Potential for high resolution microscintigraphy using polycapillary optics

Patrick Richard Conlon

University at Albany, State University of New York, pat.r.conlon@gmail.com

The University at Albany community has made this article openly available. Please share how this access benefits you.

Follow this and additional works at: https://scholarsarchive.library.albany.edu/legacy-etd

Part of the Optics Commons, and the Radiology Commons

Recommended Citation

https://scholarsarchive.library.albany.edu/legacy-etd/860

This Master's Thesis is brought to you for free and open access by the The Graduate School at Scholars Archive. It has been accepted for inclusion in Legacy Theses & Dissertations (2009 - 2024) by an authorized administrator of Scholars Archive. Please see Terms of Use. For more information, please contact scholarsarchive@albany.edu.
Potential For High Resolution Microscintigraphy using Polycapillary Optics

by

Patrick Conlon

A Thesis
Submitted to the University At Albany, State of New York
In Partial Fulfillment of
the Requirements for the Degree of
Master of Science

College of Arts & Science
Department of Physics
2013
Potential For High Resolution Microscintigraphy using Polycapillary Optics

by

Patrick Conlon

COPYRIGHT 2013
Abstract

Scintigraphy, also known as nuclear imaging, is the process of imaging an object that has been labeled with a radioactive material. A novel technique employing polycapillary optics for very high – resolution scintigraphy is presented. The small channel size and angular selectivity of polycapillary optics allow them to act as multiple-hole collimators and be used with high – resolution detectors. The ability of the optics to work with high resolution detectors allow the system to discriminate against scatter, thus negating the need for energy sensitive detectors, which are known to have poor resolution. Therefore the use of polycapillary optics presents the opportunity to both reject scatter and increase resolution. Measurements were performed to determine the effects of increasing source – to – detector distances and optic – to – detector distances compared to those used in previous works, as well as increasing the length of the optics. The images exhibited promising signal – to – background ratios while still displaying sub – millimeter resolutions, even with large amounts of tissue – equivalent material in place. Lastly, a Markov Chain Monte Carlo algorithm was developed to estimate the resolution of images, determine parameters of the brachytherapy seeds employed to simulate patient dose, and determine theoretical signal – to – background ratios, all of which showed fair agreement with experimental results.
I would first like to thank my Advisor, Professor Carolyn MacDonald, for all of her help and patience, without whom I would not have been able to complete this thesis, and her diligence in working on this with me for many years.

Also, I would like to thank all of the members of the Center for X-ray Optics (CXO) for all of their help and friendship during my time at CXO.

I would like to thank my parents and friends for their support during my time working on this project and for helping me to keep my composure and sanity at times.

Lastly my mother, Susan, for her patience and confidence that I would one day complete this paper. While there were times I may not have enjoyed it, I would have never completed this without her pushing and driving me to complete this.
Table of Contents

1. Introduction ......................................................................................................................... 1

2. Theory .................................................................................................................................... 7
   2.1 Radioactive Decay ........................................................................................................... 7
   2.2 Incoherent Scattering .................................................................................................... 11
   2.3 Computed Radiography ............................................................................................... 12
   2.4 Polycapillary Optics ..................................................................................................... 13
       2.4.1 Polycapillary Resolution ......................................................................................... 14
       2.4.2 Sensitivity ............................................................................................................. 17
       2.4.3 Septa Penetration ................................................................................................. 19
   2.5 Bayesian Data Analysis ................................................................................................. 21
       2.5.1 Computational Problem of Nested Sampling ......................................................... 22
       2.5.2 Basic Principles of Nested Sampling .................................................................... 23

3. Experimental Results ......................................................................................................... 26
   3.1 Intensity Measurements and Calculations with $^{125}$I Brachytherapy Seeds ............. 27
   3.2 Seed Images ................................................................................................................... 31
   3.3 Image Resolution ........................................................................................................... 35
       3.3.1 Resolution Calculations Using Full Width Half Maximum .................................. 35
       3.3.2 Resolution Calculations Using Nested Sampling ............................................... 37
       3.3.3 Trends in Resolution ............................................................................................ 46
   3.4 Image Signal to Background .......................................................................................... 46
       3.4.1 Image Signal .......................................................................................................... 47
       3.4.2 Image Background Due to Septa Penetration ....................................................... 48
       3.4.3 Image Background Due to Scattering .................................................................. 50
       3.4.4 Theoretical and Experimental Signal to Background Comparisons .................... 51

4. Conclusion ............................................................................................................................ 55

Appendix A - Table of Images and Imaging Parameters .................................................... 57

Appendix B – Nested Sampling Fitting Code ..................................................................... 63
Main ......................................................................................................................................... 63
Apply ....................................................................................................................................... 70
Prior ......................................................................................................................................... 73
logLhood ................................................................................................................................. 76
Explore ..................................................................................................................................... 78
Results ...................................................................................................................................... 84
IntensityConvRev .................................................................................................................... 87

References ............................................................................................................................... 92
1. Introduction

Scintigraphy, also known as nuclear imaging, is the process of imaging an object that has been labeled with a radioactive material, a process which has been greatly hindered by poor resolution.

In nuclear imaging, collimators are used to create an image.\(^1\) Imaging without the use of a collimator would result in a process similar to that of situation depicted in Figure 1a, where the image produced would not present any usable information due to the fact that radiation is emitted isotropically creating an large and blurred image of the source. However, even with the use of a collimator, the majority of photons are scattered by surrounding tissue, creating a much broader source with approximately the same intensity as the real source, as shown in Figure 1b.\(^2\) Empirical images displaying this may be observed in Figure 2. Compton scattered photons will be at a lower energy, thus making energy discrimination between primary and scatter possible by using energy sensitive detectors.\(^3\) Unfortunately, while this technique works, energy sensitive detectors...
tend to have poor spatial resolution, approximately 3 mm, when compared to radiographic detectors, which have spatial resolution of about 0.05 mm. In addition, the use of energy-sensitive detectors typically requires the use of radioactive isotopes with relatively high photon energies in order to discriminate between the scattered and direct photons.

In this work, a different technique using polycapillary optics has been employed to reject scatter and increase resolution. Polycapillary optics (as shown in Figure 3), are arrays of capillary tubes which range in diameter from 2 – 50 μm. Polycapillary arrays are composed of glass or lead glass; they work well as a scatter grid because of their ability to act as a multiple-hole collimator. For polycapillary arrays, the walls of the holes (the septa) are reflective. X rays striking the interior walls at grazing incidence are guided along through the hole by total external reflection. An x ray is reflected from the smooth surface by total reflection if the grazing angle of incidence, θ, is less than
the critical angle, $\theta_c$. The critical angle is inversely dependent on photon energy, and is approximately 1.6 mrad, 0.09°, for lead glass at 27 keV.\textsuperscript{5,6,7,8}

Polycapillary optics, in conjunction with a high resolution detector, stand to provide a very crucial role in the advancement of the field of microscintigraphy through their ability to accomplish two tasks simultaneously, acting as a collimator while also increasing resolution. This increase in resolution is made possible by the small channel size and the angular selectivity provided by the small critical angle necessary for transmission through the capillaries. This restricts the field of view of the pixels in the image,\textsuperscript{2} thus allowing the image to be broken into an array of many more pixels.
A simple hypothetical example with the geometry shown in Figure 1b can be used to better explain process described above. Assuming the scatter is uniform and as strong as the radiation from the source, consider an image that can be resolved into an 8 x 8 array of pixels. If the radioactive hot spots are distributed over a 4 x 4 central part of the array, then the direct counts from these spots are distributed over 16 pixels. The counts from the scatter are then distributed over 64 pixels, so that the scatter count rate is 1/4 of the signal. This is most likely insufficient scatter discrimination, and would therefore require energy discrimination. Also, the distribution within the hot region may not be uniform, and may contain features that are not unobservable with poor resolution. If the 8 x 8 array was replaced by an 800 x 800 array, these small features might be observable.

The larger number of small pixels, in addition to...
to improving resolution could also provide a reduction in scatter background. If the each pixels from the 8 x 8 array were divided into a 10 x 10 array, and if the radioactive features covered 20% of the original central area, then only 1/5\textsuperscript{th} of the central area, 1/20\textsuperscript{th} of the of the 640,000 pixels receive direct signal counts. Assuming equal emission rates, the signal count rate per pixel would be 20 times larger than the scatter count rate per pixel, even in the absence of energy discrimination.\textsuperscript{2} An illustration of this process can be seen in Figure 4.

A schematic diagram of the polycapillary collimator and experimental setup used is this work are displayed in Figure 5. Using this setup, measurements were taken with three polycapillary optics. Two of the optics were new fabricated optics and the third was an optic that was used in previous experiments. The newly drawn optics were longer than the optic previously used. The new optics also had a larger percentage of lead in the glass. The optics were used to image an array of 10 \textsuperscript{125}I Brachytherapy seeds.

In previous work\textsuperscript{2}, it was shown that polycapillary optics with a non-energy sensitive detector were a feasible method for use in microscintigraphy. The 20-mm thick glass polycapillary parallel hole collimator used with a computed radiography image plate demonstrated promising resolution, limited by the pixel
size of the image plate. The work demonstrated the ability of polycapillary arrays to effectively “dilute” scatter through the patient without the use of an energy sensitive detector. However, the images showed significant background, mostly due to septal penetration.

Some questions that will be investigated in this work are the effects of increasing optic to detector distance, increasing the length of the optic, increasing lead concentration in the optic, as well as increasing the object-to-collimator distance, as well as, scatter rejection with larger amounts of tissue-equivalent scatter material. These questions will be investigated as the belief is that increasing the length of the optic and increasing lead concentration will help to increase the signal – to – background ratio as it was relatively low in previous work. Also, the investigation of performance of the optics at larger distances from the object is to show the feasibility of polycapillary optics in nuclear imaging by conducting at distances that are reasonable for clinical use.
2. Theory

2.1 Radioactive Decay

X rays are a form of electromagnetic radiation with wavelengths ranging from .01 nm to 10 nm with energies ranging from approximately 100 eV to 100 keV. The energy of an x ray can be calculated by

\[ E = \frac{hc}{\lambda}, \]  

(1)

where \( h \) is Planck’s constant, \( c \) is the speed of light, and \( \lambda \) is the wavelength.\(^9\)

For these experiments, x-ray generation was from a radionuclide. The experiments used Iodine 125, which is a proton-rich radionuclide seeking to reduce its number of protons through the means of electron capture.

Electron capture is a means of decay which occurs when a nucleus is proton rich, but has less than 1.022 MeV of energy, the energy required for positron emission. In such cases, an electron from the K or L shell is captured and the proton is converted to a neutron coupled with the emission of a neutrino,

\[ p^+ + e^- \rightarrow n^0 + \nu_e. \]  

(2)

After this process has occurred, the atom has transitioned to a different element with one less proton and one more neutron. This newly formed element is in an excited state and must release energy in order to transition down to its ground state. This is done by either the emission of a gamma ray or internal conversion. The process of internal conversion and gamma ray emission are in competition with each other. For gamma ray emission the process is simple; a gamma ray of
the necessary energy is released from the nucleus in order to bring the nucleus out of the excited state. In internal conversion the energy of the transition between two nuclear energy levels is transferred to an orbital electron. The orbital electron is ejected from the atom with an energy equal to the energy of the transition minus the binding energy of the electron. Ejection from the K shell is the most probable, however ejection from different electron shells can occur. Both internal conversion and electron capture leave vacancies in the electron shells.  

Once electron capture and gamma ray emission or internal conversion have occurred, the newly created vacancies in the electron shells must be filled. These “holes” in the electron shells will now be filled by electrons from higher energy levels, creating either x rays or Auger electrons. Since these experiments were not done in vacuum, Auger electrons are quickly absorbed and not detected. For x-ray emission, the vacancies in the lower electron shells are filled by electrons from the higher electron shells creating characteristic x rays. 

Characteristic radiation is produced when an electron from a higher shell drops down to replace a vacancy. In order for this higher energy electron to occupy a place in a lower shell, it must release energy, which is done in the form of a photon. The energy of the resultant photon is 

$$ E_{characteristic} = E_o \left[ \frac{(Z - \sigma_1)^2}{n_1^2} - \frac{(Z - \sigma_2)^2}{n_2^2} \right], \quad (3) $$
where $E_0$ is the initial energy of an electron in the ground state ($E_0 = 13.6$ eV), $\sigma_1$ and $\sigma_2$ are the screening constants of the electrons, $n_1$ and $n_2$ are the energy levels of the two electrons, and $Z$ is the atomic number. Characteristic x-ray lines are named in the form $X\alpha$, where:

1. $X$ is the energy level which is being filled (example: K, L, M, …)
2. If the electron filling the vacancy is from the energy level directly above, $a$ is denoted as $\alpha$. If the electron is from 2 energy levels above, $a$ is denoted as $\beta$.

An example of this is if a vacancy in the K shell ($n = 1$) is filled by an electron in the L shell ($n = 2$), this would be a $K\alpha$. If a vacancy in the K shell were filled by an electron from the M shell ($n = 3$), the x ray would be a $K\beta$.

After electron capture occurs $^{125}\text{I}$ decays into $^{125}\text{Te}$, with 35.5 keV of energy to release, which is done either by the emission of a 35.5 keV gamma ray or with an internal conversion electron carrying 3.68 eV, the difference between 35.5 keV and one binding energy. The decay scheme for $^{125}\text{I}$ can be seen in Figure 6.

Most typically, the electron capture occurs with a K shell ($n = 1$) electron creating a vacancy in that shell, which is usually filled by an electron from the L shell, producing a $K\alpha$ characteristic x ray of Tellurium, with an energy.
of 27 keV. For the decay of $^{125}\text{I}$, there are approximately $6.67 \times 10^{-2}$ gamma rays produced per decay as compared with $7.41 \times 10^{-1}$ K\(\alpha\) characteristic x rays, which is approximately 11 times larger than the production of gamma rays. The spectra for the brachytherapy seeds used for these experiments can be seen in Figure 7.\(^{10}\)

It is important to note that, $^{125}\text{I}$ does not only produce gamma rays and K\(\alpha\) radiation. The isotope also produces radiation at other wavelengths such as the K\(\beta\) peak, the peak between the gamma ray peak and K\(\alpha\) peak in the figure below. For $^{125}\text{I}$, there are 1.46 counts per decay. The counts per decay was determine by summing the relative counts per decay for all particles with energies that are detectable by the Canberra HPGe detector.\(^{10}\)

![Figure 7. A spectrum of the brachytherapy seeds used for the experiments in this work. The plot is of x-ray counts vs. x-ray energy.](image)
2.2 Incoherent Scattering

Incoherent scattering, also called Compton scattering, is an interaction between a photon and a loosely bound electron. When this interaction occurs, radiant energy is absorbed by the electron (forcing it to recoil) and a photon with the remaining energy is emitted in a different direction, to conserve momentum. A schematic diagram of this interaction can be seen in Figure 8. Because of the loss in energy, the resultant photon will have a longer wavelength than the incident photon. This change in wavelength is dependant upon angle of scattering for a given target particle. Considering conservation of energy and momentum, this change in wavelength is

$$\lambda_f - \lambda_i = \Delta\lambda = \frac{h}{m_e c} (1 - \cos \theta)$$

where $h$ is Plank’s constant, $m_e$ is the rest mass of an electron, $c$ is the speed of light, $\lambda_i$ and $\lambda_f$ are the wavelengths of the incident and scattered photons, and $\theta$ is the angle between the direction of the incident and scattered photons.
2.3 Computed Radiography

Computed Radiography (CR) image plates use a phosphor of BaFBr and BaFI. Because of this mixture, the material is often referred to as barium fluorohalide. Typical image plates are composed of approximately 85% BAFBr and 15% BaFI, activated with a small quantity of Europium (Eu). This activation process, called doping, creates defects in the BaFBr crystals that allow electrons to be trapped more efficiently.

When an x ray’s energy is absorbed by the BaFBr phosphor the absorbed energy excites electrons associated with the Europium atoms, causing divalent Europium atoms (Eu$^{2+}$) to be oxidized and changed to the trivalent state (Eu$^{3+}$). The excited electrons become mobile, and some fraction of them interacts with a so-called F-center. An F-center is a crystallographic defect in which an anionic vacancy is filled by one or more electrons. The F-center traps these electrons in a high-energy, metastable state, where they can remain for very many hours, with some fading over time. The latent image that exists on the image plate after x-ray exposure, but before readout exist as billions of

![Figure 9. The process of image storage and retrieval on a computed radiography plate.](image)
electrons trapped in F-centers. The number of trapped electrons per unit area of the imaging plate is proportional to the intensity of the x-rays incident at each location during the exposure.

When an image plate is read, the plate is translated across a moving stage and scanned by a laser beam. When the red laser light scans the exposed image plate the red light is absorbed at the F-center, where the energy of the red light is transferred to the electron trapped in the F-center. The photon energy of the red light is less than that of the blue-green emission. However, the electron gains enough energy to reach the conduction band, enabling it to become mobile again. Many of these electrons then become de-excited by releasing blue-green light as they become reabsorbed by the trivalent Europium atoms, converting them back to the bivalent state. The blue-green emission is easily distinguished from the scattered red light in the reader. The process is illustrated in Figure 9.4

### 2.4 Polycapillary Optics

Polycapillary optics are bundles of small glass tubes, which can be used for multiple applications. These optics can be used to guide x rays given certain conditions, as sketched

![Figure 10. Schematic diagram of polycapillary optic.](image)

Since the angle of incidence for photon A is greater than the critical angle it will be absorbed by the septa and not transmitted through the optic, while photon A’s angle of incidence is less than the critical angle and will be reflected, allowing it to be transmitted through.
in Figure 10. An x-ray will be transmitted through a polycapillary if the angle of incidence between the x-ray photon and the capillary wall is less than the critical angle for total reflection. If the angle of incidence is greater than the critical angle then the photon will be absorbed. The performance of the optic is also dependent on the energy of the incident photon as well as the composition of the glass, and septal thickness, which affects fractional open area and thus efficiency. The efficiency of the optic can also be plagued by defects in manufacturing which may result in absorption of transmitted photons which would otherwise be reflected.  

2.4.1 Polycapillary Resolution

For an ideal conventional parallel-hole collimator, the object resolution (the size of a radioactive source from which the transmitted radiation could fall on a single point at the detector) would be given, as shown in Figure 11a, by

\[ R_{\text{parallel}} = d + 2z \tan \phi = (L_e + z) \frac{d}{L_e} \]  

where \( d \) is the diameter of the hole, \( z \) is

**Figure 11.** a) Resolution of an ideal parallel-hole collimator. b) A long polycapillary collimator allows reflection up to the critical angle, \( \theta_c \), which allows both paths, (dotted and solid lines) down the channel.
the object to collimator distance, $L_o$ is the effective length of the collimator which may be less than the geometric length because the septal material is penetrated by gamma rays,$^{11}$

$$L_e = L_{\text{actual}} - \frac{2}{\mu}, \quad (6)$$

where $\mu$ is the attenuation coefficient.$^{20}$

With polycapillary optics, due to the reflectiveness of the septa, paths such as the ones show in Figure 11b are made possible, as long as the angle of incidence, $\theta$, is less than the critical angle, $\theta_c$. The resolution becomes

$$R_{\text{longpoly}} \sim d + 1.5z \tan \theta_c \approx d + 1.5z \theta_c. \quad (7)$$

The factor of 1.5 is an empirical function of photon energy, derived from studies of focal spot sizes for focusing polycapillary optics.$^{7,8,9}$ $\theta_c$ can be calculated as

$$\theta_c = \sqrt{2 \delta}, \quad (8)$$

where $\delta$ is the index of refraction of the material. Due to waviness of the glass surface, actual paths inside the channels are more complex than show in Figure 11b.$^{7,8,9,12}$ The factor would in be closer to two if the all the rays were totally reflected up to the critical angle; however, in reality, rays with incident angles close to the critical angle are not totally reflected by the septa. Optics for which the reflectivity dominates the acceptance angle are called long polycapillary optics. This occurs when the acceptance angle, $\alpha$, $^{18}$

$$\alpha = \tan^{-1} \frac{d}{L_e}, \quad (9)$$
shown in Figure 12 is less than the critical angle
\[ \alpha < \theta_c. \] (10)

Since all three of the optics used in this work have an \( \alpha \) less than 1.6 mrad, the inequality (10) is satisfied. For the remainder of this work we shall make the approximations that the acceptance angle for short polycapillary optics \( (\alpha > \theta_c) \) is

\[ \alpha \approx \frac{d}{L_e}, \] (11)

and for long polycapillary optics \( (\alpha < \theta_c) \) is

\[ \alpha \approx 1.5 \theta_c. \] (12)

For a parallel-hole collimator, the field of view (FOV) is determined by the area spanned by the collimator array, which is proportional to the number of elements in the array, and is independent of the resolution.²
2.4.2 Sensitivity

The sensitivity, the fraction of photons that are transmitted through a collimator, is the product of two factors; a solid angle factor, and the area fill factor.\(^{12}\) The solid angle factor is dependant upon the acceptance angle, \(\alpha\), as seen in Figure 12. If a ray transmitted from a source lying on the centerline of the optic is to hit the optic at an angle less than the acceptance angle, the ray must fall with in the radius \(V\) of the centerline.

\[
V = z \tan \alpha \approx z \alpha \Rightarrow V_{shortparallel} \approx \frac{d}{L_e} . \tag{13}
\]

Thus the solid angle factor, the fraction of the \(4\pi\) emissions that are within the acceptance angle is,

\[
\Omega = \frac{\pi V^2}{4\pi z^2} = \frac{1}{4} \alpha^2 \Rightarrow \Omega_{shortparallel} = \frac{1}{4} \left( \frac{d}{L_e} \right)^2 . \tag{14}
\]

The solid angle factor is reduced by \(f\), the fraction of the front face of the collimator which is open,\(^{13}\)

\[
f = 4K \left( \frac{d}{d + t} \right)^2 . \tag{15}
\]
Where \( t \) is the distance between holes, which in the case of polycapillary arrays will be the septal thickness, and \( K \) is a geometric factor depending on the shape of the holes, which is approximately 0.25. When \( t \ll d \), as is usually the case with polycapillary optics,

\[
f = 4K\left(\frac{d}{d + t}\right)^2 = 4K\left(\frac{1}{1 + \frac{t}{d}}\right)^2 = 4K\left(1 - \frac{t}{d}\right)
\]

Taking the product of the fractional open area and the solid angle factor gives the total geometric sensitivity for an ideal parallel-hole collimator to be,

\[
S \approx \frac{1}{4} \alpha f
\]

Since both the resolution and sensitivity depend on \( d \) and \( L \), the sensitivity depends on the resolution, \( R \)^2. Following the approximation given by Equation 12,

\[
S_{\text{longpoly}} = \frac{1}{4} \alpha^2 f \approx \frac{1}{4} \theta_e^2 f
\]

and making the approximation given by Equation (13),

\[
S_{\text{shortpoly}} = \frac{1}{4} \alpha^2 f \approx \frac{1}{4} \left(\frac{d}{L_e}\right)^2 f = \frac{1}{4} \left(\frac{R}{L_e + z}\right)^2 f
\]
thus, for long polycapillary arrays, the sensitivity is independent of resolution. Therefore, for optics with a small d, a very high-resolution collimator, the sensitivity of a polycapillary optic will be higher than that of a conventional parallel-hole collimator.\textsuperscript{2}

For long polycapillary arrays, the effective fractional open area ($f_{\text{effec}}$) may be less than the geometric value if the optic is slightly bent, or if there is partial absorption of photons near the critical angle due to waviness or roughness in the septa. Also, $f_{\text{effec}}$ may be lower because the reflectivity of the septa is slightly less than unity for angles just below the critical angle or because all materials have a non-zero absorption coefficient.\textsuperscript{5}

2.4.3 Septa Penetration

Background in images made with polycapillary arrays using radiographic detectors comes from two sources, from rays which cut through the optic rather than being properly absorbed in the collimator and from Compton scatter which were not sufficiently “diluted” by the small pixel size. Parallel-hole collimators are conventionally designed so that the minimal path length through the septa gives adequate stopping power. The wall thickness is typically chosen so that the transmission for the design, the minimum path length, is some small percentage such as five percent. The minimum path length, $w$, is generally drawn as the path through a wall of thickness $t$ that crosses two holes is,\textsuperscript{15}
\[
\frac{t}{w} = \frac{t + 2d}{\sqrt{L^2 + (t + 2d)^2}} \approx \frac{t + 2d}{L} \Rightarrow w \approx L \frac{t}{t + 2d},
\] (20)

where the last approximation comes from Equation (16) for small septal width, \(t/d\).

Thus, the penetration \((\beta)\) depends exponentially on the length of the optic and the fraction of the face of the optic that is filled, \(1 - f\), and is defined as,

\[
\beta = e^{-\mu L'(1-f_{effc})} \approx \beta_o \equiv e^{-\mu L(1-f_{effc})}.
\] (21)

Where \(L'\) is some path length through the collimator as shown in Figure 5.

For this work the photon energy was 27 keV and the polycapillary was constructed of borosilicate glass with a mixture of lead oxide. From traditional calculations the absorption length is expected to be approximately 0.25 mm depending on the exact concentration and density.\(^{14}\) However, the geometry of the septa is complex, as the holes are rounded hexagons alternating with small circles, as shown in Figure 3. To obtain an estimate of the required collimator length, rather than using minimum path length approximations, measurements of penetration with a pencil beam input were made for polycapillary collimators using collimators containing 0% and 30% PbO by weight using an external x-ray source and an energy resolving photon counting detector.\(^{15}\)
2.5 Bayesian Data Analysis

In this work, the use of Bayesian data analysis and nested sampling algorithms were employed in order to verify the resolution, total flux, and background levels of the images taken in this work. Additionally, the algorithm was used to determine thickness of the iodine coating within brachytherapy seeds, which the manufacturer, Best Medical, did not supply.

The nested sampling algorithm used in this work relies on Bayes’ theorem, which can be expressed as

\[
\text{prob}(\text{hypothesis} \mid \text{data}, \mathcal{I}) = \frac{\text{prob}(\text{data} \mid \text{hypothesis}, \mathcal{I}) \times \text{prob}(\text{hypothesis} \mid \mathcal{I})}{\text{prob}(\text{data} \mid \mathcal{I})}. \tag{22}
\]

Where the vertical bar ‘\( \mathcal{I} \)’ means ‘given’ (so that all items to the right of this are regarded as true) and \( \mathcal{I} \) is all previous knowledge about the system.

The power of Bayes’ theorem lies in the fact that it relates the quantity of interest, the probability that the hypothesis is true given the data, to the term that we have a better chance of being able to assign, the probability that we would have observed the measured data if the hypothesis is true.

The various terms of Bayes’ theorem have formal names. The quantity \( \text{prob}(\text{hypothesis} \mid \mathcal{I}) \), is called the prior probability, \( \pi \); it represents the state of knowledge about the truth of the hypothesis before the current data has been analyzed. This is modified by the experimental measurements through the likelihood function \( \mathcal{L} \), \( \text{prob}(\text{data} \mid \text{hypothesis}, \mathcal{I}) \), and yields the posterior probability \( P \), \( \text{prob}(\text{hypothesis} \mid \text{data}, \mathcal{I}) \), representing our state of knowledge.
about the truth of the hypothesis in light of the data. Finally, the evidence $Z$, $\text{prob} (\text{data} \mid I)$, is a normalizing factor and can be thought of a way of updating the posterior probability.$^{16}$

### 2.5.1 Computational Problem of Nested Sampling

In the case of nested sampling we will use the concept of Bayes’ theorem to estimate the values of unknown parameters of the brachytherapy seeds imaged in this work and to test the validity of the model developed for determining the resolution of the images acquired.

For a given system with parameters $x$ and data $D$ one can examine the probability of the hypothesis that parameters $x$ are correct and obtaining the given data using the product rule as,

$$\text{prob} (D \mid x,I) \text{prob} (x \mid I) = \text{prob} (x,D \mid I) = \text{prob} (D \mid I) \text{prob} (x \mid D,I)$$

$$\mathcal{L} (x) \pi (x) = \text{Joint} = Z \mathcal{P} (x)$$

Likelihood $\times$ Prior $= \text{Joint} = \text{Evidence} \times \text{Posterior}$

inputs $\Rightarrow \ldots \Rightarrow$ outputs

We must also define how to calculate the evidence, which can be done using the equation

$$Z = \iiint \ldots \int \mathcal{L} (x) \pi (x) \, dx. \quad (23)$$

Additionally we will define the function $\xi (\lambda)$, which is the proportion of prior with likelihood greater than $\lambda$,

$$\xi (\lambda) = \iiint \ldots \int d\xi = \int \ldots \int \pi (x) \, dx, \quad (24)$$
in which the element of prior mass is
\[ d\xi = \pi(x) \, dx. \] (25)

Using equations (24) and (25), the evidence of equation (23) is the sum over the likelihood and the prior can then be rewritten as
\[ Z = \int_{\xi_{\text{min}}}^{\xi_{\text{max}}} L(\xi) \, d\xi, \] (26)

and is the area enclosed in Figure 13.

### 2.5.2 Basic Principles of Nested Sampling

Nested sampling uses a collection of \( n \) sets of the parameters \( x \), randomly sampled from the prior, \( \pi \), but also subject to the constraint \( L(x) > L^* \) preventing the likelihood from falling below the current limiting value \( L^* \).

In terms of \( \xi \), the objects are uniformly sampled subject to the constraint \( \xi < \xi^* \), where \( \xi^* \) corresponds to \( L^* \).

At the onset, sampling is uniform over the entire set, meaning that \( \xi^* = 1 \) and \( L^* = 0 \). The idea is to then iterate inwards on \( \xi^* \) and in turn, upwards on \( L^* \), in
order to locate and quantify the small region of high likelihood as can be see in Figure 13.

Upon entrance to an iteration n objects (an object is a set of parameters) are held with the restriction \( \xi < \xi^* \) as shown in Figure 14a. The worst of these objects, being the one with the lowest likelihood and therefore the highest \( \xi, \) is selected; this object will lie about one part in \( n \) less than \( \xi^* \). Iteration proceeds by using the worst object’s \((\xi, L)\) as the new constraints \((\xi^*, L^*)\). Once the new constraints have been set, the worst object is then discarded, as it no longer meets the restraints. There are now \( n - 1 \) surviving objects, still disturbed uniformly over \( \xi \) but confined to smaller domain; bounded by the new \( \xi^* \) constraint. The next step is to generate a replacement object, sampled uniformly over the prior but constrained within this reduced domain. Using Markov Chain methods, this is accomplished by replacing the worst object with a randomly selected copy of one of the surviving objects. However, an independently sampled new object is needed, which is

**Figure 14.** An iteration process in which the worst object is replaced inside the contracted domain. a) The four dots represent the likelihood for parameter sets within the constrained by \( \xi^* \) and therefore \( L^* \). The worst of the data sets is determined as the parameter set closest to the \( \xi^* \) constraint, and therefore has the lowest likelihood. b) The worst object is then removed from the set leaving 3 objects (\( n - 1 \)) and the \( \xi^* \) constraint is reduced to the \( \xi \) value from the now removed object. c) A new object is then created by uniformly sampling from the prior but constrained within the reduced domain.
accomplished by moving sufficiently far away such that memory of the starting point is lost, but not so far that the new object is outside of the constraints. The process of moving sufficiently far away from the copy is accomplished by taking twenty random “steps” away from copy. Having done this, the iteration again holds n objects restricted to $\xi < \xi^*$, just as upon entry, except the domain is shrunken by 1 part in n. This process is illustrated in Figure 14.

Successive iterations generate a sequence of discarded objects on the edges of progressively smaller nested domains. At iterate k,

$$L_k = L^*$$

(27)

and

$$\xi_k = \xi^* = \prod_{j=1}^k t_j,$$

(28)

where $t = \xi / \xi^*$. Ignoring uncertainty, we can state each log t to be -1/n so that $\xi_k = \exp(-k/n)$.

The evidence is evaluated by associating with each object in the sequence a width $h = \Delta \xi$, and hence a vertical strip of area $A = h \cdot L$, whence

$$Z \approx \sum_k A_k,$$

(29)

where

$$A_k = h_k L_k$$

(30)

and where the simplest assignment of the width is

$$h_k = \xi_{k-1} - \xi_k.$$

(31)
3. Experimental Results

The seeds were placed in 0.8 mm grooves machined 3 mm apart in a 25 x 25 x 3 mm Lucite block. A second 25 x 25 x 3 mm un-grooved Lucite block was placed on top of the block containing the seeds and was then sealed shut using plastic tape. The seed arrangement can be seen in Figure 15, and the experimental set up may be seen in Figure 5. Encouraging images of the seeds were obtained using all three optics. Most of the experiments were conducted using Optic # 4812; similar experiments were carried out using Optic # 4932, which has larger channel sizes, to observe the effect on resolution and background as well as the feasibility of larger channels, which are easier to manufacture. In addition, images were taken with Optic # A326 in order to compare results with previous work. The dimensions of these optics can be found in Table I. Both Optic # 4812 and 4932 were longer than Optic # A326 and higher percentage of lead concentration in order to cut down on penetration through the optic and increase signal–to–background ratios.

Figure 15. The seeds are arranged in alternating rows of 2 and 3, and the seeds are approximately 6 mm apart. a) Empirical image of seed arrangement. b) Schematic diagram of seed arrangement.
3.1 Intensity Measurements and Calculations with $^{125}$I Brachytherapy Seeds

Using the geometry shown in Figure 5 the seed array was placed a distance $z$ from the collimator. The seeds were small titanium capsules 5 mm in length, 0.8 mm wide\(^{17}\), which incased a plastic cylinder with $^{125}$I deposited on it, which were donated by Best Medical. A schematic diagram of a brachytherapy seed can be seen in Figure 16. When originally obtained the seeds were labeled by Best as having a strength of 0.3 mCi per seed. The half-life of $^{125}$I is 60 days.

A sheet of lead with a 20 mm diameter aperture was placed between the seed array and the collimator to ensure that no X rays went around the collimator, and an image plate was placed behind the collimator.

The $^{125}$I spectrum and intensity were measured with a large area, non-imaging, Canberra HPGe, High Purity Germanium, detector with and without a collimator in place. To prevent dead time

<table>
<thead>
<tr>
<th>Optic #</th>
<th>Outer Diameter (mm)</th>
<th>Channel Diameter ($\mu$m)</th>
<th>Length (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4812</td>
<td>28</td>
<td>26</td>
<td>30</td>
</tr>
<tr>
<td>4932</td>
<td>12.6</td>
<td>45.8</td>
<td>30</td>
</tr>
<tr>
<td>A326</td>
<td>30</td>
<td>30</td>
<td>20</td>
</tr>
</tbody>
</table>

Table 1. A listing of the optics used in these experiments and their dimensions.
in measuring the intensity and spectrum, a second aperture was placed directly in front of the detector with a diameter of 14 millimeters.

From simple geometry, in the absence of a collimator, the count rate \( C_0 \), in the geometry of Figure 5 can be calculated as

\[
C_0 = 0.97NF \frac{\pi r^2}{4\pi y^2},
\]

(32)

where \( N \) is the number of sources, \( F \) is the emission rate in counts per second, \( r \) is radius of the limiting aperture, and \( y \) is the source to detector distance. Experimentally \( y \) is taken as,

\[
y = y_{\text{meas}} + y_D,
\]

(33)

where \( y_{\text{meas}} \) is the measured source to detector distance, and \( y_D \) is the distance from the outside case of the detector to actual sensitive area of the detector. It is important to notice that the unit for \( F \) is counts per second, where as the seed strength is given in units of Curies, which is decays per second. This is significant because for many isotopes there maybe more than one photon emitted per decay, as is the case for \(^{125}\)I.

Using Equations (32) and (33) with a source to detector distance of 252 mm, a \( y_D \) of 5 mm, and an aperture radius of 3.5 mm, the expected number of counts in a 100 second collection time was calculated. The calculated and measured counts

<table>
<thead>
<tr>
<th>Counts</th>
<th>Measured ( y = 257 , \text{mm} )</th>
<th>Calculated ( y = 257 , \text{mm} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( 197411 \pm 444 )</td>
<td>( 199714 \pm 447 )</td>
</tr>
</tbody>
</table>

Table II. Measured and calculated number of counts for a 100 s collection time with a 7 mm radius aperture a source to detector distance of 257 mm and no optic in place.
can be seen in Table II. The seeds were received on March 28\textsuperscript{th}, 2008 and spectrum measurements were taken on July 15\textsuperscript{th}, 2008. In order to compensate for the reduction in strength of the seeds, a decay factor of 0.274 was used. The uncertainty in measurements can be attributed to Poisson statistics and can be taken as

$$\Delta N = \sqrt{N}.$$  \hspace{1cm} (34)

The percent error between the measured and calculated values is 1.15 percent. In addition to Poisson statistics, which gives a small percentage of uncertainty due to the large number of counts, there is some error attributed to air absorption, scatter, and the fact that the Canberra HPGe detector has a stated efficiency of 97\%. Alternatively, when calculating the seed strength from measured number of counts, the seeds had a measured strength of $300 \pm 1 \mu \text{Ci}$ per seed as compared with the given strength of 300 $\mu \text{Ci}$ per seed.

The primary counts through the collimator will be

$$C_{primary} = NFS = NF \frac{1}{4} \alpha^2 f_{effc},$$  \hspace{1cm} (35)

where the sensitivity, $S$, was substituted from Equation (17), $\alpha$ is the acceptance angle, and $f_{effc}$ is the effective fractional open area. Since all the optics used in this work are long polycapillary optics,

$$C_{primary} = NF \frac{1}{4} \alpha^2 f_{effc} \approx NF \frac{1}{4} \Omega_c^2 f_{effc},$$  \hspace{1cm} (36)
where the substitution for $\alpha$ follows from Equation (18), and $\theta_c$ is the critical angle.  

In addition to the primary intensity, the detector will also collect air scatter and penetrating radiation, which is radiation that has not been sufficiently blocked by the lead glass. The amount of penetrating radiation can be calculated by

$$C_{\text{penetrate}} = \beta C_o = \beta NF \frac{\pi r^2}{4\pi y^2}. \tag{37}$$

Where $\beta$ is the penetration, and $C_o$ is the count rate with no optic in place. The substitution for $C_o$ is taken from Equation (32).

Combining these Equations (35) and (37) the total count rate through the collimator is

$$C_{\text{total}} = C_{\text{primary}} + C_{\text{penetrate}} = NF \frac{1}{4} \alpha^2 f_{\text{effec}} + \beta \left(NF \frac{\pi r^2}{4\pi y^2}\right). \tag{38}$$

The penetration was assumed to be dominated by rays which pass through the collimator with a path length $L' \sim L$ so that $\beta \sim \beta_o. \tag{39}$

In order to approximate a value for the penetration, spectra were taken using a molybdenum x-ray source and a Canberra HPGe detector with and without optic #4812 and 4932, spectra were not necessary for optic A326 as the penetration for this optic had already been calculated. The penetration for the two optics was approximated as

$$\beta_o = \frac{C_{\text{with}}}{C_{\text{without}}}, \tag{39}$$
where $C_{\text{with}}$ is the number of counts with the optic in place and $C_{\text{without}}$ is the number of counts without the optic in place. For the measurement done with the optic in place, the optic was placed in such a way that it was not aligned with the source as to ensure observed counts would not be from normal transmission through the capillaries. For optic # 4812 the penetration was found to be $(2.0 \pm 0.3) \times 10^{-5}$ and for optic # 4932 the penetration was found to be $(2.8 \pm 0.4) \times 10^{-5}$. The relatively high error in both results can be attributed to Poisson statistics because the number of counts for both experiments with the optics in place was very low. Additionally, when calculating the theoretically expected value for $\beta$ for both optics using Equation (21), the value was found to be $2.0573 \times 10^{-30}$; this result would imply that the counts observed with both optics in place for the penetration calculations were actually in fact due in fact not to cut through but rather Compton scatter from the air and the glass of the optics.

### 3.2 Seed Images

Using the set up shown in Figure 5, images were taken of the seed array with the three optics listed in Table I with varied exposure times, source to optic distances, optic to image plate distances, with different thicknesses of tissue equivalent material between the seed array and the optic, as shown in Appendix A. The image plate used in these experiments was a Fujifilm high sensitivity computed radiography plate with 50 µm pixels. The image plate was read using a Fujifilm image plate reader and the images were then analyzed using Fujifilm Image Gauge software. A resulting image taken using optic # 4812 can be seen
in Figure 17 with a profile through one of the seeds. As can be seen in the image, some of the seeds are not completely visible, because the aperture had to be made smaller than the seed array to prevent x rays from going around the optic. Figure 18 shows a simulated image of a source buried within a patient, by employing 20 mm of tissue equivalent material between the seeds and the collimator. Figure 19 shows an image taken with optic # 4932. Only part of a second seed is visible, again this is due to the fact that the aperture in front of the optic had to be make smaller than the optic to minimize cut through on the edges of the optic. Lastly, Figure 20 shows an image taken with optic # A326. This image was taken for comparison, as this optic was used in previous works. There are some distortions evident in this image, which can be attributed to the fact that this optic was created by gluing together a group of smaller polycapillary arrays. It may also be observed from the images below that the iodine is not evenly distributed around the plastic cylinders. It should also be noted that in the images below, the numbered lines drawn through one seed in each of the figures are the tools actually used to generate the intensity profiles. The lines shown in the images below however are simply drawn into the picture to show the process. However, it can be observed from these “dummy” profile lines that the task of drawing the profile through the seed proved to be a very difficult task as it is very difficult to draw the profile perfectly perpendicular to the seed. Additionally, due to the variable deposition of the iodine, it was difficult to consistently draw a profile through the area of a seed with the highest concentration of iodine.
Lastly, in all of the images below it can be seen that there is a double peak at the top of the intensity profiles. This is because the intensity, and thus the number of counts (x rays detected), in the image is proportional to the thickness of the iodine coating. As shown in Figure 21 the iodine is deposited on a cylinder; as a result, there is a larger amount of iodine at the edges of the seed, thus increasing the number of counts and causing the double peaks.

![Figure 17](image1.png)

**Figure 17.** Image taken with optic # 4812 with a seed-to-optic distance of 89 mm, an optic to image plate distance of 5 mm, and 3 mm of tissue equivalent material. The maximum signal value was 0.932 PSL and the average background signal in the image was 0.011 PSL. The Signal to Background ratio for the image was 87. From the image it is clear that radionuclide is not evenly distributed through the seed. An intensity profile was taken horizontally through the seed in the area of the highest concentration of the radionuclide with 50 µm pixels.

![Figure 18](image2.png)

**Figure 18.** Image of $^{125}$I brachytherapy seeds with optic # 4812. An intensity profile through one of the seeds can be seen to the right of the image. For this image there was 24 mm of tissue equivalent material in place, a seed to optic distance of 54 mm, an optic to image plate distance of 38 mm, and a seed to image plate distance of 150 mm. The measured signal to background ratio of this image was 30. The image was taken over a 17-hour exposure time.
Figure 19. Image taken with optic # 4932 with a seed to optic distance of 72 mm, an optic to image plate distance of 20 mm, 3 mm of tissue equivalent material, and a 17 hour exposure time. The maximum signal value was 0.894 PSL, the average background value was 0.012 PSL and the Signal to Background ratio was 75. Only one seed was imaged because the outer diameter of the optic was 12.6 mm and a smaller aperture was used to minimize the background radiation in the image making it more difficult to align the seed array with the optic.

Figure 20. Image taken with optic # A326 with a seed to optic distance of 95 mm, optic to image plate distance of 25 mm, 3 mm of tissue equivalent material, and a 6 hour exposure time. The maximum signal value was 0.287 PSL, the average background value was 0.007 PSL and the Signal to Background ratio was 38. The image above is slightly distorted because this optic was created by gluing multiple polycapillary optics together.
3.3 Image Resolution

The sensitivity of the image plate used in these experiments is suboptimal for 27 keV X rays and also had a fairly large, 50 µm readout pixel size, but it still serves to show the feasibility of microscintigraphy with a non-energy sensitive detector.

Two methods were employed to determine the resolution of the images obtained. The first techniques used were traditional calculations using full width half maximums, however, additionally a nested sampling Markov Chain Monte Carlo Algorithm was also used.

3.3.1 Resolution Calculations Using Full Width Half Maximum

In order to estimate the resolution of the images, intensity profiles of features were obtained using the Fujifilm Image Gauge software. For a first estimate the full width half maximum of the derivative of the intensity profile through a seed was taken as the resolution of the image. The resolutions of the images were in fair agreement with theoretical calculations theoretical calculations from equation (7). Results from the fits can be seen in Table III. This is an overestimate of the

![Figure 21. Cross sectional views of a brachytherapy seed for the geometries used to derive equation (40). a) The geometry for the thickness of the iodine coating when the position for the profile is between the two edges of the plastic cylinder. b) The geometry for the thickness of the iodine coating when the position for the profile is between the titanium capsule and the plastic cylinder.](image)
actual blur, as the true intensity profile is not a step function, however the
resolution was limited by the large pixel size of the image plate, and the
distribution of radioactive material within the seeds.\textsuperscript{2} While the results were
equally encouraging, the parameter values for the fit were chosen manually so resolution
calculations were also done using a Markov Chain Monte Carlo algorithm.

<table>
<thead>
<tr>
<th>Filename</th>
<th>Gaussian Resolution (mm)</th>
<th>Theoretical Resolution (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optic_close_ip_4812_2hr_0520</td>
<td>0.216</td>
<td>0.213</td>
</tr>
<tr>
<td>Optic_close_ip_4812_2hr_0610</td>
<td>0.2</td>
<td>0.213</td>
</tr>
<tr>
<td>Optic_close_ip_4812_4hr_0521</td>
<td>0.216</td>
<td>0.213</td>
</tr>
<tr>
<td>Optic_close_ip_4812_4hr_0611</td>
<td>0.25</td>
<td>0.213</td>
</tr>
<tr>
<td>Optic_close_ip_4812_6hr_0522</td>
<td>0.25</td>
<td>0.213</td>
</tr>
<tr>
<td>Optic_close_ip_4812_6hr_0612</td>
<td>0.225</td>
<td>0.213</td>
</tr>
<tr>
<td>Optic_close_ip_4812_12hr_0612</td>
<td>0.216</td>
<td>0.213</td>
</tr>
<tr>
<td>Optic_close_ip_4812_17hr_0611</td>
<td>0.233</td>
<td>0.213</td>
</tr>
</tbody>
</table>

\textbf{Table III.} A comparison of the FWHM values from derivative fits
to intensity profiles from experiments and the theoretically
expected resolutions from equation (7).

\textbf{Figure 22.} A derivative fit plotted with data from “Optic\_close\_to\_ip\_4812\_2hr\_0520.xls”. The fit is in
good agreement with the edge of the profile, however the values for the fitting parameters were chosen
manually.
3.3.2 Resolution Calculations Using Nested Sampling

A simulated intensity profile was generated to account for the actual profile, as well as to determine the seed coating thickness (the thickness of the $^{125}$I impregnated on the seed), because Best Medical did not provide this information. The intensity was taken as proportional to the apparent thickness $L$, of the iodine impregnated coating on the seed as shown in Figure 21,

$$I(x) = \begin{cases} 
2F\left(\frac{D'}{2}\right)^2 - (x-x_o)^2 - \left(\frac{D'}{2} - c\right)^2 - (x-x_o)^2 & |x-x_o| < \left(\frac{D'}{2} - c\right) \\
2F\left(\frac{D'}{2}\right)^2 - (x-x_o)^2 & \frac{D'}{2} - c \leq |x-x_o| < \frac{D'}{2}, \\
\text{background} & \text{else}
\end{cases}$$

(40)

where $x_o$ is the location of the center of the bead, $F$ is the flux of the seeds, $c$ is the seed coating thickness, $\sigma$ is the resolution of the image, and $D'$ is the effective diameter of the bead due to blur. $D'$ is calculated as

$$D' = D + \Delta D,$$  

(41)

where $D$ is the actual bead diameter and $\Delta D$ is given by

$$\Delta D = (h + z)\theta_c.$$  

(42)

Where $h$ is the optic to detector distance, $z$ is the seed to optic distance, and $\theta_c$ is the critical angle of the optic. The intensity profile was then convoluted with a Gaussian profile,
\[ H(x) = G(x) * I(x) = \int_{-\infty}^{\infty} G(\tau)I(x - \tau)d\tau, \quad (43) \]

where

\[ G(x) = e^{-\frac{(x-x_0)^2}{2\sigma^2}}. \quad (44) \]

A plot of the intensity profile from Equation (40) can be seen in Figure 23. Also, a plot of the intensity profile after being convolved with Equation (44) can be seen in Figure 24.

\[ \]

**Figure 23.** A plot of a simulated intensity profile generated using Equation (40). The equations were derived using the geometries shown in Figure 21.
The thickness of the iodine coating and resolution were taken as fitting parameters, the diameter of the seeds was determined by measuring it with a set of calipers and was then confirmed using the Best Medical Website. The diameter of the seeds was found to be 0.8 mm. Additionally, the position of the center of the seed in the intensity profile, the background noise in the image, and the Flux of the seeds were taken as fitting parameters in order to obtain more precise values for $c$ and $\sigma$.

In order to accurately determine the values for the fitting parameters, the use of a Monte Carlo Markov Chain Nesting Sampling Algorithm was employed. The program is designed so that it runs until the difference in the evidence between iterations is less than $1 \times 10^{-10}$. The difference is chosen to be such a
small value to ensure that sufficiently accurate values for the parameters are found; this is confirmed by plotting the log of the likelihood function vs. the number of iterations as well as the log weight vs. the number of iterations, which can be seen in Figure 26 and Figure 25. A copy of the computer code used to perform the calculations can be referred to in Appendix B.

The program works by initially calling the function Main2(). Which initialize some of the variables and is the hub for the algorithm. From there, the function Apply() is then called to define structs Try, Samples, and Objects. The struct Objects holds 100 samples and are the samples that are used through most of the program and the process as discussed in Section 2.5.2. The struct Try is used in finding a new sample to replace \( \xi^* \), which is the worst sample held in Objects again as discussed in Section 2.5.2. Samples is the struct which is used to hold the value of a sample from Objects which is used at the end of the program to calculate the weighted average values of the fitting parameters. Within each struct the variables c, back (the background noise in each individual image), sigma (the variable for \( \sigma \)), xo (the seed center, \( x_0 \)), logWt (which is the log of the weight as defined by
Equation(30)), and logL (the log of the likelihood) are defined for each image that is calculated in the program. Additionally, \textit{Apply()} sets minimum and maximum values for all of the parameters. Lastly, the function determines the number of data points that are in each profile to be analyzed.

From there, the function \textit{Prior()} is then called. \textit{Prior()} is used to set the struct \textit{Objects} to a set of random uniformly distributed sample values for all of the parameters; this is done by multiplying the range of acceptable values for each parameter by a random number and then adding that value to the minimum value for the parameter. Once initial values are set for each parameter, the function \textit{IntensityConvRev()} is called which generates a simulated intensity profile using the starting and ending positions of the intensity profile (which were defined in \textit{Apply()}), the number of samples in the image, and the now sampled values for the iodine coating, the background noise, and the resolution of the image. The profile created by \textit{IntensityConvRev()} uses equations (40), (43), and (44).
simulated intensity profile is then passed to the function \( \text{logLhood()} \) along with variance calculated from each intensity profile. The log of the likelihood is then calculated for the parameter set by,

\[
L = \text{prob}(x \mid \mu, \sigma) = \frac{1}{\sigma \sqrt{2\pi}} \exp\left[ -\frac{(x - \mu)^2}{2\sigma^2} \right].
\] (45)

Therefore following from the joint probability, the log of the likelihood for all sets is,

\[
\log L = \log_e \left[ \text{prob}(x_k \mid \mu, \sigma) \right] = \sum_{j=1}^{M} \left( \log_e \left( \frac{1}{\sigma_j \sqrt{2\pi}} \right) - \sum_{k=1}^{N} \left( \frac{x_{jk} - \mu_{j,k}}{2\sigma_j^2} \right) \right),
\] (46)

where \( x \) is the intensity profile generated by the sampled parameter set, \( \mu \) is the intensity profile from experiment, \( \sigma \) is the noise in the image, approximated as the average background from the experiment, \( \sigma \) is the square root of \( \sigma \), \( N \) is the number of data points in the intensity profile from the image, and \( M \) is the number of data sets from images are being compared. All of this information is then returned to \( \text{Main2()} \). From there a while loop is then implemented which runs until the difference in the log of the evidence from iteration to iteration is greater than \( 10^{-10} \). Within the while loop, the worst sample in Objects is found and set as the limiting constraint. The Information and log evidence are then calculated as well as the log weight for the worst sample in Objects, after which the struct Samples is updated with the values for the parameters with the values
from the worst sample from Objects. This is done because a sample needs to be selected from each iteration.

Once Samples has been updated another sample from Objects is randomly sampled and copied to replace the worst sample in Objects and the value of $L^*$ is updated to be the log likelihood of the worst object which has just been replaced. So that the replacement sample is not just a copy of a surviving object, the function $Explore()$ is then called and the now copied object is passed to the function in order to do a “random walk” away from the copy. Within $Explore()$, the object is copied into the struct Try, additionally initially a step size is defined as 10% of the range for each of the parameters. This step size is then multiplied a normalized random number generator, which is then added to the value of one of the parameters in Try, once it is confirmed that this updated value for the parameter is within the bounds for the parameter, the simulated intensity profile is generated and again the log of the likelihood is calculated. If the log likelihood is greater than $L^*$ then the Object value is overwritten and is set equal to the value of the parameters in Try. This process is repeated 20 times for each parameter in order to move the value of parameter sufficiently far away from the original value of the copied object. Also, the step size is varied after every iteration such that the success for every iteration is about 50%. Once this process is completed, the updated value of the object is returned back to $Main2()$. This process is continued until the difference between log of the evidence between iterations is $1 \times 10^{-10}$; once this criteria is met, the while loop is
ended. The values for the evidence and information are then outputted, and the function \textit{Results()} is called. Within \textit{Results()}, the weighted averages are calculated for all the parameters, and plots are generated overlaying the intensity profiles obtained from experiments with the estimated values of the parameters for the image. Also, plots for log of the weight vs. the number of iterations, which is defined as the evidence in Equation (32), and the log of the likelihood vs. the number of iterations. These plots can be seen in Figure 26 and Figure 25.

The seed coating after adjusting for blur from distance 0.291 ± 0.028 mm, the resolution was 0.226 ± 0.018 mm. Average values for the background and Flux are not listed, as these variables varied with time because the source used was a radionuclide and therefore decreased with time. A list of the full results can be seen in Table IV. The results for the resolution were in fair agreement with the theoretical results from Equation (7). It should be noted that the results for the flux had very high levels of uncertainty. In all cases the uncertainty was higher than the results themselves. This is believed to be because the peaks of many of the images were so noisy that the algorithm had difficulty distinguishing between noise and actual results causing the high uncertainties. Graphs comparing intensity profiles from experiments and simulations can be seen in Figure 27.
### Table IV

A table of results analysis from fits that were done using the MCMC algorithm, where the theoretical resolutions from the final column were calculated using Equation (7).

<table>
<thead>
<tr>
<th>Filename</th>
<th>Seed Diameter with Blur (mm)</th>
<th>Simulated Seed Coating with Blur (mm)</th>
<th>Simulated Seed Coating without Blur (mm)</th>
<th>Simulated Flux (AU)</th>
<th>Simulated Background (AU)</th>
<th>Simulated Resolution (mm)</th>
<th>Theoretical Resolution (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optic_close_ap_cloth_17hr_0424</td>
<td>0.9205</td>
<td>0.462 ± 0.003</td>
<td>0.3048</td>
<td>0.117 ± 0.040</td>
<td>0.0002 ± 0.0001</td>
<td>0.012 ± 0.002</td>
<td>0.132</td>
</tr>
<tr>
<td>Optic_close_ip_4812_2hr_0520</td>
<td>0.9572</td>
<td>0.455 ± 0.009</td>
<td>0.2978</td>
<td>0.129 ± 0.035</td>
<td>0.003 ± 0.006</td>
<td>0.115 ± 0.008</td>
<td>0.213</td>
</tr>
<tr>
<td>Optic_close_ip_4812_2hr_0610</td>
<td>0.9572</td>
<td>0.403 ± 0.012</td>
<td>0.2458</td>
<td>0.248 ± 0.049</td>
<td>0.006 ± 0.002</td>
<td>0.069 ± 0.014</td>
<td>0.213</td>
</tr>
<tr>
<td>Optic_close_ip_4812_4hr_0521</td>
<td>0.9572</td>
<td>0.401 ± 0.001</td>
<td>0.2438</td>
<td>0.211 ± 0.057</td>
<td>0.005 ± 0.0005</td>
<td>0.100 ± 0.004</td>
<td>0.213</td>
</tr>
<tr>
<td>Optic_close_ip_4812_4hr_0611</td>
<td>0.9572</td>
<td>0.463 ± 0.003</td>
<td>0.3058</td>
<td>0.133 ± 0.036</td>
<td>0.004 ± 0.0006</td>
<td>0.133 ± 0.007</td>
<td>0.213</td>
</tr>
<tr>
<td>Optic_close_ip_4812_6hr_0522</td>
<td>0.92052</td>
<td>0.399 ± 0.001</td>
<td>0.2418</td>
<td>0.240 ± 0.048</td>
<td>0.006 ± 0.002</td>
<td>0.082 ± 0.004</td>
<td>0.213</td>
</tr>
<tr>
<td>Optic_close_ip_4812_6hr_0612</td>
<td>0.9572</td>
<td>0.454 ± 0.008</td>
<td>0.2968</td>
<td>0.246 ± 0.049</td>
<td>0.006 ± 0.0001</td>
<td>0.086 ± 0.008</td>
<td>0.213</td>
</tr>
<tr>
<td>Optic_close_ip_4812_12hr_0612</td>
<td>0.9572</td>
<td>0.463 ± 0.003</td>
<td>0.3058</td>
<td>0.186 ± 0.043</td>
<td>0.003 ± 0.0004</td>
<td>0.112 ± 0.004</td>
<td>0.213</td>
</tr>
<tr>
<td>Optic_close_ip_4812_17hr_0611</td>
<td>0.9572</td>
<td>0.451 ± 0.003</td>
<td>0.2938</td>
<td>0.165 ± 0.040</td>
<td>0.003 ± 0.0003</td>
<td>0.108 ± 0.002</td>
<td>0.213</td>
</tr>
<tr>
<td>Optic_close_ip_4932_2hr_0507</td>
<td>0.93755</td>
<td>0.454 ± 0.003</td>
<td>0.31645</td>
<td>0.160 ± 0.039</td>
<td>0.002 ± 0.0002</td>
<td>0.122 ± 0.004</td>
<td>0.193</td>
</tr>
<tr>
<td>Optic_close_ip_4932_2hr_0508</td>
<td>0.92445</td>
<td>0.450 ± 0.002</td>
<td>0.32555</td>
<td>0.163 ± 0.040</td>
<td>0.004 ± 0.0005</td>
<td>0.094 ± 0.004</td>
<td>0.173</td>
</tr>
<tr>
<td>Optic_close_ip_4932_17hr_0514</td>
<td>0.8917</td>
<td>0.388 ± 0.004</td>
<td>0.2963</td>
<td>0.279 ± 0.052</td>
<td>0.004 ± 0.0005</td>
<td>0.089 ± 0.002</td>
<td>0.154</td>
</tr>
<tr>
<td>Optic_close_ip_4932_17hr_0516</td>
<td>0.8917</td>
<td>0.403 ± 0.018</td>
<td>0.3113</td>
<td>0.250 ± 0.049</td>
<td>0.004 ± 0.002</td>
<td>0.084 ± 0.020</td>
<td>0.124</td>
</tr>
</tbody>
</table>

**Figure 27.** A plot showing the overlay of data with a simulation created using the values obtained from the MCMC algorithm explained above for the data file “Optic_close_ip_4812_02hr_0520.xls”.

45
3.3.3 Trends in Resolution

No discernible trend was found in resolution as a function of any of the parameters that were varied through experimentation, which were seed to optic distance, optic to detector distance, or seed to detector distance. This may be in large part because the apparent resolution was broadened due to the difficulty of drawing the intensity profile exactly perpendicular to the seed.

3.4 Image Signal to Background

The measured signal to background ratio (SBR), when measured with an imaging detector, is the ratio of the primary signal to the background radiation

$$ SBR = \frac{I_{\text{primary}}}{I_{\text{background}}}. $$ (47)

The measured SBR was determined from the intensity profile from the measured image, but since the intensity peak is a combination of both signal and background, the SBR was taken as

$$ SBR = \frac{I_{\text{peak}} - I_{\text{adjacent}}}{I_{\text{adjacent}}}, $$ (48)

where $I_{\text{peak}}$ is the peak intensity and $I_{\text{adjacent}}$ is the intensity in the adjacent pixel.
3.4.1 Image Signal

For theoretical calculations the primary count rate on a single pixel aligned with a single seed is the total count rate divided by the total number of pixels aligned with aligned with the seed,

\[ I_{primary} = \frac{C_{primary} e^{-\mu_{tot} \Lambda}}{B}, \]  \hspace{1cm} (49)

where the exponential factor is taken to account for attenuation of the primary counts by some intervening material of thickness \( \Lambda \) with the attenuation coefficient \( \mu_{tot} \), and \( B \) is the number of pixels per seed. \( B \) can be approximated by

\[ B = \frac{D' \ell'}{P^2}, \] \hspace{1cm} (50)

where \( P \) is the pixel dimension, 0.05 mm for this work, \( \ell' \) is the effective height of the seed, and \( D' \) is the effective diameter of the seed due to blur. The area of the seed can be approximated as a rectangle because of the fact that the seed is cylinder that is then projected onto the two-dimensional detector plane. The effective diameter and length of the seed are,

\[ D' = D + \Delta D \] \hspace{1cm} (51)

\[ \ell' = \ell + \Delta \ell, \] \hspace{1cm} (52)

where \( D \) is the actual seed diameter and \( \ell \) actual height of the seed as shown in Figure 16 and \( \Delta D \) and \( \Delta \ell \) are given by
\[ \Delta D = (h + z)\theta_c \]  

(53)

and

\[ \Delta \ell = (h + z)\theta_c . \]  

(54)

Where \( h \) is the height of the seed plane from the detector, \( z \) is the distance of the seed plane from the collimator, and \( \theta_c \) is the critical angle of the optic.

Equations (51) and (52) applies only for long polycapillaries, as the blur does not increase within holes due to the fact that rays reflect down the optic channels as shown in Figure 11a. Substituting \( C_{primary} \) from Equation (35) yields

\[ I_{primary} = \frac{C_{primary}e^{-\mu_{tot}\Lambda}}{NB} = F \frac{\alpha^2 \bar{f}_{effec} e^{-\mu_{tot}\Lambda}}{B} \]  

(55)

for the primary intensity. Because nuclear medicine imaging is count rate limited, a significant contribution to image noise is Poisson noise,

\[ \Delta I = \sqrt{I}, \]  

(56)

where \( I \) is the measured number of counts in a particular pixel and counting interval.

### 3.4.2 Image Background Due to Septa Penetration

For background calculations, the background has two components

\[ I_{back} = I_{scatter} + I_{penetrate} . \]  

(57)
The penetration intensity is different for different image pixels, depending on the
distance $y_m$ between the pixel and the $m^{th}$ seed as shown in Figure 5. The
penetration intensity per pixel due to the $m^{th}$ neighboring seed is

$$I_{\text{penetrate},m} = \beta_m \frac{FP^2}{4\pi y_m^2} e^{-\mu_{tot}^A},$$  \hspace{0.5cm} (58)

where $F$ is the flux of the seed, $P$ is the pixel size, and the last term is for the
attenuation rays due to passing through an intervening material. From Figure 5,
the distance $y_m$ is

$$y_m = \sqrt{(L + z + h)^2 + x_m^2},$$  \hspace{0.5cm} (59)

where $x_m$ is the transverse distance. If the distance $y_o$ is the distance from the
seed to the pixel it is aligned with

$$y_o = L + z + h,$$  \hspace{0.5cm} (60)

then distance $L'_m$ with the collimator is from Figure 5 to be

$$L'_m = L \frac{y_m}{y_o}.$$  \hspace{0.5cm} (61)

The penetration $\beta_m$ is then as

$$\beta_m = e^{-\mu(1-f_{\text{eff}})L \frac{y_m}{y_o}} = \left(\beta_o\right) \frac{y_m}{y_o}.$$  \hspace{0.5cm} (62)
Therefore, the expected total penetration intensity for a pixel is

\[ I_{\text{penetrate}} = Fe^{-\mu a} \frac{P^2}{4\pi} \text{Sum}, \]  

(63)

where

\[ \text{Sum} = \sum_{m=1}^{N-1} \left( \frac{\beta_m}{\gamma_o} \right)^{y_m/y_o} \]  

(64)

Due to the fact that the seeds were actually cylinders placed in grooves 3 mm apart, the seeds were approximated as point sources at the center of the seeds which are 3 mm apart. This value was used as the \( x_m \) in order to calculate the \( y_m \) values.

### 3.4.3 Image Background Due to Scattering

If the scatter flux is uniform, as would be the case if there is a high degree of multiple scattering, as is the case in scintigraphy, then the background due to scattering is equal to the total scatter flux, \( F_{\text{scatter}} \), spread over the entire image,

\[ I_{\text{scatter}} = \frac{F_{\text{scatter}}}{\Pi} S = F_{\text{scatter}} \frac{\alpha^2 f_{\text{effec}}}{4\Pi}. \]  

(65)
Where S is the sensitivity and is given by Equation (17), $\Pi$ is the total number of pixels, and $f_{\text{effec}}$ is the effective fractional open area of the optic. The total scatter flux is unknown, so to estimate the scatter flux, it was assumed that

$$F_{\text{scatter}} = NF\left(1 - e^{-\mu_{\text{scatter}}\Lambda}\right)e^{-\mu_a\Lambda},$$

(66)

where $\Lambda$ is the thickness of the scattering material, including the glass of the collimator, $\mu_a$ is the photoelectric absorption coefficient, and

$$\mu_{\text{scatter}} = \mu_{\text{tot}} - \mu_a.$$

(67)

### 3.4.4 Theoretical and Experimental Signal to Background Comparisons

The experimental SBR values which can be seen in Appendix A and were calculated using the variant of Equations (47) and (48),

$$SBR = \frac{I_{\text{peak}} - I_{\text{background}}}{I_{\text{background}}},$$

(68)

Where $I_{\text{peak}}$ is the maximum PSL value for the image and $I_{\text{background}}$ is the average value of the background PSL for the image. Values for the signal to background ratio for the two new optics, numbers 4812 and 4932, ranged from 5 to 110. However it is important to not that not all of these images were taken under optimal conditions. During the time images were taken, it was first determined that of the various types of image plates available, the Fujifilm ST-V image plates yielded optimal results. Additionally, further tests were done to determine whether or not the use of a cassette (a thin metal casing used to house the
image plate) produced the best results; tests confirmed that the best results came when not using the cassette. Also, the decision was made to put the entire experimental setup underneath a blackout cloth was necessary. The purpose of the blackout cloth was to block any external light from interacting with the image plate and reducing the PSL values from the experiments. Lastly, after looking at initial images it was discovered that there was cut through on the edges of the optic which may have been contributing to higher than expected values for background counts in the images; as a result a layer of lead tape was wrapped around both optics in order to help absorb any x rays which may have penetrated through the edges of the optic. Once all of these additional measures were put in place to help optimize imaging conditions, the SBR ranges for all optics can be seen in Table V.

<table>
<thead>
<tr>
<th>Optic Number</th>
<th>Minimum SBR</th>
<th>Maximum SBR</th>
</tr>
</thead>
<tbody>
<tr>
<td>4812</td>
<td>20.8</td>
<td>110</td>
</tr>
<tr>
<td>4932</td>
<td>20</td>
<td>109</td>
</tr>
<tr>
<td>A326</td>
<td>15.3</td>
<td>45.5</td>
</tr>
</tbody>
</table>

Table V. Table of Signal to Background Ratio ranges for all optics used in this work.

It was initially anticipated that SBR would decrease as the distance from the brachytherapy seeds to the image plate varied, $y_o$, and the SBR would increase as the optic to image plate distance, $h$, increased. However, it was found that there was no discernable correlation between either. This is possible for multiple reasons. First, it was difficult to draw the intensity profiles through the seeds in the images. As discussed in Section 3.2 the iodine is not evenly
distributed through out the seed, so it was difficult to draw the profile through the part of the seed with the highest PSL value. The difficulty drawing the profiles through the “hottest” part of the seed made it difficult to find the maximum value for each seed, which in turn could have affected the values for the SBR. Additionally, the amount of background in the images is higher than expected. The increased amount of background can be most likely be attributed to air scatter.

It should also be noted that the measured SBR ranges shown in Table V are fairly large. This may be due to the fact that while it was determined which type of image plate was best to use for exposures, there were multiple image plates of that type. The large fluctuation could in part be due to inconsistencies between the various image plates used for exposures. The use of multiple plates was to in an attempt to get as many images captured in the short period of time for which the \(^{125}\)I brachytherapy seeds were viable.

Additionally, when the experimental result for the penetration was used in the theoretical SBR formulas discussed above, the results were less than the results obtained experimentally, however they are on the right order of magnitude, which seems to imply that the experimental penetration is an overestimate for the actual penetration. The minimum result for the theoretical SBR using the new optics that were tested in these experiments was 24.3 and the maximum was 89.3. As previously stated, this underestimate is most likely
because the value for the penetration that was used in these calculations was the
higher than expected as it had large contributions from air scatter.

Also, calculations for the expected SBR were calculated using the nested
sampling algorithm. The SBR was calculated as

\[
SBR = \frac{\text{flux} - \text{back}}{\text{back}},
\]

where \(\text{flux}\) is the result obtained for the flux and \(\text{back}\) is the result obtained for the
background from the simulation. The results for the SBR from these calculations
can be seen in Table VI. As can be seen from the results, the uncertainty in the
results is very high. As discussed in Section 3.3.2, this is due to the fact that the
results for the flux had high levels of uncertainty.

<table>
<thead>
<tr>
<th>Filename</th>
<th>Experimental SBR</th>
<th>Theoretical SBR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optic_close_ap_cloth_17hr_0424</td>
<td>92.2 ± 0.009</td>
<td>584.0 ± 2388</td>
</tr>
<tr>
<td>Optic_close_ip_4812_2hr_0520</td>
<td>51.8 ± 0.136</td>
<td>42.0 ± 125</td>
</tr>
<tr>
<td>Optic_close_ip_4812_2hr_0610</td>
<td>37.9 ± 0.212</td>
<td>40.3 ± 93.8</td>
</tr>
<tr>
<td>Optic_close_ip_4812_4hr_0521</td>
<td>42.8 ± 0.283</td>
<td>41.2 ± 115</td>
</tr>
<tr>
<td>Optic_close_ip_4812_4hr_0611</td>
<td>46.0 ± 0.121</td>
<td>32.3 ± 93.1</td>
</tr>
<tr>
<td>Optic_close_ip_4812_6hr_0522</td>
<td>57.3 ± 0.312</td>
<td>39.0 ± 83.7</td>
</tr>
<tr>
<td>Optic_close_ip_4812_6hr_0612</td>
<td>39.8 ± 0.195</td>
<td>40.0 ± 86.7</td>
</tr>
<tr>
<td>Optic_close_ip_4812_12hr_0612</td>
<td>41.5 ± 0.242</td>
<td>61.0 ± 149</td>
</tr>
<tr>
<td>Optic_close_ip_4812_17hr_0611</td>
<td>60.0 ± 0.462</td>
<td>54.0 ± 138</td>
</tr>
<tr>
<td>Optic_close_ip_4932_2hr_0507</td>
<td>109 ± 0.441</td>
<td>79.0 ± 205</td>
</tr>
<tr>
<td>Optic_close_ip_4932_2hr_0508</td>
<td>63.7 ± 0.282</td>
<td>39.8 ± 103</td>
</tr>
<tr>
<td>Optic_close_ip_4932_17hr_0514</td>
<td>74.9 ± 0.433</td>
<td>68.8 ± 137</td>
</tr>
<tr>
<td>Optic_close_ip_4932_17hr_0516</td>
<td>81.6 ± 0.591</td>
<td>61.5 ± 153</td>
</tr>
</tbody>
</table>

Table VI. A comparison of SBR values obtained experimentally and using the nested sampling
algorithm.
4. Conclusion

In conclusion, while discernable trends in signal to background were not detectable, expectations about the performance of the two new optics which were used were as expected in that they did produce better signal – to – background ratios than the optic used in previous work and displayed results above 100. This result proves promising as the distance these experiments were done at much larger distances than used previously. Expected values for the SBR were difficult to obtain as air scatter dominated the experimentally obtained value for the penetration, causing it to be lower than expect. The lower than expected value of the penetration in turn also had adverse effects on the expected values for the SBR as well. These difficulties in calculating theoretical values for SBR do not however discount the marked increases shown in SBR. Modifications should be made within experimental setups in order to help reduce air scatter in further experiments in order to more adequately estimate penetration through optics. Also, further work should be done to better identify “hot spots” in seeds in order to maximize SBR calculations.

Calculation to approximate resolution also should promising results. While approximations using the full width half maximum of the derivative of the profile and of a Gaussian fit both were overestimates, both techniques still displayed resolutions of 0.6 mm and below. Additionally, approximations of the resolution using a Markov Chain Monte Carlo algorithm displayed resolutions of approximately 0.1 mm. The MCMC algorithm also displayed promising results in
its ability to approximate the thickness of the iodine layer within the brachytherapy capsules.

The experiments presented demonstrate the feasibility of the use of polycapillary optics in nuclear imaging without the need of energy discrimination. They have shown their ability to produce sub-millimeter resolution while sufficiently diluting scatter.
# Appendix A - Table of Images and Imaging Parameters

<table>
<thead>
<tr>
<th>File name</th>
<th>Seed to IP</th>
<th>Seed to optic</th>
<th>Seed to aperture</th>
<th>Aperture to optic</th>
<th>Optic to IP</th>
<th>Signal</th>
<th>Background</th>
<th>SBR</th>
<th>Resolution</th>
<th>Collection Time</th>
<th>Special Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optic#: 4812</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Image_with_optic_13_hour_exposure</td>
<td>175</td>
<td>95</td>
<td>79</td>
<td>8</td>
<td>50</td>
<td>0.193</td>
<td>0.0059</td>
<td>32.5</td>
<td>13 hours</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Overnight_exposure_21hr</td>
<td>174</td>
<td>103</td>
<td>87</td>
<td>8</td>
<td>41</td>
<td>0.265</td>
<td>0.014</td>
<td>19.5</td>
<td>21 hours</td>
<td></td>
<td>Image plate laid sideways, BASIII IP</td>
</tr>
<tr>
<td>Image_w_optic_ipsideways</td>
<td>158</td>
<td>103</td>
<td>87</td>
<td>8</td>
<td>25</td>
<td></td>
<td></td>
<td></td>
<td>10 minutes</td>
<td>10 minutes</td>
<td>Seeds covered with lead apron, BASIII IP</td>
</tr>
<tr>
<td>With_optic_Apron_1hr</td>
<td>171</td>
<td>103</td>
<td>87</td>
<td>8</td>
<td>38</td>
<td>0.034</td>
<td>0.0017</td>
<td>19.5</td>
<td>1 hour</td>
<td></td>
<td>Seeds covered with lead apron, BASIII IP</td>
</tr>
<tr>
<td>With_optic_Apron_40min</td>
<td>171</td>
<td>103</td>
<td>87</td>
<td>8</td>
<td>38</td>
<td></td>
<td></td>
<td></td>
<td>40 minutes</td>
<td>40 minutes</td>
<td>Seeds covered with lead apron, BASIII IP</td>
</tr>
<tr>
<td>With_optic_Apron_50min</td>
<td>171</td>
<td>103</td>
<td>87</td>
<td>8</td>
<td>38</td>
<td></td>
<td></td>
<td></td>
<td>50 minutes</td>
<td>50 minutes</td>
<td>Seeds covered with lead apron, BASIII IP</td>
</tr>
<tr>
<td>With_optic_Apron_30min_20mm</td>
<td>171</td>
<td>103</td>
<td>87</td>
<td>8</td>
<td>38</td>
<td></td>
<td></td>
<td></td>
<td>30 minutes</td>
<td>30 minutes</td>
<td>Seeds covered with lead apron, BASIII IP</td>
</tr>
<tr>
<td>With_Optic_Apron_15hr</td>
<td>147</td>
<td>79</td>
<td>60</td>
<td>11</td>
<td>38</td>
<td>0.544</td>
<td>0.0046</td>
<td>118</td>
<td>15 hours</td>
<td></td>
<td>Seeds covered with lead apron, BASIII IP</td>
</tr>
<tr>
<td>With_Apron_Optic_close_to_IP</td>
<td>153</td>
<td>118</td>
<td>60</td>
<td>50</td>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td>3 hours</td>
<td>3 hours</td>
<td>Optic close to IP &amp; W Lead Apron, BASIII IP</td>
</tr>
<tr>
<td>W_Apron_Optic_close_IP_1hr</td>
<td>153</td>
<td>118</td>
<td>60</td>
<td>50</td>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td>1 hour</td>
<td></td>
<td>Optic close to IP &amp; W Lead Apron, BASIII IP</td>
</tr>
<tr>
<td>Optic_close_to_IP_lead_1hr</td>
<td>153</td>
<td>118</td>
<td>60</td>
<td>50</td>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td>1 hour</td>
<td></td>
<td>Optic close to IP, Lead Apron, Lead around optic,</td>
</tr>
<tr>
<td>W_optic_close_IP_lead_17hr</td>
<td>153</td>
<td>118</td>
<td>60</td>
<td>50</td>
<td>5</td>
<td>0.175</td>
<td>0.005</td>
<td>35</td>
<td>17 hour</td>
<td>Optic close to IP, Lead Apron, &amp; Lead around optic, BASIII IP</td>
<td></td>
</tr>
<tr>
<td>---------------------------</td>
<td>-----</td>
<td>-----</td>
<td>----</td>
<td>----</td>
<td>---</td>
<td>--------</td>
<td>--------</td>
<td>----</td>
<td>---------</td>
<td>-----------------------------------------------</td>
<td></td>
</tr>
<tr>
<td>Removed_0411</td>
<td>138</td>
<td>70</td>
<td>60</td>
<td>2</td>
<td>38</td>
<td>0.61</td>
<td>0.046</td>
<td>12</td>
<td>20 hours</td>
<td>Optic close to IP, Lead Apron, &amp; Lead around optic, BASIII IP</td>
<td></td>
</tr>
<tr>
<td>Optic_close_ap_21 hr</td>
<td>138</td>
<td>70</td>
<td>60</td>
<td>2</td>
<td>38</td>
<td>0.1055</td>
<td>0.0041</td>
<td>25</td>
<td>21 hours</td>
<td>ST-V IP, No Cassette</td>
<td></td>
</tr>
<tr>
<td>Optic_close_ap_17hr_0417</td>
<td>138</td>
<td>70</td>
<td>60</td>
<td>2</td>
<td>38</td>
<td>0.1234</td>
<td>0.0085</td>
<td>14.5</td>
<td>17 hours</td>
<td>ST-V IP, No Cassette</td>
<td></td>
</tr>
<tr>
<td>Optic_close_ap_4hr_0417</td>
<td>138</td>
<td>70</td>
<td>60</td>
<td>2</td>
<td>38</td>
<td>0.0486</td>
<td>0.0018</td>
<td>27</td>
<td>4 hours</td>
<td>HR-V IP, No Cassette</td>
<td></td>
</tr>
<tr>
<td>Optic_close_ap_17hr_0418</td>
<td>138</td>
<td>70</td>
<td>60</td>
<td>2</td>
<td>38</td>
<td>0.3387</td>
<td>0.0081</td>
<td>42</td>
<td>17 hours</td>
<td>ST-V IP, No Cassette</td>
<td></td>
</tr>
<tr>
<td>Optic_close_ap_cloth_2hr_0418</td>
<td>133</td>
<td>65</td>
<td>55</td>
<td>2</td>
<td>38</td>
<td>0.149</td>
<td>0.006</td>
<td>25</td>
<td>0.2165</td>
<td>2 hours</td>
<td></td>
</tr>
<tr>
<td>Optic_close_ap_cloth_17hr_0423</td>
<td>137</td>
<td>69</td>
<td>55</td>
<td>6</td>
<td>38</td>
<td>0.909</td>
<td>0.0097</td>
<td>93.5</td>
<td>0.2165</td>
<td>17 hours</td>
<td></td>
</tr>
<tr>
<td>Optic_close_ap_cloth_4hr_0423</td>
<td>122</td>
<td>54</td>
<td>40</td>
<td>6</td>
<td>38</td>
<td>0.283</td>
<td>0.0053</td>
<td>53.5</td>
<td>0.2165</td>
<td>4 hours</td>
<td></td>
</tr>
<tr>
<td>Optic_close_ap_cloth_17hr_0424</td>
<td>122</td>
<td>54</td>
<td>40</td>
<td>6</td>
<td>38</td>
<td>0.955</td>
<td>0.0087</td>
<td>109.5</td>
<td>0.2165</td>
<td>17 hours</td>
<td></td>
</tr>
<tr>
<td>Optic_close_ap_cloth_4hr_0424</td>
<td>122</td>
<td>54</td>
<td>40</td>
<td>6</td>
<td>38</td>
<td>0.182</td>
<td>0.0031</td>
<td>57</td>
<td>0.2165</td>
<td>4 hours</td>
<td></td>
</tr>
<tr>
<td>Optic_close_ap_cloth_17hr_0425</td>
<td>157</td>
<td>89</td>
<td>75</td>
<td>6</td>
<td>38</td>
<td>1.14</td>
<td>0.0109</td>
<td>105</td>
<td>0.2165</td>
<td>17 hours</td>
<td></td>
</tr>
</tbody>
</table>

See above (image plate may not completely erased)
<table>
<thead>
<tr>
<th>Experiment</th>
<th>ST-V IP, No Cassette, Black cloth, Lead over optic, IP sideways</th>
<th>ST-V IP, No Cassette, Black cloth, Lead around optic, IP sideways</th>
<th>ST-V IP, No Cassette, Black cloth, Lead OVER optic, IP sideways</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optic_close_ip_cloth_4hr_0501</td>
<td>124</td>
<td>89</td>
<td>75</td>
</tr>
<tr>
<td>Optic_close_ip_cloth_17hr_0502</td>
<td>124</td>
<td>89</td>
<td>75</td>
</tr>
<tr>
<td>New Optic being used: # 4923</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Optic_close_ip_4932_19hr_0506</td>
<td>133</td>
<td>93</td>
<td>75</td>
</tr>
<tr>
<td>Optic_close_ip_4932_2hr_0507</td>
<td>135</td>
<td>85</td>
<td>75</td>
</tr>
<tr>
<td>Optic_close_ip_4932_19hr_0508</td>
<td>135</td>
<td>85</td>
<td>75</td>
</tr>
<tr>
<td>Optic_close_ip_4932_2hr_0508</td>
<td>125</td>
<td>75</td>
<td>65</td>
</tr>
<tr>
<td>Optic_close_ip_4932_17hr_0509</td>
<td>125</td>
<td>75</td>
<td>65</td>
</tr>
<tr>
<td>Optic_close_ip_4932_2hr_0509</td>
<td>125</td>
<td>75</td>
<td>65</td>
</tr>
<tr>
<td>Optic_close_ip_4932_0513</td>
<td>115</td>
<td>65</td>
<td>55</td>
</tr>
<tr>
<td>Observation</td>
<td>Dose Rate (R/h)</td>
<td>Absorption (mm)</td>
<td>Distance (cm)</td>
</tr>
<tr>
<td>-------------</td>
<td>----------------</td>
<td>----------------</td>
<td>--------------</td>
</tr>
<tr>
<td>Optic_close_ip_4932_17hr_0514</td>
<td>115</td>
<td>65</td>
<td>55</td>
</tr>
<tr>
<td>Optic_close_ip_4932_2hr_0514</td>
<td>100</td>
<td>50</td>
<td>40</td>
</tr>
<tr>
<td>Optic_close_ip_4932_17hr_0516</td>
<td>100</td>
<td>50</td>
<td>40</td>
</tr>
<tr>
<td>Optic_close_ip_4932_17hr_0520</td>
<td>100</td>
<td>50</td>
<td>40</td>
</tr>
<tr>
<td>Optic #: 4812</td>
<td>150</td>
<td>95</td>
<td>85</td>
</tr>
<tr>
<td>Optic_close_ip_4812_2hr_0520</td>
<td>150</td>
<td>95</td>
<td>85</td>
</tr>
<tr>
<td>Optic_close_ip_4812_4hr_0521</td>
<td>150</td>
<td>95</td>
<td>85</td>
</tr>
<tr>
<td>Optic_close_ip_4812_6hr_0522</td>
<td>150</td>
<td>95</td>
<td>85</td>
</tr>
<tr>
<td>Optic_close_ip_4812_6hr_0522_2</td>
<td>150</td>
<td>95</td>
<td>85</td>
</tr>
<tr>
<td>Optic_close_ip_4812_18hr_0603</td>
<td>150</td>
<td>95</td>
<td>85</td>
</tr>
<tr>
<td>-------------------------------</td>
<td>-----</td>
<td>----</td>
<td>----</td>
</tr>
<tr>
<td>Optic_close_ip_4812_2hr_0603</td>
<td>150</td>
<td>95</td>
<td>85</td>
</tr>
<tr>
<td>Optic_close_ip_4812_17hr_0604</td>
<td>150</td>
<td>95</td>
<td>85</td>
</tr>
<tr>
<td>Optic_close_ip_4812_2hr_0610</td>
<td>150</td>
<td>95</td>
<td>85</td>
</tr>
<tr>
<td>Optic_close_ip_4812_2hr_0610_2</td>
<td>150</td>
<td>95</td>
<td>85</td>
</tr>
<tr>
<td>Optic_close_ip_4812_17hr_611</td>
<td>150</td>
<td>95</td>
<td>85</td>
</tr>
<tr>
<td>Optic_close_ip_4812_17hr_611_2</td>
<td>150</td>
<td>95</td>
<td>85</td>
</tr>
<tr>
<td>Optic_close_ip_4812_12hr_612</td>
<td>150</td>
<td>95</td>
<td>85</td>
</tr>
<tr>
<td>Optic_close_ip_4812_12hr_0612_2</td>
<td>150</td>
<td>95</td>
<td>85</td>
</tr>
<tr>
<td>Optic_close_ip_4812_4hr_0611</td>
<td>150</td>
<td>95</td>
<td>85</td>
</tr>
<tr>
<td>Experiment</td>
<td>Coating</td>
<td>Linac</td>
<td>Target</td>
</tr>
<tr>
<td>------------</td>
<td>---------</td>
<td>-------</td>
<td>--------</td>
</tr>
<tr>
<td>Optic_close_ip_4812_4hr_0611_2</td>
<td>150</td>
<td>95</td>
<td>85</td>
</tr>
<tr>
<td>Optic_close_ip_4812_6hr_0612</td>
<td>150</td>
<td>95</td>
<td>85</td>
</tr>
<tr>
<td>Optic_close_ip_4812_6hr_0612_2</td>
<td>150</td>
<td>95</td>
<td>85</td>
</tr>
<tr>
<td>Scatter Experiments using 4812</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Optic_close_ip_scatter_0618</td>
<td>150</td>
<td>95</td>
<td>85</td>
</tr>
<tr>
<td>Scatter_24mm_0620</td>
<td>150</td>
<td>95</td>
<td>85</td>
</tr>
<tr>
<td>Scatter_50mm_0624</td>
<td>150</td>
<td>95</td>
<td>85</td>
</tr>
<tr>
<td>Scatter_41mm_0625</td>
<td>150</td>
<td>95</td>
<td>85</td>
</tr>
</tbody>
</table>
Appendix B – Nested Sampling Fitting Code

Main

% Main2.m

% NESTED SAMPLING MAIN PROGRAM

% This is the main nested sampling algorithm. Other m-files are called to
% implement particular applications.

% Usage:   Main2()

% Where:

% This program is located in the top directory in the path that also
% contains:

%   apply.m    application initialization
%   Prior.m    sample from prior probability
%   logLhood.m  calculate log Likelihood
%   Explore.m  evolve a simulation object
%   Results.m  summarize the posterior probability
%   IntensityConvRev.m Calculates the intensity profile
%   for a cylindrical brachytherapy
% Originally written in C
% Modified:
%       Patrick Conlon
%       June 2011
%       Converted to Matlab

function Main2()

% Initialize Variables

global xo;       % seed center
global srange;   % start of interval
global erange;   % end of interval
global n;        % number of elements in data
global Data;     % data

n = 140;
xo = 3.85;
srange = 0;
erange = 6.95;
[Obj, Samples, Try] = apply();

% Obj: Simulation Objects
% Samples: Simulation Samples for Posterior
% Try: Simulation Object for Trials
% n: Number of Simulation Objects

%Data =

% Evidence Z, initially 0 (use exp(-realmax))
logZ = -realmax;

% Information H, initially 0
H = 0;

% Set prior objects
Obj = Prior(Obj,xo,srange,erange);

% Outermost interval of prior mass
logwidth = log(1.0 - exp(-1.0/n));

% Plot Prior
% figure;
% hold on;
% plot(Data,'b');
% plot(Obj,'r');
% hold off;

% Nested Sampling Loop

diff = 1;
counter = 0;

while abs(diff) > 1e-7
    worst = 1;
counter = counter + 1

    % Find Worst Object
    for i = 1:n
        if Obj(i).logL < Obj(worst).logL
            worst = i;
        end
    end

    % Weight = width * Likelihood

Obj(worst).logWt = logwidth + Obj(worst).logL;

% Update evidence Z and information H
if (logZ > Obj(worst).logWt)  % implement PLUS here
    logZnew = logZ + log(1 + exp(Obj(worst).logWt - logZ));
else
    logZnew = Obj(worst).logWt + log(1 + exp(logZ - Obj(worst).logWt));
end

diff = logZ - logZnew;
logZ = logZnew;

%Update Posterior Samples
Samples(counter).d = Obj(worst).d;
Samples(counter).c = Obj(worst).c;
Samples(counter).sig = Obj(worst).sig;
Samples(counter).logL = Obj(worst).logL;
Samples(counter).logWt = Obj(worst).logWt;
% Kill Worst Object in favor of a copy of a different survivor

    copy = ceil(n * rand());  % choose an object between 1 and n

while ((copy == worst) && n>1)

    copy = ceil(n * rand());  % choose another object

end

logLstar = Obj(worst).logL;  % new likelihood constraint

Obj(worst) = Obj(copy);  % overwrite worst object

% Evolve copied object within constraint

Obj(worst) = Explore(Obj(worst), Try, logLstar);

% Shrink Interval

logwidth = logwidth - 1.0/n;

% If iteration is a multiple of 200 plot Objects

if (rem(counter,200) == 0)

    plot(Data,'b');

    plot(Obj(worst),'r');

    hold off;

end
if (rem(counter,200) == 0)

    figure;
    hold on;

    worstData = IntensityConvRev(Obj(worst).d, xo, Obj(worst).sig,
    Obj(worst).c, n, srange, erange);

    plot (worstData,'r');

    plot (Data,'b');

end

%Put Samples.logL value for current iteration into an array for
%graphing later

logL(counter) = Samples(counter).logL;

end

% Exit with evidence Z and posterior Samples
disp(['Number of iterates = ' num2str(counter)]);

disp(['Evidence: ln(Z) = ' num2str(logZ) ' + ' num2str(H/log(2.))])

disp(['Information: H = ' num2str(H) ' nats = ' num2str(H/log(2)) ' bits'])

Results(Samples,counter,logZ);

%Plot iterates vs. LogL
iterates = 1:length(logL);
figure;
plot(iterates,logL);

return

Apply

% apply.m
% Usage:   [Obj, Samples, Try] = apply
%
% % Where:
% %    Obj     Structure array for n simulation objects
% %    Samples Structure array for simulation samples
% %    Try     Structure array for trial simulation
% %    n       Number of Simulation Objects
%
%
% Problem:   To determine the width of a brachytherapy seed and the size
of the coating of the titanium capsule as well as the resolution of the image.

% Inputs to the Problem:

% Prior(d) is random value for the diameter of the seed and
% Prior(c) is random value for the seed coating of the seed and
% Prior(sig) is random value for the resolution of the image.

% Likelihood is \( L(d, c, \sigma) = \frac{N}{2} \sum [F(i)^2 - D(i)^2] \)

% Outputs to the Problem:

% Evidence is \( Z = \int L(d, c, \sigma) \text{ Prior}(d, c, \sigma) \text{ d}(d)\text{d}(c)\text{d}(\sigma) \)
% Posterior is \( P(d, c, \sigma) = \frac{L(d, c, \sigma)}{Z} \) estimating characteristics of brachytherapy seed

% Originally written in C

% Modified:

% Patrick Conlon
% June 2011
% Converted to Matlab

function [Obj, Samples, Try] = apply()

global Data;  % data

global n;
% read in data from excel file

columnData = xlsread('Optic_close_to_ip_4923_0507.xls');

MAX = max(columnData);

Data = columnData/MAX;

% Skilling defines the structure here, which is necessary in C
% We actually create the objects here

% Define the fieldnames
% This is all you will have to change if you want to change the structure
fieldnames = {'c', 'sig', 'd', 'logL', 'logWt'};

% set up the Objects and Samples
f = size(fieldnames,2);

cObj = cell([n, f]);

Obj = cell2struct(cObj, fieldnames, 2);

cSamples = cell([5000, f]);

Samples = cell2struct(cSamples, fieldnames, 2);

cTry = cell([1, f]);
Try = cell2struct(cTry, fieldnames, 2);

return

Prior

% Prior.m
% Prior sets an object according to the prior
%
% Usage:
%    Obj = Prior(Obj);
%
% Where:
%    Obj is the object being set using the Matlab structure array
%          defined by struct
%
% Originally written in C
% Created:
%        Patrick Conlon
function Obj = Prior(Obj,xo,srange,erange)

global n;

% Nested Loop
for i = 1:n
    OK = 0;
    % Generate random values for c, d, and sig
    while OK == 0
        % Assign value for Obj.c
        Obj(i).c = .1*abs(randn());
    end
    % Check to make sure c is within a reasonable range of values
    while Obj(i).c < 0.06 || Obj(i).c > 0.11
        % Obj(i).c = abs(0.2 - 1 * abs(randn()));
        Obj(i).c = .1*abs(randn());
    end

    % Assigning values for Obj.sig
    Obj(i).sig = abs(0.5 - 1 * abs(randn()));
Obj(i).sig = .22*abs(randn());

%Check to make sure sig is within a reasonable range of values
while Obj(i).sig < 0.15 || Obj(i).sig > 0.3
  Obj(i).sig = abs(0.5 - 1 * abs(randn()));
  Obj(i).sig = .22*abs(randn());
end

%Assign value for Obj.d
%  Obj(i).d = abs(2 - 1 * abs(randn()));
Obj(i).d = abs(randn());

%Check to make sure d is within a reasonable range of values
while Obj(i).d < 0.5 || Obj(i).d > 1.5
  Obj(i).d = abs(2 - 1 * abs(randn()));
  Obj(i).d = abs(randn());
end

OK = 1;

end
F = IntensityConvRev(Obj(i).d,xo,Obj(i).sig,Obj(i).c,n, srange, erange);
maximum = max(F);
sim = F/maximum;

%Assign log likelihood value
Obj(i).logL = logLhood(sim);
end

return

logLhood

%logLhood.m
% Calculates the Log of the Likelihood for a student t distribution
%
% Usage: logL = logLhood(F)
%
% Inputs to the Problem:
%  F  A 1 x N matix of hypothesized data image of brachytherapy
%  seed
% Outputs to the Problem:
% logL      The Log of the Likelihood of the hypothesized data
%
% Created:
%           Patrick Conlon

function logL = logLhood(F)

global Data;  % data
% global n;
% global srange;
% global erange;
% global xo;

% find length of vectors for the hypothesized data and actual data
a = length(F);
b = length(Data);

% determine which vector is larger
if a > b
    m = b;
end
if a < b
    m = a;
end

%initialize variable
x = 0;

%calculate \( \sum (F(j) - Data(j)^2) \)
for j = 1:m
    x = x + (F(j) - Data(j))^2;
end

%Calculate log likelihood
logL = -(m)/2 * log(x);

return

Explore

%explore.m
% Evolves the simulated object. Returns 20 new Objects for possible
% characteristics of brachytherapy seed
% Usage: [Obj, Try, logLstar] = Explore(Obj, Try, logLstar)
%
% Where:
%    Obj      Structure array for n simulation objects
%    Try      Structure array for trial simulation
%    LogLstar Likelihood constraint
%
% Inputs to the Problem:
%   Obj      Structure array for n simulation objects
%   Try      Structure array for trial simulation
%   LogLstar Likelihood constraint

% Outputs to the Problem:
%   Obj      Structure array for n simulation objects
%   Try      Structure array for trial simulation
%   LogLstar Likelihood constraint
%
% Originally written in C

% Modified:
%    Patrick Conlon
%    June 2011
function [Obj, Try, logLstar] = Explore(Obj, Try, logLstar)

%global Data;   % data
global n;
global srange;
global erange;
global xo;

%Initialize Variables
step = .2;
reject = 0;
accept = 0;

%Nested Sampling Loop
for l = 0:20

    %Loop to explore space.

    %Sets value for diameter
    Try.d = Obj.d + step*randn();
%Tests if d value is OK
while Try.d < 0.5
    Try.d = Obj.d + step*abs(randn());
end

while Try.d > 1.5
    Try.d = Obj.d - step*abs(randn());
end

%Sets value for c
Try.c = Obj.c + step*randn();

%Tests if c value is OK
while Try.c < .05
    Try.c = Obj.c + step*abs(randn());
end

while Try.c > .11
    Try.c = Obj.c - step*abs(randn());
end
%Sets value for sig
Try.sig = Obj.sig + step*abs(randn());

%Tests if sig value is OK
while Try.sig < 0.05
    Try.sig = Obj.sig + step*abs(randn());
end

while Try.sig > 0.5
    Try.sig = Obj.sig - step*abs(randn());
end

%Generate seed profile for new assigned variables
F = IntensityConvRev(Try.d, xo, Try.sig, Try.c, n, srange, erange);
maximum = max(F);
sim = F/maximum;

%Set likelihood value for the points
Try.logL = logLhood(sim);

%If likelihood value > logLstar, then set Try value as Obj, and update step and
reject values

    if Try.logL > logLstar

        Obj = Try;

        accept = accept + 1;

    else

        reject = reject + 1;

    end


%Update Step size

if accept > reject

    step = step * exp(1/accept);

else

    step = step/exp(1/reject);

end

end

return
Results

%results.m

% GNU General Public License software: Copyright Sivia and Skilling 2006

% This function is a matlab implementation of the Lighthouse Problem

% Calculates the average x, y, and r values and their standard deviations
% for the tumor

% Usage:   [a, x, b, y, c, r] = Results(Samples,counter,logZ)

% Where:
%   Samples   Structure array for simulation samples
%   counter   Iteration counter
%   logZ      Log of Evidence
%   a         Standard deviation of x
%   x         Average x value for location of tumor
%   b         Standard deviation of y
%   y         Average y value for location of tumor
%   c         Standard deviation of r
%   r         Average r value for size of tumor
% Inputs to the Problem:
%   Samples   Structure array for simulation samples
%   counter   Iteration counter
%   logZ      Log of Evidence

% Outputs to the Problem:
%   x         Standard deviation of d
%   d         Average d value for seed
%   y         Standard deviation of sig
%   sig       Average sigma value for image
%   z         Standard deviation of c
%   c         Average c value for seed

% Originally written in C
% Modified:
%       Patrick Conlon
%       June 2011
%       Converted to Matlab

function [x, c, y, sig, z, d] = Results(Samples,counter,logZ)

%Initialize Variables to zero

c = 0;
cc = 0;
sig = 0;
ss = 0;
d = 0;
dd = 0;

% Calculate Values
i = 0;
while i < counter
    i = i + 1;
    w = exp(Samples(i).logWt - logZ);
    c = c + w * Samples(i).c;
    cc = cc + w * (Samples(i).c)^2;
    sig = sig + w * Samples(i).sig;
    ss = ss + w * (Samples(i).sig)^2;
    d = d + w * Samples(i).d;
    dd = dd + w * (Samples(i).d)^2;
end

% Calculate standard deviations
z = sqrt(cc - c^2);
y = sqrt(ss - sig^2);
x = sqrt(dd - d^2);

% Display results
disp(['mean(c) = ' num2str(c) ' stddev(c) = ' num2str(x)])
disp(['mean(sig) = ' num2str(sig) ' stddev(sig) = ' num2str(y)])
disp(['mean(d) = ' num2str(d) ' stddev(d) = ' num2str(z)])

return

**IntensityConvRev**

% IntensityConvRev.m
% Generates an array of x-ray intensities created by a cylindrical
% brachytherapy seed, where the seed coating, the diameter of the seed, and
% sigma can all be set. The center of the seed, xo, can also be set.

% Usage: I = IntensityConv(d, xo, sig, c, n, srange, erange)

% input:
% d - seed diameter
% xo - seed center
% sig - sigma (resolution of image)
% c - seed coating thickness
% n - number of data points at end of simulation
% srange - start distance from right side of interval
% erange - end distance from left side of interval

% output:
% I - 1 x n array of intensities

function H = IntensityConvRev(d, xo, sig, c, n, srange, erange)

% calculate how many data points needed to create an array of length n after convolution
if mod(n,2) == 1
    s = (n+1)/2;
end

if mod(n,2) == 0
    s = n/2;
end

% Calculate the distance between data points
dist = (erange - srange)/s;

%Initialize vectors for intensity and gaussian functions
I = zeros(s,1);  %intensity vector
G = zeros(s,1);  %gaussian vector

%Calculate radius of Iodine 125 cylinder without metal capsul
r = d/2 - c;

%Initialize variables
pos = srange;
j = 1;

while pos < erange

    TF = 1;

    %if x position is before beginning of seed set intensity to zero
    if pos > xo + d/2
        I(j) = 0;
        TF = 0;
    end
%if x position is after end of seed set intensity to zero
if pos < xo - d/2
    I(j) = 0;
    TF = 0;
end

%if x position is within two edges of seed, calculate intensity
if pos > xo - r && pos < xo + r
    I(j) = 2*(sqrt((r+c)^2 - (xo - pos)^2) - sqrt(r^2 - (xo - pos)^2));
    TF = 0;
end

%if x position is at one of edges of seed, calculate intensity
if TF == 1
    I(j) = 2*sqrt((r + c)^2 - (xo - pos)^2);
end

%calculate gaussian function for position
G(j) = exp(-((pos - xo)^2/(2*sig^2)));
j = j + 1;

%update position
pos = pos + dist;
end

%calculate convolution
H = conv(I,G);

return
References

10. Annals of ICRP pg 2