Collaborate in Learning and Teachers’ right to appropriately ask them not to**)

   It is encouraged that homework be done collaboratively, even hand-in assignments. However, take-home problem sets (or TESTS) are to be done solely by yourself without help from anyone. You may contact me for clarification or if you need a push in the right direction. Any work that is done with others and handed in will be considered academic dishonesty and will receive a failing grade. If you have any questions about whether or not you can collaborate on an assignment, please ask!

6. **REGARDING TEST CORRECTIONS** – (**Students’ Right to Learn from Their Mistakes**) Corrections will be due the third class period after the assessments are handed back to you. You will gain back 1/3 of the points you lost. Please make an appointment to review the errors you made. However, only one of these new grades will count in your trimester average. However, you are encouraged to do as many as you like. I expect there to be something written about what you have learned about the problems you corrected. There are no corrections on quizzes.

7. **Students’ Right to get Extra Help** - Please do not hesitate to email with a question on homework at any time before 11:00 p.m. you may make an appointment for extra help then. For other times, please see my schedule.
Appendix E: Participant Analysis Maps: Created in software MaxQDA, representing the connections between the coding of the four or five documents for each student participant.
Appendix F: Outline of Teacher Professional Development and Training for Teaching with PBL

I. Curriculum Mapping exercise: take current geometry curriculum and looked at what we thought could go and what we wanted to add from the Exeter materials. Refer to “map” for our own writing, taped up on the walls while we began to go through the Exeter materials.

II. Needed to remove problems that were not in our curriculum, over the heads of our students, (kept some for challenge). As we went through the problems, we saw places where there were holes in the scaffolding of the concepts and wrote problems that would introduce or review material.

III. Discussion paradigm shift to problem-solving being the focus of the course. Worked on the use of a problem in class discussion. How was this problem relevant to new material? How much detail was needed at every step of the “thread” of problems on that topic? We did this both on the problem level and at the “big picture” or “thread” level. We spent time doing “mock” classes where we would focus on a problem or group of problems and would see how students would react to the problem and how we could react as the teacher.

IV. Articles read and discussed:
   a. Saphier “Five Student Beliefs that Liberate Learning v. Limit Learning”
   c. Savery, “Overview of Problem-Based Learning: Definitions and Distinctions”
   e. Ellis, “Initiating and Eliciting in Teaching: A Reformulation of Telling”

V. Discussions
   a. ways in which the teacher is integral in sending students these messages and how we could be sure to liberate learning in our classrooms.
   b. Specific ways to create the classroom culture we desire – teachers’ responsibilities
1. establish a connected, caring and personally accountable classroom environment
2. nurture the learners’ inquiry processes, maintaining positive self-images and commitment
3. imbue the learning process with the excitement and challenge of seeking comprehension
4. raise alternatives in order to stimulate the process of evidence gathering and critique
5. clarify different intellectual roles …identify their appropriateness to certain settings or activities and provide opportunities to engage them (legitimate the students’ participation in this kind of community)

VI. Analyzed and discussed examples from our classes, and some from my background, of different types of learners and how they react to this type of curriculum.

VII. Discussions of alternative assessment, homework, etc. all to be consistent with the values of the course

VIII. Reorganized how technology is used in the course. Wrote GeoGebra labs that were embedded in the curriculum.