Essays on the macroeconomics of labor markets

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Essays on the Macroeconomics of Labor Markets

by

Arindam Mandal

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Abstract

This dissertation is a collection of three essays on the theoretical and the empirical aspects of the labor search theory.

Essay 1 presents a two sector search model to explain the observed positive correlation between productivity and unemployment in the United States. The paper concludes that the positive correlation between unemployment and productivity can be explained by substantial sectoral shifts in the economy along with productivity changes. Traditional one sector search framework cannot account for changes in vacancies and unemployment caused by sector specific shocks because there is no mechanism by which changes in one sector can spillover to other sectors. Whereas economies are multi sector and often changes in one sector spillover to other sectors through changes in wages and hence in turn changes resource allocations and preferences. These intersectoral spillover effects are captured in a two sector search framework. I have analyzed the model under the random search and the sector specific search framework.

In essay 2, the two sector model developed in essay 1, is extended by introducing endogenous flow capital. Interestingly, with perfectly mobile capital markets, the productivity shocks have no impact on the vacancies and the unemployment both under the random search and the sector specific search frameworks. The standard results with respect to change in consumer preferences and bargaining power of the workers remains same.
Finally, in the last essay, I estimated the matching function for the United States using a new set of data called Job Opening and Labor Turnover Survey (JOLTS) collected by the Bureau of Labor Statistics. Matching function is one of the key elements of the modern search theory and its stability over time has important policy implications. Using the JOLTS data from the period December 2000 to March 2009, I have shown that the standard Constant Returns to Scale assumption for the Matching function only holds for the non-recessionary time periods. During recessions, the Matching function exhibits decreasing returns to scale.
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Essay 1:

Vacancies and Unemployment in a Two Sector Search Framework
Abstract

The paper develops a two-sector search model to analyze the behavior of vacancies and unemployment as a result of persistent exogenous shocks. I will analyze both the random search and the sector specific search versions of the model. Since the mid 1980s, the United States economic data shows positive correlation between unemployment and labor productivity. Standard one sector search models cannot explain this counter intuitive fact. Shimer (2005) showed that one sector search and matching models cannot generate the observed business cycle fluctuations in vacancies and unemployment as a result of exogenous productivity shocks of plausible nature. The two sector search models are slightly better in explaining the relationship between vacancies and unemployment as a result of productivity shocks in comparison to one sector version. In a two sector search model, exogenous productivity shock to one sector can spillover to other sectors through changes in consumer demand as a result of change in relative wages across sectors. Furthermore, preference changes and variations in bargaining power of the workers can have substantial impact on the movements of vacancies and unemployment. The model also been able to generate the counter intuitive positive correlation between unemployment and labor productivity as observed in the United States.
1 Introduction

The purpose of this paper is to develop a two sector search model to explain the relationship between vacancies, unemployment, labor productivity and the sectoral shifts in the economy. In the United States, during the period 1948 to 1984, the contemporaneous correlation between detrended labor productivity and the unemployment is negative 0.29. From 1985 to 2008, however, the contemporaneous correlation is 0.1. This positive correlation becomes stronger since 2000. Standard neoclassical economic theory predicts that there is a negative correlation between productivity and unemployment (Fig: 1.1 and Fig: 1.2). As labor productivity increases, marginal product of labor becomes positive and therefore, firms employ more workers to reap the profits from increased productivity and hence unemployment in the economy falls. This relationship clearly cannot explain the trends in the data shown since mid 1980s. Along with productivity increases, during the same period the United States economy experienced substantial shifts across sectors in the economy. For example, the share of the manufacturing sector in the GDP declined from 25% in 1984 to 18% in 2007, whereas the share of the non-government services sector increased from 56% to 66%. Despite productivity increases, movements of labor across sectors can generate substantial short term unemployment as a result of adjustments in the labor market in response to sectoral shifts. The two sector search framework can explain the positive contemporaneous correlation between labor productivity and the unemployment in the economy, given that productivity changes are accompanied by sectoral shifts across the economy.

Additional purpose of this paper is to understand the extent a two-sector search model can better explain the relationship between vacancies and unemp-
ployment than the one sector model. What are the implications of preference changes on vacancies and unemployment? How does change in the bargaining power of the worker can affect vacancies and unemployment?

Literature on search and matching emerged as the standard theory of "equilibrium unemployment". By equilibrium, I mean both the firms and the workers maximize their expected payoffs, and wages are determined to exploit all the benefits of trade. Contrary to traditional Walrasian models, where trade takes place in a centralized market without any informational or trading frictions, in reality trades take place between two individuals, and it is a time consuming process to locate a trading partner. In the labor market, both the employers and the potential employees are imperfectly informed about the whereabouts of potential employees and the employers respectively. Hence, both the parties commit a substantial amount of time and resources to search for the potential match. The potential trades are matched using a stochastic matching technology and once together, their terms of trade are determined instantaneously by a bargaining process. Though match can be beneficial for both the employers and the workers, but a match can breakup, if gains of breakup outweigh the gains of match. Once a match breakup, again both the workers and the employers has to spend time and resources to look for another potential match. The existence of imperfect information along with costs associated with search explains the existence of unemployment in standard competitive market structure.

The model in this paper is based upon Mortensen-Pissarides (1994)\textsuperscript{1}. They showed that analytically their search and matching model is qualitatively consistent with some of the empirical regularities displayed by job creation and job destruction over the business cycle. The standard Mortensen-Pissarides search

\textsuperscript{1}See Diamond (1984) for more.
model is one sector search framework, which means workers look for jobs only in one particular sector. Though one sector search framework can explain the labor market frictions as a result of informational disadvantage, but it cannot explain the existence of unemployment as a result of sectoral adjustments in the economy. Sectoral adjustments affect the derived demand for labor in a particular sector, and it takes time for the labor to get adjusted for these changes. These adjustments take time and hence can cause substantial unemployment in the economy in the short run.

Traditional one sector search framework cannot account for changes in vacancies and unemployment caused by sector specific changes because there is no mechanism by which changes in one sector can spillover to other sectors. Therefore, one sector search models are limited in terms of transmission of shocks in the system. Whereas economies are multi-sector and often changes in one sector spillover to other sectors through changes in wages and hence in turn changes in resource allocations and preferences. These intersectoral spillover effects are captured in a two sector search framework.

The first major work to account for the impact of sectoral changes on unemployment started with Lilien (1982), who using sectoral data showed that periods where changes in employment differ more across sectors tend to be periods of aggregate downturns. With more disaggregated firm-level data, Davis and Haltiwanger (1990,1992) show that job reallocation across establishments is countercyclical. With worker-level PSID data, Loungani and Rogerson (1989) get a similar relationship between sectoral shifts and aggregate employment. With state-level data, Davis et al. (1995) find that a reallocation of military expenditures across states, holding aggregate expenditures fixed, causes a temporary
increase in unemployment, which they interpret as consistent with the view that reallocation costs matter. With national income accounts data Campbell and Kuttner (1996) show that downturns in output are associated with accelerations in the downward trend in the relative size of manufacturing. More recently Phelan and Trejos (2000) showed that sectoral reallocations generate responses that are qualitatively similar to productivity shocks.

Abraham and Katz (1986) document a positive correlation between vacancies and unemployment, which is supposed to contradict Lilien’s hypothesis. Others have pointed out the difficulty in reconciling the importance of sectoral shifts with the strong comovements across sectors observed in the United States at business cycle frequency.

Based upon Real Business Cycle (RBC) framework, majority of the literature explains the employment, unemployment and vacancy fluctuations over the business cycle through supply side shocks. Shimer (2005) showed that the standard one sector search and matching models cannot generate the observed business-cycle frequency fluctuations in unemployment and vacancies in response to productivity shocks of plausible nature. In the United States, the standard deviation of vacancy to unemployment ratio is almost 23 times more than the standard deviation of labor productivity.

In the paper, with a two sector search and matching model, I explore the sectoral and aggregate dynamics of the effect of changes in productivity, preference and bargaining power of the workers on employment, unemployment and vacancies. I assume consumers have Cobb-Douglas preferences, which introduces a non-transferrable utility function in the model. I discussed two variants of the two sector model - random search and sector specific search. In the random
search, unemployed worker search for jobs irrespective of any sectoral preference, whereas in sector specific search, workers do look for jobs only in one of the two sectors\textsuperscript{2}.

The paper is structured as follows: section 2 describes the model environment. Section 3 and 4 describes the model under random search and sector specific search framework, respectively. Section 5 discusses comparative statics using calibration and numerical analysis. Some final conclusions are presented in section 6.

2 Model Environment

2.1 Preferences and Technology

I am assuming a continuous time, infinite horizon framework. There are two sectors of production with a large number of firms associated with each sector. Firms create as many vacancies as they like subject to a zero \textit{ex-ante} profit condition. The (endogenous) mass of vacancies is $v = v_1 + v_2$, where $v_i$ is the mass of vacancies in sector $i$. Once a firm creates a vacancy, it incurs an exogenous flow cost $c_i$, in units of its own output, to maintain a job throughout its life. Jobs are either vacant or filled. Filled jobs break up at the exogenous Poisson arrival rate $\delta$.

Capital adjusted labor is used to produce two nonstorable final goods that are then sold in a competitive market. An employed worker in sector $i$ produce $\phi_i$ units of good $i$, where $\phi_i$ can be interpreted as per unit labor productivity.

\textsuperscript{2}For more on two sector models see Acemoglu (1997), Navarro (2004).
The model economy is populated by an infinitely lived continuum of workers with mass one, where workers are \textit{ex-ante} identical. They derive utility from the consumption of the final goods and maximize the present discounted value of their utility. Preferences of all agents are defined over the final consumption goods alone. Workers have a utility function over consumption \( U = y_1^\alpha y_2^{1-\alpha} \), where \( y_i \) is the consumption of the good produced in sector \( i \).\(^3\) Both workers and firms are risk neutral with a common discount rate \( r \).

### 2.2 Matching Technology

A \textit{matching technology} gives the outcome of the investments of resources by firms and workers in the trading process as a function of the inputs. It is a modeling device that captures the implications of the costly trading process without the need to make the heterogeneities and other features that give rise to it explicitly. Unemployed workers and vacancies are assumed to meet each other randomly according to a function \( M(u, v) \), where \( u \) is the number of unemployed and \( v \) is the measure of vacancies. In the random search, both types of vacancies have the same probability of meeting workers, and it is the total number of vacancies and unemployment that enters the matching function. In the sector specific search, though functional form of the matching function remains same across sectors, but arguments in the function e.g. vacancies and unemployment are sector specific.

We assume that the meeting function is characterized by constant returns to scale so that

\[
M(u, v) = M \left( 1, \frac{v}{u} \right) u = \lambda(\theta)u, \quad \text{where} \quad \theta = \frac{v}{u}
\]

\(^3\)Compared with the standard one sector models, this introduces non-linearity in the utility function which means that the usual Hosios [1990] condition does not hold.
We assume that \( M(u, v) \) is increasing in both arguments and concave, e.g. \( M'(\theta) > 0 \) and \( M''(\theta) < 0 \). The contact rate of vacancies by a worker searching for employment is \( \frac{M(u, v)}{\lambda(\theta)} = \lambda(\theta) \) and the rate at which vacancies meet workers is \( \frac{M(u, v)}{\lambda(\theta)} = \lambda(\theta) \), where \( \theta \) denotes the labor market tightness. I also make the standard Inada type assumptions on \( M(u, v) \); which ensure that \( \lim_{\theta \to 0} \lambda(\theta) = 0 \); \( \lim_{\theta \to 1} \lambda(\theta) = 1 \); \( \lim_{\theta \to 0} \frac{\lambda(\theta)}{\theta} = 0 \) and \( \lim_{\theta \to 0} \frac{\lambda(\theta)}{\theta} = 1 \).

The model has random search technology, which implies aggregate variables known but the whereabouts of individual vacancies/workers unknown.

### 2.3 Institutions

In equilibrium, because of the existence of entry/exit externality\(^4\), filled jobs generate total returns, which are strictly greater than the expected returns of the searching firms and the workers. If the firms and the workers get separated, both need to go through a costly process of job search. Hence any realized match generates surpluses. If there is a surplus, it needs to be allocated between the firms and the workers by some rule. Based upon the literature, I am assuming match surpluses are divided by the Nash bargaining\(^5\). Only those matches are formed,\(^4\)

---

\(^4\)Entry/exit externality arises because of the dependence of the functions \( \lambda(\theta) \) and \( \frac{\lambda(\theta)}{\theta} \) on the relative number of traders. The presence of an additional worker (firm) makes it easier (harder) for vacancies to find workers but harder (easier) for workers to find jobs. For example, during a short interval of time \( \delta t \), there is a positive probability \( \left(1 - \frac{\lambda(\theta)}{\theta} \delta t\right) \) that the hiring firm will not find a worker and another positive probability \( (1 - \lambda(\theta) \delta t) \) that an unemployed worker will not find a job, whatever be the set of prices. Hence, whenever there is a successful match, it does generate some trading or entry/exit externalities. The existence of this externality has important implications for the efficiency of the equilibrium (Hosios, 1990).

\(^5\)John Nash (1950) did not analyze the bargaining process, but took as given four simple axioms and showed that his solution is the unique outcome satisfying these axioms. Based upon Rubinstein (1982), one can provide a game-theoretic description of the bargaining process along the lines of that has a unique subgame perfect equilibrium see e.g. Osborne and Rubinstein (1990).
which are mutually advantageous relative to the alternative of continuing unmatched. That is, the workers’ and firms’ choices constitute a Nash equilibrium in the sense that they are value maximizing, taking as given the actions of the other agents. Exogenous bargaining power of the worker is $\beta$ and that of the firm is $(1 - \beta)$.

Workers get paid in the good they help produce. Since, utility of the employed workers depend upon the consumption of both the goods, therefore employed workers exchange a part of their earnings for the good produced in the other sector. Exchange takes place in a centralized competitive market in which the relative price $p$ of good 2 in terms of good 1 is taken as given by all the participants (Good 1 is the numeraire good). Unemployed workers get nothing and spend all their time looking for work.

3 Model Analysis - Random Search

In the model with random search, workers look for jobs in a single market. The model economy consists of employed and unemployed workers, where total number of workers employed in each sector is denoted by $e_i$ and the pool of unemployed workers looking for a job is denoted by $u$. Since the number of workers is normalized to one, therefore $u + e_1 + e_2 = 1$. The sector specific wage rate is denoted by $w_i$. Any snapshot of the economy finds workers of three types defined by their sector of choice and their employment status. Matches are formed between unemployed workers and vacancies whenever the joint surplus that would be realized by the match is nonnegative. I will solve the model via series of Bellman equations and steady state equilibrium conditions.
3.1 Labor Market - Firm Side

Job creation takes place when a firm and a worker meet and agree to an employment contract. Before this, firms must open a new job vacancy and search for potential employees, whereas unemployed workers look for potential employers. The employment contract specifies only a wage rule that gives the wage rate at any moment in time. Hours of work are fixed and either side can break the contract at any time without incurring any termination cost.

Furthermore, for convenience, I will assume that firms are sufficiently small, so that each can create one vacant job when it enters the market. The number of jobs is endogenous and determined by profit maximization condition. The market under consideration is perfectly competitive and hence there are free entry and exit of firms. Therefore, profit maximization requires that in equilibrium the profit from one more vacancy should be zero.

For sector $i$, let us denote by $J^F_i$ the present discounted value of expected profit from an occupied job and by $J^V_i$ the present discounted value of expected profit from a vacant job. With a perfect capital market, an infinite horizon and when no dynamic changes in parameters are expected, $J^V_i$ satisfies the Bellman equation

$$ rJ^V_i = \frac{\lambda(\theta)}{\theta} (J^F_i - J^V_i) - c_i, \text{ where } i = 1, 2 $$

(1)

Intuitively, a job is an asset of the firm. The valuation of the asset is such that the capital cost, $rJ^V_i$, is equal to the rate of return on the asset. The vacant job costs $c_i$ per unit time, where cost is measured in terms of the goods in the particular sector. The vacancy changes state according to a time invariant Poisson process with rate $\frac{\lambda(\theta)}{\theta}$. The change of state yield net return $(J^F_i - J^V_i)$. 

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Free entry condition drives the profits from additional vacancies to zero. Therefore, the equilibrium condition for supply of a vacant job is \( rJ_i^V = 0 \). Hence from equation (1)

\[ J_i^F = \frac{c_i\theta}{\lambda(\theta)} \quad (2) \]

This equation states that the expected profit from a new job is equal to the expected cost of hiring a worker.

We can derive flow equations for occupied jobs \( J_i^F \) in the same way we derived flow equations for the vacant jobs. The flow capital cost of the job is \( rJ_i^F \). Filled job yields a net return \((\phi_i - w_i)\). A match between an employed worker and the firm can breakup and leads to expected loss \( \delta(J_i^V - J_i^F) \). Hence flow equation for \( J_i^F \) is

\[ rJ_i^F = \phi_i - w_i + \delta(J_i^V - J_i^F), \text{where } i = 1, 2 \quad (3) \]

Though the firm takes the interest rate and product value as given, wage rate is determined by the bargain between the meeting firm and the worker. Therefore, by using equations (2) and (3) we derive the equation

\[ \phi_i - w_i - \frac{(r + \delta)c_i\theta}{\lambda(\theta)} = 0 \quad (4) \]

Intuitively above equation corresponds to marginal condition for the demand for labor. \( \phi_i \) is the output per person employed and \( \frac{(r + \delta)c_i\theta}{\lambda(\theta)} \) is the expected capitalized value of the firm’s hiring cost.
3.2 Goods Market

In this subsection, I will discuss the consumption goods market. Employed workers go to the centralized goods markets to spend their income to buy goods produced in the other sector. Each worker maximizes utility subject to the income constraints. Depending upon the sector in which the worker is employed, the worker’s sector specific income constraints are \( y_{11} + p y_{12} = w_1 \) for sector 1 and \( y_{21} + p y_{22} = p w_2 \) for sector 2, where \( y_{ij} \) is consumption by the sector \( i \) worker of the output of sector \( j \) and \( w_i \) is the wage. The solution to the consumer’s problem yields the following demand conditions.

\[
\begin{align*}
  y_{11} &= \alpha w_1 \\
  y_{12} &= \frac{(1 - \alpha) w_1}{p} \\
  y_{21} &= p \alpha w_2 \\
  y_{22} &= (1 - \alpha) w_2
\end{align*}
\]

In the economy, total supply of goods must be equal to total demand. In equilibrium, it is determined by the following market clearing conditions

\[
\begin{align*}
  e_1 w_1 &= e_1 y_{11} + e_2 y_{21} \\
  e_2 w_2 &= e_1 y_{12} + e_2 y_{22}
\end{align*}
\]

Intuitively the above equations state that the value of total wages paid to workers in each sector is equal to the value of total output in that sector measured in goods of that sector.

From the market clearing conditions and the demand equations, we can derive
the equilibrium price for good 2 in terms of good 1 as follows

\[ p = \frac{(1 - \alpha)e_1 w_1}{\alpha e_2 w_2} \]

Given the demand equations and price, the indirect utility derived by the workers from consumption is denoted by

\[ z_1(w_1, w_2) = \alpha w_1^{\alpha} \left( \frac{w_2 e_2}{e_1} \right)^{(1-\alpha)} \]  \hspace{1cm} (5)

\[ z_2(w_1, w_2) = (1 - \alpha) \left( \frac{w_1 e_1}{e_2} \right)^\alpha w_2^{(1-\alpha)} \]  \hspace{1cm} (6)

Indirect utility of the employed workers not only depends upon the wage of the sector in which the worker is employed, but also on the wage in the other sector. Workers get paid in terms of the goods they produce. Since workers consume both the goods, therefore wages of other sectors also affect the utility of the employed workers. This particular fact that utility of the workers depends upon the wages in both the sectors has important welfare implications. Even if productivity in a particular sector is going up, it may cause disproportionately more increase in utility in the other sector.

### 3.3 Labor Market - Worker Side

Workers normally influence the equilibrium outcome through their job search and their influence on wage determination. In this model, the size of the labor force and each worker’s intensity of search are fixed. So the only influence that workers have on the equilibrium outcome is through wages. A typical worker earns \( w_i \) when employed and searches for a job when unemployed. I assume opportunity cost of unemployment is zero.
Let $J^U$ and $J^E_i$ denote the present discounted value of the expected income stream of an unemployed and an employed worker in sector $i$. The unemployed worker can become employed with probability $\lambda(\theta)$. Hence we have

$$rJ^U = \lambda(\theta)((\psi J^E_1 + (1 - \psi)J^E_2) - J^U)$$

where $\psi$ denotes the proportion of total vacancies in sector 1. The above equation states that the flow value of unemployment in any sector is equivalent to the expected gain from finding a job and realizing a flow value $rJ^E_i$.

Similarly, for the employed worker, the flow equations are

$$rJ^E_i = z_i(w_i) + \delta(J^U - J^E_i)$$

The above equation states that the flow value of being employed is equal to value derived from the wage which is given by indirect utility $z(w_i)$ and the net value derived from being unemployed if the current job break up.\(^6\)

By solving equations (7) and (8) and then by substituting equation (5) and (6), we get

$$rJ^U = \frac{\lambda(\theta) \left[ w_1^\alpha w_2^{(1 - \alpha)} \left( \psi \alpha \left( \frac{e_2}{e_1} \right)^{(1 - \alpha)} + (1 - \psi)(1 - \alpha) \left( \frac{e_1}{e_2} \right)^\alpha \right) \right]}{(r + \lambda(\theta) + \delta)}$$

\(^6\)The value of being employed or permanent income of employed workers $rJ^E_i$ is different from the value derived from wage e.g. $z(w_i)$ because of the risk of unemployment. Without on the job search, employed workers stay in their jobs for as long as $J^E_i > J^U$. The necessary and sufficient condition for this to hold is $z(w_i)$ should be more than the unemployment income earned, which assumed to be zero in this model (see Pissarides, 2000).
3.4 Wage Determination

In equilibrium, occupied jobs yield a total return that is strictly greater than the sum of the expected returns of a searching firm and a searching worker. If the firm and the worker separate, each will have to go through an expensive process of search before meeting another partner. Hence a realized job match yields some pure economic rent, which is equal to the sum of the expected search costs of the firm and the worker. Wage levels need to share this economic rent, in addition to compensating each side for its costs from forming the job. We assume that the monopoly rent is shared according to the Nash solution to a bargaining problem. Since workers and firms are risk neutral and have the same discount rate, Nash bargaining implies that $w_i$ be chosen so that

$$ (1 - \beta)(J_i^F - J_i^U) = \beta(J_i^F - J_i^V)z_i'(w_1, w_2) $$ (10)

We can derive the wage equations in implicit form by solving equations (3), (8) and (10)

$$ (1 - \beta) \left( \alpha w_1 \left( \frac{w_2 e_2}{e_1} \right)^{(1-\alpha)} - r J_i^U \right) = \beta(\phi_1 - w_1)z_1'(w_1, w_2) $$ (11)

$$ (1 - \beta) \left( (1 - \alpha) \left( \frac{w_1 e_1}{e_2} \right)^\alpha w_2^{(1-\alpha)} - r J_i^U \right) = \beta(\phi_2 - w_2)z_2'(w_1, w_2) $$ (12)

3.5 Steady State under Random Search

In steady state mean rate of unemployment and employment are constant. Therefore, flow out of unemployment from any sector equal to the flow back into un-
employment to that sector. The flow out of unemployment is \( u\lambda(\theta) \) and flow into unemployment is \( \delta(1 - u) \). Therefore, in steady state

\[
\frac{u}{\lambda(\theta) + \delta} = \delta
\]

The above equilibrium condition states that in steady state, for given \( \delta \) and \( \theta \), there is a unique unemployment rate.

On the same note, in steady state, flows in and out of jobs should be same. Total flows in to jobs is defined as \( \frac{\lambda(\theta)\nu}{\theta} \psi \) and \( \frac{\lambda(\theta)\nu(1 - \psi)}{\theta} \) for sector 1 and sector 2, respectively. Total flow out of job for each sector is \( \delta e_i \). Therefore in steady state

\[
\begin{align*}
\frac{\lambda(\theta)}{\theta} \nu \psi &= \delta e_1 \\
\frac{\lambda(\theta)}{\theta} \nu (1 - \psi) &= \delta e_2
\end{align*}
\]

Taking the ratio of the above two equations we get in steady state

\[
\frac{\psi}{(1 - \psi)} = \frac{e_1}{e_2} \quad (14)
\]

In steady state, ratio of the workers in each sector should be equal to the ratio of vacancies created in each sector.

By replacing (14) in (9) and then replacing the resulting equation in equations (11) and (12) we get the wage equations

\[
w_1 = \frac{\beta \phi_1 \alpha^2}{(1 - \beta)\psi \left( \frac{\alpha}{\psi} - \frac{\lambda(\theta)}{(r + \lambda(\theta) + \delta)} \right) + \beta \alpha^2} \quad (15)
\]
\[
\begin{align*}
\nu_2 &= \frac{\beta \phi_2 (1 - \alpha)^2}{(1 - \beta)(1 - \psi) \left( \frac{(1-\alpha)}{1-\psi} - \frac{\lambda(\theta)}{(r+\lambda(\theta)+\delta)} \right)} + \beta (1 - \alpha)^2
\end{align*}
\]

3.6 Equilibrium under Random Search

**Definition 1** An equilibrium is defined as labor market tightness \( \theta \), wages \( w_i \), and the proportion \( \psi \) of vacancies in each sector, such that equations (4), (15) and (16) are satisfied for both the sectors.

A steady-state equilibrium is a collection of variables \( \{\theta, w_i, \psi\} \) that satisfy the following conditions:

First, all matches that are mutually advantageous relative to the alternative of continuing unmatched. The actions of the workers’ and firms’ constitute a Nash equilibrium in the sense that they are value maximizing, taking as given the actions of the other agents.

Second, since this is a long-run model with free entry/exit of vacancies. Therefore, firm vacancy creation satisfies zero-value conditions.

Finally, the appropriate steady-state labor market flow conditions hold. This will require that the flow of workers into and out of unemployment be equal. Similarly, flows into and out of jobs should be same.

4 Model Analysis - Sector Specific Search

It is seldom the case that the unemployed workers look for jobs randomly across sectors. Often, job search takes place within a particular sector. Based upon
worker’s experience and skills, job search is a sector specific activity. Though in the model, all the workers are homogeneous and there are no on the job skill acquisition, still it is possible to analyze the model based upon sector specific search. Workers look for jobs in the sectors where they have the highest discounted value of being employed. Model assumptions remain same except that the matching function varies across sectors, since unemployment and vacancies are sector specific.

Hence probability of vacancies meeting workers varies across sectors. The contact rate of vacancies by a worker searching for employment in sector $i$ is $M(u_i, v_i) = \lambda(\theta_i)$ and the rate at which vacancies meet workers in sector $i$ is $\frac{M(u_i, v_i)}{v_i} = \frac{\lambda(\theta_i)}{\theta_i}$, where sector specific labor market tightness is denoted by $\theta_i = \frac{v_i}{u_i}$.

The goods market conditions remain same as in equations (5) and (6). Firms solve the following labor market equations

$$r J_i^V = \frac{\lambda(\theta_i)}{\theta_i} (J_i^F - J_i^V) - c_i$$
$$r J_i^F = \phi_i - w_i + \delta (J_i^V - J_i^F)$$

The intuition for above equations are exactly same as in the random search framework. Solving the above set of equations we get

$$\phi_i - w_i - \frac{(r + \delta)c_i \theta_i}{\lambda(\theta_i)} = 0$$

(17)

As in random search, the decision to supply labor by the workers is determined
by the following equations

\[ rJ_i^U = \lambda(\theta_i)(J_i^E - J_i^U) \]
\[ rJ_i^E = z_i(w_i) + \delta(J_i^U - J_i^E) \]

Solving above equations we get

\[ rJ_i^U = \frac{\lambda(\theta_i)z_i(w_i)}{(r + \delta + \lambda(\theta_i))} \tag{18} \]

Wage is determined by the following Nash bargaining conditions

\[ (1 - \beta)(z_i(w_i) - rJ_i^U) = \beta (\phi_i - w_i) z_i'(w_i) \tag{19} \]

4.1 Steady State under Sector Specific Search

In the model, workers are homogeneous across sectors and so do the firms. Worker’s decision in which sector to work is based upon relative value of being unemployed across sectors. In steady state, workers are indifferent to work in either sectors. Therefore, in steady state \( rJ_i^U = rJ_i^L \), which implies

\[ \frac{e_1}{e_2} = \frac{\lambda(\theta_1)\alpha(r + \delta + \lambda(\theta_2))}{\lambda(\theta_2)(1 - \alpha)(r + \delta + \lambda(\theta_1))} \tag{20} \]

The set of wage equations are

\[ w_1 = \frac{\beta \phi_1 \alpha(r + \delta + \lambda(\theta_1))}{(1 - \beta)(r + \delta) + \beta \alpha(r + \delta + \lambda(\theta_1))} \tag{21} \]
\[ w_2 = \frac{\beta \phi_2 (1 - \alpha)(r + \delta + \lambda(\theta_2))}{(1 - \beta)(r + \delta) + \beta (1 - \alpha)(r + \delta + \lambda(\theta_2))} \tag{22} \]
Also in steady state, flow out of unemployment in each sector is equal to flow into unemployment. Therefore

\[
u_1 \lambda(\theta_1) = \delta(\pi - u_1) \tag{23}\]
\[
u_2 \lambda(\theta_2) = \delta((1 - \pi) - u_2) \tag{24}\]

where \(\pi\) is the proportion of total pool of workers in sector 1 and \((1 - \pi)\) is the proportion of workers in sector 2.

Similarly, flows into and out of jobs should satisfy

\[
\frac{\lambda(\theta_1)}{\theta_1} v_1 = \delta e_1 \tag{25}\]
\[
\frac{\lambda(\theta_2)}{\theta_2} v_2 = \delta e_2 \tag{26}\]

Therefore from equation (23) - (26) we get

\[
\frac{e_1}{e_2} = \frac{\lambda(\theta_1)\pi(\lambda(\theta_2) + \delta)}{\lambda(\theta_2)(1 - \pi)(\lambda(\theta_1) + \delta)} \tag{27}\]

In steady state

\[
\frac{\alpha(r + \delta + \lambda(\theta_2))}{(1 - \alpha)(r + \delta + \lambda(\theta_1)))} = \frac{\pi(\lambda(\theta_2) + \delta)}{(1 - \pi)(\lambda(\theta_1) + \delta)} \tag{28}\]

### 4.2 Equilibrium under Sector Specific Search

**Definition 2** An equilibrium is defined as labor market tightness \(\theta_i\), wages \(w_i\) and the proportion \(\pi\) of workers looking for job in each sector, such that equations (17), (21), (22) and (28) are satisfied for both the sectors.
A steady-state equilibrium is a collection of variables \( \{\theta_i, w_i, \pi\} \) that satisfy the following conditions:

First, all matches that are mutually advantageous relative to the alternative of continuing unmatched. The actions of the workers’ and firms’ constitute a Nash equilibrium in the sense that they are value maximizing, taking as given the actions of the other agents.

Second, since this is a long-run model with free entry/exit of vacancies. Therefore, firm vacancy creation satisfies zero-value conditions.

Third, workers are indifferent to work in either sectors. Therefore value of being unemployed across sectors is equal in steady state.

Finally, the appropriate steady-state labor market flow conditions hold. These will require that the flow of workers into and out of unemployment be equal. Similarly, flows into and out of jobs should be same.

5 Calibration and Numerical Analysis

In this section I will present the results of the numerical simulations of the random and the sector specific search models. It shows the comparative statics for the decentralized model only. The model is simulated using the parameter configurations based upon the practice in the literature.

Parameter values are chosen based upon Shimer (2005) and Job Opening and Labor Turnover Survey (JOLTS) data. Without loss of generality, labor productivity \( \phi_i \) has been normalized to one. According to the JOLTS data,
between 2001 and 2007, separation rate averaged around 0.033 every month. Consistent with Shimer (2005) estimates, this means that on an average a job lasted for 2.5 years before it broke up. The model is solved assuming a yearly time period, hence I set the job breakup rate $\delta$ as 0.4 per year. The discount rate $r$ is assumed to be 5 percent per year, which is standard in the literature. The literature generally assumes a Cobb-Douglas matching function of the form $m(\theta) = \overline{L}\theta^{1-\eta}$, where $\overline{L}$ is the matching intensity and the elasticity parameter is $\eta$. Based upon Shimer (2005) and Petrongolo and Pissarides (2001), I set the elasticity parameter $\eta$ as 0.5. Shimer (2005) showed that if $\eta = \beta$, then "Hosios (1990) Rule", the decentralized equilibrium maximizes a well posed social planner’s problem. Therefore, bargaining power of the workers has been set to 0.5. The matching intensity $\overline{L}$ has been set to 7.2.

In the following sections, I have compared the behavior of the two sector search model under random and sector specific search framework. I have also compared the model under standard one sector search framework\(^8^*.\)

### 5.1 Productivity changes

Whether reasonable permanent productivity changes alone can explain the high standard deviation in the vacancy-unemployment ratio as found in the United States data is still an open question in the search literature. Shimer(2005) showed that standard one sector search and matching model cannot generate the observed business-cycle-frequency fluctuations in unemployment and job vacancies in response to productivity changes of plausible magnitude. The standard deviation of the vacancy-unemployment ratio is almost 23 times as large as the

\(^8^*\)See Appendix 3 for one sector search framework.
standard deviation of average labor productivity, while the search model predicts that the two variables should have nearly the same volatility.

Shimer’s (2005) model was a single sector model. In one sector model, any increase in labor productivity relative to the flow cost of creating vacancies make unemployment relatively more expensive than vacancies. Therefore, the market starts substituting vacancies for unemployment by creating more vacancies. As a result the unemployment rate goes down and this leads to reduction in unemployment duration, which implies raising workers’ wage. Higher wages absorb a part of the positive productivity gains and hence eliminate the incentive for vacancy creation. Therefore, productivity changes dampen the effect of productivity increase on unemployment and vacancy rate.

The basic issue in the literature is the lack of propagation of the productivity changes in the models. In the two sector model, this issue can be handled in a better way. The model has two separate goods producing sectors and consumer utility depends upon the consumption of both the goods. Any increase in labor productivity in one of the sectors relative to the flow costs of creating vacancies, raise the wages in that sector and hence dampen the effect of productivity increase on unemployment and vacancy rate. But unlike one sector model, we have second round effects in the two sector models. Any wage increase, change the demand for goods produced by the other sector, since consumer utility depends upon consumption of both the goods. This increase in demand for goods of the other sector, increases the price for the goods produced in that sector. Therefore, firms tend to produce more in the other sector too because of increased demand. This leads to creation of more vacancies. Finally, any productivity changes to one sector spillover to other sectors.
Table 1.1(a,b,c) shows the effects of productivity changes on the model economies. In the two sector model, productivity change only impacts one of the sectors. Hence one percent change in two sector model is equivalent to 0.5 percent change in one sector model. All the models are simulated using the same parameter values. Changes in $\theta$ as a result of productivity changes are comparatively less in case of one sector model than in two sector models. For example, a 0.5 percent change to productivity in one sector model, changes $\theta$ by only 0.55 percent, whereas in case of two sector models, an one percent productivity change changes $\theta$ by 0.61 percent and 1.16 percent in random search and sector specific search models respectively.

One important thing to note is that in the sector specific search model, productivity changes to sector 1 have no impact on vacancy, unemployment and wages of sector 2. Any changes in relative demands for goods by the workers as a result of productivity changes is mediated through changes in the relative prices. As productivity in sector 1 increases, there is relative shortage of sector 2 goods and hence price for good 2 increases.

5.2 Preference changes or Sectoral Shifts

What happens if there is change in consumer preferences? In modern economies, it is not unlikely that overtime consumer preferences change. In the model, it is possible to introduce effects of consumer preferences across sectors, which is not possible in standard one sector Mortensen-Pissarides models. Change in consumer preferences can be introduced through changes in parameter $\alpha$ in the utility function, where $\alpha$ proportion of good 1 consumed by the employed workers. Details of the preference changes are shown in Table 1.2a and Table
In the random search model (Table 1.2a), increase in the relative share of the demand for sector 1 good cause a fall in $\theta$, which implies falling vacancies and rising unemployment. Though in aggregate vacancies has fallen and unemployment has gone up, but dynamics are very different across the sectors. In sector 1 vacancies increased, whereas in sector 2 it declined. The decrease in sector 2 vacancies are not offset by the increase in sector 1 vacancies. Table (1.2a) shows that because of shift in preference towards sector 1 goods, employment increased in sector 1, whereas falling relative demand for sector 2 goods caused decrease in employment in sector 2. Again, increased employment is sector 1 cannot offset falling employment in sector 2 and hence total employment in the economy declined. As a result total unemployment in the economy increased. Though wages remained same across sector but, wages in sector 2 defined as $p \times w_2$ decreased relative to wage in sector 1. This decline in wages is due to fall in price of good 2 as a result of preference shift to good 1.

In the sector specific search model (Table 1.2a), increase in $\alpha$ leads to decrease in $\theta_1$ and the increase in $\theta_2$. Decrease in $\theta_1$ is attributable to more than proportionate increase in unemployment $u_1$ compared to vacancies $v_1$. Similarly increase in $\theta_2$ is due to relatively larger decrease in unemployment $u_2$ compared to vacancies $v_2$. It is counter intuitive that unemployment has gone up in the sector 1 which experienced favorable shift in demand, whereas unemployment has declined in sector 2 whose demand for output decreased relatively. The economic reason is that, because of the favorable shift in demand for sector 1 goods, wages have gone up in sector 1 relative to sector 2 and hence proportionately more workers started searching job in sector 1 compared to sector 2. This is evident
from the increase in the parameter \( \pi \) which is the pool of workers looking for jobs in sector 1.

Shift in demand towards sector 1, caused increase in unemployment in the economy because of increase in matching frictions as a result of sectoral reallocation of workers. This result has particular policy implications. Economies go through structural change overtime. For example, in the United States, relative share of the services sector has gone up compared to the manufacturing sector in the last couple of decades. This unbalanced shift towards services sector might lead to disproportionately more reallocation of workers to these sectors, which might lead to increased unemployment as these booming sectors might not be able to absorb all the workers. As a result unemployment might go up in the economy.

5.3 Changes in Relative Bargaining Power

In the search literature, wages are determined by Nash bargaining, where bargaining power of the workers and firms are assumed to be constant overtime and typically assumed to be 0.5. There is no reason to believe that both the firms and the workers have the same bargaining power. Technically match will take place for any bargaining power strictly greater than zero e.g. for any \( \beta > 0 \). Because wages are settled through Nash bargaining, hence any change in the bargaining power of the worker will have influence on the equilibrium wages and hence as a result on vacancies and unemployment.

The impact of changes in bargaining power of the workers \( \beta \) on wages, \( \theta \), vacancies, employment and unemployment are exactly same in both the random
search and the sector specific search framework (see Table 1.3a, 1.3b). As \( \beta \) increases, so does the wages go up. Increase in wages decreases the value of filled vacancies and hence reduce the number of vacancies created. Therefore, overall unemployment in the economy increases. So the adverse effect of increase in bargaining power is increasing unemployment.

5.4 Productivity Changes along with Sectoral Shifts

In this section, I will study the effects of productivity changes along with sectoral shifts. The United States economy has experienced substantial sectoral shifts in the economy. Often these shifts accompanied with productivity changes. Studying effects of productivity changes without accounting for sectoral shifts will not give the complete picture. For example, since mid 1980s in the United States, there is positive correlation between labor productivity and unemployment. This counter intuitive fact can be explained if we account for sectoral shifts along with productivity changes. From Tables 1.4a and 1.4b, we can see that in both the random search and the sector specific search, sectoral shifts generated negative correlation between productivity and unemployment. This again reinforces the fact that sectoral shifts can generate substantial short run unemployment because of the adjustment time needed for the labors to get employed in the new sectors.

6 Conclusion

In this paper, I built a two sector search model with non-transferrable utilities. The general conclusion is that productivity changes alone are not enough to ex-
plain the observed relationship between vacancies and unemployment. Compare to one sector model, two sector models generate slightly more variation in the movement of vacancies and unemployment for given technology change but not enough variation to fit the business cycle data. In comparison to the random search model, the sector specific search generates substantially more variance in vacancies and unemployment. But the observed variances in vacancies and unemployment in sector specific search are for the sector that gets affected by productivity changes and not for the economy as a whole.

Comparative statics results show that preference changes and the changes in bargaining power can generate substantial movements in vacancies and unemployment. Given that sectoral shifts we observe in economies over time, it is not unlikely that preferences shift overtime. Also I have shown that changing bargaining power can generate substantial movements in vacancies and unemployment. This model is also able to explain the observed positive correlation between unemployment and productivity since mid 1980s. If the productivity changes are accompanied by substantial sectoral shifts, the model can generate the positive correlation between unemployment and productivity.

References


Once match between the employer and the worker takes place, it generates a matching surplus. In the model, match surplus is distributed according to Nash bargaining solution. Nash bargaining solution is the $w_i$ that maximizes the weighted product of the worker’s and the firm’s net return from the job match. Therefore, the wage rate for the job satisfies

$$w_i = \arg \max (J_i^E - J_i^U)\beta(J_i^F - J_i^V)^{(1-\beta)}$$
Using (3) and (8)

\[ w_i = \arg \max \left( \frac{\phi_i - w_i - rJ_i^V}{r + \delta} \right)^\beta \left( \frac{z_i(w_1, w_2) - rJ_i^U}{r + \delta} \right)^{(1-\beta)} \]

Maximizing above expression with respect to \( w_i \) gives

\[(1 - \beta)(z_i(w_i) - rJ_i^U) = \beta (\phi_i - w_i) z_i'(w_1, w_2)\]

**Appendix 2: Model Analysis - One Sector Random Search**

The analysis of one sector search models are much simpler compared to two sector search. In one sector search, workers get what they produce and wages reflect workers’ utility. Workers as consumers do not have to decide which goods to consume to maximize utility, or in other words, there is no demand side in the case of one sector model. The logic and rationale for the one sector model follow exactly as in the two sector model, I developed in the previous section.

Value of creating new vacancies and the value to fill the existing vacancies are denoted as

\[ rJ^V = \frac{\lambda(\theta)}{\theta} (J^F - J^V) - c \]  \hspace{1cm} (29)

\[ rJ^F = \phi - w + \delta(J^V - J^F) \]  \hspace{1cm} (30)

Similarly, workers decision whether to remain unemployed or to take employment
is determined by the following sets of equations.

\[ rJ^U = \lambda(\theta)(J^E - J^U) \]  \hspace{1cm} (31)

\[ rJ^E = w + \delta(J^U - J^E) \]  \hspace{1cm} (32)

Note in the equation for flow value of being employed, value received by the employed workers is denoted by wage \( w \) and not by the indirect utility function as in the case of two sector model.

We assume that the monopoly rent is shared according to the Nash solution to a bargaining problem. Since workers and firms are risk neutral and have the same discount rate, Nash bargaining implies that \( w \) be chosen so that

\[ (1 - \beta)(J^E - J^U) = \beta(J^F - J^V) \]  \hspace{1cm} (33)

By substituting \( rJ^E \) and \( rJ^F \) from 29 and 32 into 33, and by imposing the equilibrium condition \( J^V = 0 \), I derive the wage equation

\[ w = rJ^U + \beta(\phi - rJ^U) \]  \hspace{1cm} (34)

Workers receive their reservation wage \( rJ^U \) and a fraction \( \beta \) of the net surplus that they create by accepting the job. The wage equation can be simplified by making use of 33 and the equilibrium condition for jobs, \( J^F = \frac{c\theta}{\lambda(\theta)} \) from 30, to substitute \((J^E - J^U)\) out of 31. This gives the following expression for \( rJ^U \):

\[ rJ^U = \frac{\beta c\theta}{(1 - \beta)} \]

which, when substituted in to the wage equation 34 to get the aggregate wage
equation which holds in equilibrium.

\[ w = \beta(\phi + \epsilon \theta) \]  \hspace{1cm} (35)

By using 29 and 30 and the steady state condition \( J^V = 0 \), in equilibrium

\[ \phi - w - \frac{(r + \delta)c\theta}{\lambda(\theta)} = 0 \] \hspace{1cm} (36)

The steady state equilibrium \( \theta \) and \( w \) is obtained by solving equations (35) and (36).
Table 1.1 a:
One Sector Random Search Model with Productivity Change

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Table 1.1 b:
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*ur = Unemployment Rate
### Table 1.2 a:
Random Search Model with Preference Change

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$*ur = Unemployment Rate$
Table 1.3 a:  
Random Search Model with Changes in Bargaining Power of Workers

<table>
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<tr>
<th>( \beta )</th>
<th>( \theta )</th>
<th>( w_1 )</th>
<th>( w_2 )</th>
<th>( v_1 )</th>
<th>( v_2 )</th>
<th>( e_1 )</th>
<th>( e_2 )</th>
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Table 1.3 b:  
Sector Specific Search Model with Changes in Bargaining Power of Workers

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*ur = Unemployment Rate
### Table 1.4 a:
Random Search Model with Productivity Changes and Sectoral Shifts

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<th>$\phi_1$</th>
<th>$\alpha$</th>
<th>$\theta$</th>
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<th>$w_2$</th>
<th>$u$</th>
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### Table 1.4 b:
Sector Specific Model with Productivity Change and Sectoral Shifts

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<th>$\alpha$</th>
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<th>$\theta_2$</th>
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<th>$w_2$</th>
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<th>$u_2$</th>
<th>$\text{ur}_1^*$</th>
<th>$\text{ur}_2^*$</th>
<th>$u$</th>
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</thead>
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<tr>
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<td>1</td>
<td>0.8948</td>
<td>0.8948</td>
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*ur = Unemployment Rate
Fig: 1.1

Productivity and Unemployment (1948-1984)

Source: Bureau of Labor Statistics
Fig: 1.2

Productivity and Unemployment (1985-2008)

Source: Bureau of Labor Statistics
Essay 2:

Two Sector Search Model with Endogenous Flow Capital
Abstract

This paper expands the scope of the two sector search model to include endogenous flow capital under both random and sector specific search framework. The model shows that if capital market is perfectly mobile, then changes in productivity in the economy will not have any impact on the employment and vacancies across sectors. On the other hand, standard results under changes in preferences and bargaining power still holds.
1 Introduction

This paper extends the two sector search and matching model Mandal (2009) to incorporate physical capital investment in jobs by firms. This introduces an intensive (along with the extensive) margin to consider the effects of changes in preferences and technologies on the employment and consumption patterns of workers as well as on the nature of job creation by the firms. In the model, the permanent productivity changes have no impact on the vacancies and unemployment. All the dynamics in the model come through adjustment in wages and capital. In the model, it is assumed that the cost of acquiring capital is linear, e.g. the firm can acquire as much capital it wants without increasing the cost of capital. This linearity in cost of capital leads to no change in unemployment and vacancies as a result of any change in productivity. Change in productivity leads to substitution of capital for labor. The model has been able to capture the steady state fact that in the long run, unemployment do not have any trend. On the other hand, change in the preference and the bargaining power of the workers, have impact on the equilibrium employment and the vacancies.

The paper introduces endogenous flow capital in a two sector search model. Firms must commit to the amount of capital to be employed before the actual matching process. Therefore, firms maximize the value of creating vacancies subject to the Nash bargaining wage constraints. Intuitively, firm need to decide how much capital to be employed per time period based upon expected possibility of the match. In case match is successful, it will lead to production but if match does not materialize, then the capital gets destroyed. So next period if the firm decides to search, then again, the firm need to decide how much capital to be employed.
The treatment of capital is varied in the literature. Capital can be introduced in the model either before the hiring process or after the hiring process. Pissarides (2000) introduced capital after the hiring process. Once the wage rate is agreed upon after a successful match, the firm decides the amount of capital to be employed per efficiency unit of labor. Pissarides’ model assumes that there is a perfect second hand market for capital goods. Therefore, the firm earns returns on the capital stock either through production or by renting it out. Hence, capital stock owned by the firm becomes part of the value of job. In this set of models, the essential features of the unemployment model remain unaltered and on the other hand the capital decision is unaffected by the existence of matching friction.

The models with *ex-ante* investments is an interesting topic. Under this, firms invest in capital before hiring. For example, in Acemoglu (1996), Acemoglu and Shimer (1999) and Masters (1998,2009), agents decide on how much capital to acquire prior to searching. Most of these models are concerned with efficiency property of the steady state vacancy to unemployment ratio ($\theta$). In this class of models, there is no $\theta$ that makes the equilibrium outcome under bargaining efficient. The value of $\theta$ that provides the right incentive for investment in human capital by workers is different from the value of that provides the right incentive for firms. However, Acemoglu and Shimer (1999) show competitive search equilibrium with ex ante investments is efficient. Another case where there is no $\theta$ such that bargaining yields the efficient outcome is discussed by Smith (1999), who assumes firms have concave production functions and hire a large number of workers, but bargain with each one individually.

Capital can also be job specific. Wong (2001) analyzed a model where firms
invest in specific capital. Capital investment in this paper is job specific. First the firm decides to spend the optimum amount of capital and then hires a worker through matching process. This job specific investment can be viewed as the cost of providing training. The idea is that, more job specific capital implies more productive workers and hence higher returns, but the higher returns come with higher cost of capital. Workers do not derive utility from acquiring skills on the job. Productivity generated by specific training is lost once a job-worker match terminates.

The paper analyzes the sectoral and aggregate dynamics of the effects of change in productivity, consumer preference and bargaining power of the workers on employment, unemployment and vacancies. I assume consumers have Cobb-Douglas preferences, which introduces a non-transferrable utility function in the model. I will discuss two variants of the two sector model - random search and sector specific search. In the random search, unemployed workers search for jobs irrespective of any sector, whereas in sector specific search, workers look for jobs only in one of the two sectors.

In the model, productivity can be increased either by hiring more capital for job specific training or by some exogenous factors. The job specific investment in capital is model driven and hence endogenous. Changes in the exogenous labor productivity do not lead to any changes in vacancies and unemployment. All the adjustments in the model is due to adjustments in capital inputs and wages and not in labor inputs. Sectoral shifts are introduced through change in consumer preferences. Any change in demand side of the market, has important implications in the labor market. In terms of policy, the model does provide interesting insights. If the capital market is flexible and firms are able to hire
optimal amount of capital, any changes in labor productivity will not affect the employment or unemployment in the economy.

The paper is structured as follows: section 2 describes the model environment. Section 3 and 4 describes the model under random search and sector specific search framework, respectively. Section 5 discusses comparative statics using calibration and numerical analysis. Some final conclusions are presented in section 6.

2 Model Environment

2.1 Demography, Preferences and Technology

I am assuming a continuous time, infinite horizon framework. There are two sectors of production with a large number of firms associated with each sector. Firms create as many vacancies as they like subject to a zero ex-ante profit condition. The (endogenous) mass of vacancies is \( v = v_1 + v_2 \), where \( v_i \) is the mass of vacancies in sector \( i \). Once a firm creates a vacancy, it incurs a flow capital cost \( k_i \), in units of its own output, to maintain a job throughout its life. Jobs are either vacant or filled. Filled jobs break up at the exogenous Poisson arrival rate \( \delta \).

Labor and capital are used to produce two nonstorable final goods that are then sold in a competitive market. Production occurs according to a neoclassical production function \( \phi_i F(K_i, N_i) \), where \( K_i \) and \( N_i \), respectively, denote aggregate capital and employment in sector \( i = 1, 2 \). The production function is strictly concave, exhibits positive but diminishing marginal productivity of capital and
labor, and constant returns to scale. I denote the capital stock per worker as $k_i$, where $k_i$ is the ratio of $K_i/N_i$. Constant returns to scale means that the output of a job/worker pair is given by $\phi_i f(k_i) = \phi_i F(K_i/N_i, 1)$, where $f'(k_i) > 0$ and $f''(k_i) < 0$.

The model economy is populated by an infinitely lived continuum of workers with mass one, where workers are ex-ante identical. They derive utility from the consumption of the final goods and maximize the present discounted value of their utility. Preferences of all agents are defined over the final consumption goods alone. Workers have an utility function over consumption $U = y_1^\alpha y_2^{1-\alpha}$, where $y_i$ is the consumption of the good produced in sector $i$.\footnote{Compared with the standard one sector models, this introduces non-linearity in the utility function which means that the usual Hosios [1990] condition does not hold.} Both workers and firms are risk neutral with a common discount rate $r$.

### 2.2 Matching Technology

A matching technology gives the outcome of the investments of resources by firms and workers in the trading process as a function of the inputs. It is a modeling device that captures the implications of the costly trading process without the need to make the heterogeneities and other features that give rise to it explicitly. Unemployed workers and vacancies are assumed to meet each other randomly according to a function $M(u, v)$, where $u$ is the number of unemployed and $v$ is the measure of vacancies. In random search, both types of vacancies have the same probability of meeting workers, and it is the total number of vacancies and unemployment that enters the matching function. In the sector specific search, though functional form of the matching function remains same across sectors, but
arguments in the function e.g. vacancies and unemployment are sector specific.

We assume that the meeting function is characterized by constant returns to scale so that

\[ M(u, v) = M \left( 1, \frac{v}{u} \right) u = \lambda(\theta) u, \quad \text{where} \quad \theta = \frac{v}{u} \]

We assume that \( M(u, v) \) is increasing in both arguments and concave, e.g. \( M'(\theta) > 0 \) and \( M''(\theta) < 0 \). The contact rate of vacancies by a worker searching for employment is \( \frac{M(u, v)}{v} = \lambda(\theta) \) and the rate at which vacancies meet workers is \( \frac{M(u, v)}{u} = \frac{\lambda(\theta)}{\theta} \), where \( \theta \) denotes the labor market tightness. I also make the standard Inada type assumptions on \( M(u, v) \), which ensure that

\[
\lim_{\theta \to 1} \lambda(\theta) = 0, \quad \lim_{\theta \to 0} \lambda(\theta) = \infty, \quad \lim_{\theta \to \infty} \frac{\lambda(\theta)}{\theta} = 0 \quad \text{and} \quad \lim_{\theta \to 0} \frac{\lambda(\theta)}{\theta} = \infty.
\]

The model has random search technology, which implies aggregate variables known but the whereabouts of individual vacancies/workers unknown.

2.3 Institutions

In equilibrium, because of the existence of entry/exit externality\(^2\), filled jobs generate total returns which is strictly greater than the expected returns of the searching firms and the workers. If the firms and the workers get separated, both of them need to go through a costly process of job search. Hence any

---

\(^2\)Entry/exit externality arises because of the dependence of the functions \( \lambda(\theta) \) and \( \frac{\lambda(\theta)}{\theta} \) on the relative number of traders. The presence of an additional worker (firm) makes it easier (harder) for vacancies to find workers but harder (easier) for workers to find jobs. For example, during a short interval of time \( \delta t \), there is a positive probability \( \left( 1 - \frac{\lambda(\theta)}{\theta} \delta t \right) \) that the hiring firm will not find a worker and another positive probability \( (1 - \lambda(\theta) \delta t) \) that an unemployed worker will not find a job, whatever be the set of prices. Hence, whenever there is a successful match, it does generate some trading or entry/exit externalities. The existence of this externality has important implications for the efficiency of the equilibrium (Hosios, 1990).
realized match generate surpluses. If there is a surplus, it needs to be allocated between the firms and the workers by some rule. Based upon the literature, I am assuming match surpluses are divided by the Nash bargaining\(^3\). All matches that are mutually advantageous relative to the alternative of continuing unmatched. That is, the workers’ and firms’ choices constitute a Nash equilibrium in the sense that they are value maximizing, taking as given the actions of the other agents. Exogenous bargaining power of the worker is \(\beta\) and that of the firm is \((1 - \beta)\).

Workers get paid in the good they help produce. Since, utility of the employed workers depend upon the consumption of both the goods, therefore employed workers exchange a part of their earnings for the good produced in the other sector. Exchange takes place in a centralized competitive market in which the relative price \(p\) of good 2 in terms of good 1 is taken as given by all the participants (Good 1 is the numeraire good). Unemployed workers get nothing and spend all their time looking for work.

### 3 Model Analysis - Random Search

In the model with random search, workers look for jobs in a single market. The model economy consists of employed and unemployed workers, where total number of workers employed in each sector is denoted by \(e_i\) and the pool of unemployed workers looking for job is denoted by \(u\). Since the number of workers

\(^{3}\text{John Nash (1950) did not analyze the bargaining process, but took as given four simple axioms and showed that his solution is the unique outcome satisfying these axioms. Based upon Rubinstein (1982), one can provide a game-theoretic description of the bargaining process along the lines of that has a unique subgame perfect equilibrium see e.g. Osborne and Rubinstein (1990).}\)
are normalized to one, therefore \( u + e_1 + e_2 = 1 \). The sector specific wage rate is denoted by \( w_i \). Any snapshot of the economy finds workers of three types defined by their sector of choice and their employment status. Matches are formed between unemployed workers and vacancies whenever the joint surplus that would be realized by the match is nonnegative. I will solve the model via series of Bellman equations and steady state equilibrium conditions.

3.1 Labor Market - Firm Side

Job creation takes place when a firm and a worker meet and agree to an employment contract. Before this, firms must open a new job vacancy and search for potential employees, whereas unemployed workers look for potential employers. The employment contract specifies only a wage rule that gives the wage rate at any moment in time. Hours of work are fixed and either side can break the contract at any time without incurring any termination cost.

Also for convenience, I will assume that firms are sufficiently small, so that each can create one vacant job when it enters the market. The number of jobs is endogenous and determined by profit maximization condition. The market under consideration is perfectly competitive and hence there are free entry and exit of firms. Therefore, profit maximization requires that in equilibrium the profit from one more vacancy should be zero.

For sector \( i \), let us denote by \( J_i^F \) the present discounted value of expected profit from an occupied job and by \( J_i^V \) the present discounted value of expected profit from a vacant job. With a perfect capital market, an infinite horizon and when no dynamic changes in parameters are expected, \( J_i^V \) satisfies the Bellman
equation

\[ rJ_i^V = \frac{\lambda(\theta)}{\theta} (J_i^F - J_i^V) - k_i, \text{ where } i = 1, 2 \]  

(1)

Intuitively, a job is an asset of the firm. The valuation of the asset is such that the capital cost, \( rJ_i^V \), is equal to the rate of return on the asset. The vacant job requires creation of \( k_i \) per unit time of capital, where capital is measured in terms of the goods in the particular sector. The vacancy changes state according to a time invariant Poisson process with rate \( \frac{\lambda(\theta)}{\theta} \). The change of state yields net return \( (J_i^F - J_i^V) \).

Free entry condition drives the profits from additional vacancies to zero. Therefore, the equilibrium condition for supply of vacant job is \( rJ_i^V = 0 \). Hence from equation (1)

\[ J_i^F = \frac{k_i\theta}{\lambda(\theta)} \]  

(2)

This equation states that the expected profit from a new job is equal to the expected cost of hiring a worker.

We can derive flow equations for occupied jobs \( J_i^F \) in the same way we derived flow equations for the vacant jobs. The flow capital cost of the job is \( rJ_i^F \). Filled job yields a net return \( (\phi_i f(k_i) - w_i) \). A match between an employed worker and the firm can breakup and leads to expected loss \( \delta(J_i^V - J_i^F) \). Hence flow equation for \( J_i^F \) is

\[ rJ_i^F = \phi_i f(k_i) - w_i + \delta(J_i^V - J_i^F), \text{ where } i = 1, 2 \]  

(3)

Though the firm takes the interest rate and product value as given, wage rate is determined by the bargain between the meeting firm and the worker. Therefore,
by using equations (2) and (3) we derive the equations

\[ \phi_i f(k_i) - w_i - \frac{(r + \delta)k_i \theta}{\lambda(\theta)} = 0 \]  

Intuitively above equation corresponds to marginal condition for the demand for labor. \( \phi_i f(k_i) \) is the output per person employed and \( \frac{(r + \delta)k_i \theta}{\lambda(\theta)} \) is the expected capitalized value of the firm’s hiring cost.

### 3.2 Goods Market

In this subsection, I will discuss the consumption goods market. Employed workers go to the centralized goods markets to spend their income to buy good produced in the other sector. Each worker maximizes utility subject to the income constraints. Depending upon the sector in which the worker is employed, the worker’s sector specific income constraints are \( y_{11} + py_{12} = w_1 \) for sector 1 and \( y_{21} + py_{22} = pw_2 \) for sector 2, where \( y_{ij} \) is consumption by the sector \( i \) worker of the output of sector \( j \) and \( w_i \) is the wage. The solution to the consumer’s problem yields the following demand conditions.

\[
\begin{align*}
y_{11} &= \alpha w_1 \\
y_{12} &= \frac{(1 - \alpha)w_1}{p} \\
y_{21} &= pw_2 \\
y_{22} &= (1 - \alpha)w_2
\end{align*}
\]
In the economy, total supply of goods must be equal to total demand. In equilibrium, it is determined by the following market clearing conditions

\[
e_1w_1 = e_1y_{11} + e_2y_{21}
\]
\[
e_2w_2 = e_1y_{12} + e_2y_{22}
\]

Intuitively the above equations state that the value of total wages paid to workers in each sector is equal to the value of total output in that sector measured in goods of that sector.

From the market clearing conditions and the demand equations, we can derive the equilibrium price for good 2 in terms of good 1 as follows

\[
p = \frac{(1 - \alpha)e_1w_1}{\alpha e_2w_2}
\]

Given the demand equations and price, the indirect utility derived by the workers from consumption is denoted by

\[
z_1(w_1, w_2) = \alpha w_1^\alpha \left(\frac{w_2 e_2}{e_1}\right)^{(1-\alpha)}
\]
\[
z_2(w_1, w_2) = (1 - \alpha) \left(\frac{w_1 e_1}{e_2}\right)^{\alpha} w_2^{(1-\alpha)}
\]

Indirect utility of the employed workers not only depends upon the wage of the sector in which the worker is employed, but also on the wage in the other sector. Workers get paid in term of the goods they produce. Since workers consume both the goods, therefore wages of other sectors also affect the utility of the employed workers. This particular fact that utility of the workers depend upon the wages in both the sectors has important welfare implications. Even if productivity in
particular sector is going up, it may cause disproportionately more increase in utility in the other sector.

3.3 Labor Market - Worker Side

Workers normally influence the equilibrium outcome through their job search and their influence on wage determination. In this model, the size of the labor force and each worker’s intensity of search are fixed. So the only influence that workers have on the equilibrium outcome is through wages. A typical worker earns $w_i$ when employed and searches for a job when unemployed. I assume opportunity cost of unemployment is zero.

Let $J^U_i$ and $J^E_i$ denotes the present discounted value of the expected income stream of an unemployed and an employed worker in sector $i$. The unemployed worker can become employed with probability $\lambda(\theta)$. Hence we have

$$r J^U_i = \lambda(\theta)((\psi J^E_1 + (1 - \psi) J^E_2) - J^U_i)$$  \hspace{1cm} (7)

where $\psi$ denotes the proportion of total vacancies in sector 1. The above equation states that the flow value of unemployment in any sector is equivalent to the expected gain from finding a job and realizing a flow value $r J^E_i$.

Similarly for the employed worker, the flow equations are

$$r J^E_i = z_i(w_i) + \delta(J^U_i - J^E_i)$$  \hspace{1cm} (8)

The above equation states that the flow value of being employed is equal to value derived from wage which is given by indirect utility $z(w_i)$ and the net
value derived from being unemployed if the current job break up.\footnote{The value of being employed or permanent income of employed workers $rJ^E_i$ is different from the value derived from wage e.g. $z(w_i)$ because of the risk of unemployment. Without on the job search, employed workers stay in their jobs for as long as $J^E_i > J^U$. The necessary and sufficient condition for this to hold is $z(w_i)$ should be more than the unemployment income earned, which assumed to be zero in this model (see Pissarides, 2000).}

By solving equations (7) and (8) and then by substituting equation (5) and (6)

$$rJ^U = \frac{\lambda(\theta)}{(r + \lambda(\theta) + \delta)} \left[ w_1^{\alpha} w_2^{1-\alpha} \left( \psi \alpha \left( \frac{e_2}{e_1} \right)^{(1-\alpha)} + (1 - \psi)(1 - \alpha) \left( \frac{e_1}{e_2} \right)^{\alpha} \right) \right]$$

(9)

3.4 Wage Determination

In equilibrium, occupied jobs yield a total return that is strictly greater than the sum of the expected returns of a searching firm and a searching worker. When a match is formed, a $f(k_i)$ is realized and the wage is determined through rent sharing over the surplus of the match. We assume that the monopoly rent is shared according to the Nash solution to a bargaining problem with threat points equal to the worker’s and firm’s continuation values, respectively. Since in order to form the job match, the firms give up $J^V_i$ for $J^E_i$ and the workers give up $J^U$ for $J^E_i$. Since workers and firms are risk neutral and have the same discount rate, Nash bargaining implies that $w_i$ be chosen so that\footnote{See Appendix 1 for details.}

$$(1 - \beta)(J^E_i - J^U) = \beta(J^E_i - J^V_i) z_i(w_1, w_2)$$

(10)
We can derive the wage equations in implicit form by solving equations (3), (8) and (10)

\[(1 - \beta) \left( \alpha w_1^\alpha \left( \frac{w_2 e_2}{e_1} \right)^{(1-\alpha)} - rJ^U \right) = \beta (\phi_1 f(k_1) - w_1) z_1'(w_1, w_2) \]  
(11)

\[(1 - \beta) \left( (1 - \alpha) \left( \frac{w_1 e_1}{e_2} \right)^\alpha w_2^{(1-\alpha)} - rJ^U \right) = \beta (\phi_2 f(k_2) - w_2) z_2'(w_1, w_2) \]  
(12)

### 3.5 Capital Market

The firms also need to determine how much flow capital to employ. This decision is taken prior to the actual match formation. Therefore, decision of the firm depends upon the value of creating vacancy \( J_i^V \) subject to the wage bargaining condition as given by the Nash bargaining constraints. Hence the firm maximizes the following with respect to \( w_i \) and \( k_i \)

\[ Max_{\{w_i, k_i\}} J_i^V = \frac{\lambda(\theta)(\phi_i f(k_i) - w_i) - \theta(r + \delta)k_i}{r(\theta(r + \delta) + \lambda(\theta))} \]

s.t. \( (1 - \beta)(z_i(w_1, w_2) - rJ^U) = \beta z_i'(w_1, w_2) (\phi_i f(k_i) - w_i - rJ_i^V) \)

By solving the above first order conditions in steady state with \( J_i^V = 0 \) and using the indirect utility functions in (5) and (6)\(^6\)

\[ f'(k_1) = \frac{\theta(r + \delta)(\beta(\alpha - 1)(\phi_1 f(k_1) - w_1) - w_1)}{\lambda(\theta)\phi_1 (\beta(\alpha - 1)((\phi_1 f(k_1) - w_1) - (1 - \beta)w_1))} \]  
(13)

\[ f'(k_2) = \frac{\theta(r + \delta)(\beta\alpha(\phi_2 f(k_2) - w_2) + w_2)}{\lambda(\theta)\phi_2 (\beta\alpha((\phi_2 f(k_2) - w_2) + (1 - \beta)w_2))} \]  
(14)

\(^6\)See Appendix 2 for details.
3.6 Steady State under Random Search

In steady state mean rate of unemployment and employment are constant. Therefore, flow out of unemployment from any sector equal to the flow back into unemployment to that sector. The flow out of unemployment is \( u \lambda(\theta) \) and flow into unemployment is \( \delta (1 - u) \). Therefore in steady state

\[
u = \frac{\delta}{\lambda(\theta) + \delta}
\]  

(15)

The above equilibrium condition states that in steady state, for given \( \delta \) and \( \theta \), there is an unique unemployment rate.

On the same note, in steady state, flows in and out of jobs should be same. Total flows in to jobs is defined as \( \frac{\lambda(\theta) \nu \psi}{\theta} \) and \( \frac{\lambda(\theta) \nu (1 - \psi)}{\theta} \) for sector 1 and sector 2, respectively. Total flow out of job for each sector is \( \delta e_i \). Therefore in steady state

\[
\frac{\lambda(\theta)}{\theta} \nu \psi = \delta e_1
\]

\[
\frac{\lambda(\theta)}{\theta} \nu (1 - \psi) = \delta e_2
\]

Taking the ratio of the above two equations we get in steady state

\[
\frac{\psi}{(1 - \psi)} = \frac{e_1}{e_2}
\]  

(16)

In steady state ratio of the workers in each sector should be equal to the ratio of vacancies created in each sector.

By replacing (16) in (9) and then replacing the resulting equation in equations
(11) and (12) we get the wage equations

\[ w_1 = \frac{\beta \phi_1 f(k_1) \alpha^2}{(1 - \beta) \psi \left( \frac{2}{\psi} - \frac{\lambda(\theta)}{(r + \lambda(\theta) + \delta)} \right) + \beta \alpha^2} \]  

(17)

\[ w_2 = \frac{\beta \phi_2 f(k_2) (1 - \alpha)^2}{(1 - \beta)(1 - \psi) \left( \frac{(1-\alpha)}{(1-\psi)} - \frac{\lambda(\theta)}{(r + \lambda(\theta) + \delta)} \right) + \beta (1 - \alpha)^2} \]  

(18)

### 3.7 Equilibrium under Random Search

**Definition 1** An equilibrium is defined as labor market tightness \( \theta \), wages \( w \), the proportion \( \psi \) of vacancies in each sector and the flow capital \( k_i \), such that equations (4), (13), (14), (17) and (18) are satisfied for both the sectors.

A steady-state equilibrium is a collection of variables \( \{\theta, w, \psi, k_i\} \) that satisfy the following conditions:

First, all matches that are mutually advantageous relative to the alternative of continuing unmatched. The actions of the workers’ and firms’ constitute a Nash equilibrium in the sense that they are value maximizing, taking as given the actions of the other agents.

Second, since this is a long-run model with free entry/exit of vacancies. Therefore, firm’s vacancy creation satisfies zero-value conditions.

Third, prior to match formation, firms decide how much flow capital to be employed based upon expected value of creating vacancies subject to the wage formation based upon the Nash bargaining conditions.

Finally, the appropriate steady-state labor market flow conditions hold. These
will require that the flow of workers into and out of unemployment be equal. Similarly, flows into and out of jobs should be same.

4 Model Analysis - Sector Specific Search

It is seldom the case that the unemployed workers look for jobs randomly across sectors. Often, job search takes place within a particular sector. Based upon worker’s experience and skills, job search is a sector specific activity. Though in the model, all the workers are homogeneous and there are no on the job skill acquisition, still it is possible to analyze the model based upon sector specific search. Workers look for jobs in the sectors where they have the highest discounted value of being employed. Model assumptions remain same except that the matching function varies across sectors, since unemployment and vacancies are sector specific.

Hence probability of vacancies meeting workers varies across sectors. The contact rate of vacancies by a worker searching for employment in sector $i$ is

$$ M(u_i, v_i) = \lambda(\theta_i) $$

and the rate at which vacancies meet workers in sector $i$ is

$$ \frac{M(u_i, v_i)}{\theta_i} = \frac{\lambda(\theta_i)}{\theta_i}, $$

where sector specific labor market tightness is denoted by $\theta_i = \frac{v_i}{u_i}$.

The goods market conditions remain same as in equations (5) and (6). Firms solve the following labor market equations

$$ rJ_i^V = \frac{\lambda(\theta_i)}{\theta_i} (J_i^F - J_i^V) - k_i $$

$$ rJ_i^F = \phi_i f(k_i) - w_i + \delta(J_i^V - J_i^F) $$
The intuition for above equations are exactly same as in the random search framework. Solving the above set of equations we get

$$\phi_i f(k_i) - w_i - \frac{(r + \delta)k_i\theta_i}{\lambda(\theta_i)} = 0 \quad (19)$$

As in random search, the decision to supply labor by the workers is determined by the following equations

$$r J^U_i = \lambda(\theta_i)(J^E_i - J^U_i)$$

$$r J^E_i = z_i(w_i) + \delta(J^U_i - J^E_i)$$

Solving above equations we get

$$r J^U_i = \frac{\lambda(\theta_i)z_i(w_i)}{(r + \delta + \lambda(\theta_i))} \quad (20)$$

Wage is determined by the following Nash bargaining conditions

$$(1 - \beta)(z_i(w_1, w_2) - r J^U_i) = \beta (\phi_i f(k_i) - w_i) z_i'(w_1, w_2) \quad (21)$$

The firms also need to determine how much flow capital to be employed. Hence the firm maximizes the following with respect to $w_i$ and $k_i$

$$Max_{\{w_i, k_i\}} J^V_i = \frac{\lambda(\theta_i)(\phi_i f(k_i) - w_i) - \theta_i(r + \delta)k_i}{r(\theta_i(r + \delta) + \lambda(\theta_i))}$$

$$s.t. \quad (1 - \beta)(z_i(w_i) - r J^U_i) = \beta z_i'(w_i) (\phi_i f(k_i) - w_i - r J^V_i)$$

Hence by solving the above first order conditions in steady state with $J^V_i = 0$
and using the indirect utility functions in (5) and (6)

\[ f'(k_1) = \frac{\theta_1 (r + \delta) (\beta(\alpha - 1)(\phi_1 f(k_1) - w_1) - w_1)}{\lambda(\theta_1) \phi_1(\beta(\alpha - 1)((\phi_1 f(k_1) - w_1) - (1 - \beta)w_1)} \]

\[ f'(k_2) = \frac{\theta_2 (r + \delta)(\beta\alpha(\phi_2 f(k_2) - w_2) + w_2)}{\lambda(\theta_2) \phi_2 (\beta\alpha((\phi_2 f(k_2) - w_2) + (1 - \beta)w_2)} \]

### 4.1 Steady State under Sector Specific Search

In the model, workers are homogeneous across sectors and so do the firms. Worker’s decision in which sector to work is based upon relative value of being unemployed across sectors. In steady state, workers are indifferent to work in either sectors. Therefore in steady state \( rJ^U_1 = rJ^U_2 \), which implies

\[
\frac{e_1}{e_2} = \frac{\lambda(\theta_1)\alpha(r + \delta + \lambda(\theta_2))}{\lambda(\theta_2)(1 - \alpha)(r + \delta + \lambda(\theta_1))}
\]

The set of wage equations are

\[
w_1 = \frac{\beta\phi_1 f(k_1)\alpha(r + \delta + \lambda(\theta_1))}{(1 - \beta)(r + \delta) + \beta\alpha(r + \delta + \lambda(\theta_1))}
\]

\[
w_2 = \frac{\beta\phi_2 f(k_2)(1 - \alpha)(r + \delta + \lambda(\theta_2))}{(1 - \beta)(r + \delta) + \beta(1 - \alpha)(r + \delta + \lambda(\theta_2))}
\]

Also in steady state, flow out of unemployment in each sector is equal to flow into unemployment. Therefore

\[
u_1 \lambda(\theta_1) = \delta(\pi - u_1)
\]

\[
u_2 \lambda(\theta_2) = \delta((1 - \pi) - u_2)
\]
where \( \pi \) is the proportion of total pool of workers in sector 1 and \( (1 - \pi) \) is the proportion of workers in sector 2.

Similarly, flows into and out of jobs should satisfy

\[
\frac{\lambda(\theta_1)}{\theta_1} v_1 = \delta e_1
\]

\[\text{(29)}\]

\[
\frac{\lambda(\theta_2)}{\theta_2} v_2 = \delta e_2
\]

\[\text{(30)}\]

Therefore from equation (27) - (30) we get

\[
\frac{e_1}{e_2} = \frac{\lambda(\theta_1) \pi \lambda(\theta_2) + \delta}{\lambda(\theta_2)(1 - \pi)(\lambda(\theta_1) + \delta)}
\]

\[\text{(31)}\]

In steady state

\[
\frac{\alpha(r + \delta + \lambda(\theta_2))}{(1 - \alpha)(r + \delta + \lambda(\theta_1))} = \frac{\pi(\lambda(\theta_2) + \delta)}{(1 - \pi)(\lambda(\theta_1) + \delta)}
\]

\[\text{(32)}\]

### 4.2 Equilibrium under Sector Specific Search

**Definition 2** An equilibrium is defined as labor market tightness \( \theta_i \), wages \( w_i \), the proportion \( \pi \) of workers looking for job in each sector and the flow capital \( k_i \), such that equations (19), (22), (23), (25), (26) and (32) are satisfied for both the sectors.

A steady-state equilibrium is a collection of variables \( \{\theta_i, w_i, \pi, k_i\} \) that satisfy the following conditions:

First, all matches that are mutually advantageous relative to the alternative of continuing unmatched. The actions of the workers’ and firms’ constitute a
Nash equilibrium in the sense that they are value maximizing, taking as given the actions of the other agents.

Second, since this is a long-run model with free entry/exit of vacancies. Therefore, firm's vacancy creation satisfies zero-value conditions.

Third, prior to match formation, firms decide how much flow capital to be employed based upon expected value of creating vacancies subject to the wage formation based upon the Nash bargaining conditions.

Fourth, workers are indifferent to work in either sectors. Therefore, value of being unemployed across sectors are equal in steady state.

Finally, the appropriate steady-state labor market flow conditions hold. These will require that the flow of workers into and out of unemployment be equal. Similarly, flows into and out of jobs should be same.

5 Calibration and Numerical Analysis

In this section I will present the results of the numerical simulations of the random and the sector specific search models. It shows the comparative statics for the decentralized model only. The model is simulated using the parameter configurations based upon the practice in the literature.

Parameter values are chosen based upon Shimer (2005) and Job Opening and Labor Turnover Survey (JOLTS) data. Without loss of generality, labor productivity $\phi_i$ have been normalized to one. According to the JOLTS data, between 2001 and 2007, separation rate averaged around 0.033 every month.
Consistent with Shimer (2005) estimates, this means that on an average a job lasted for 2.5 years before it broke up. The model is solved assuming a yearly time period, hence I set the job breakup rate $\delta$ as 0.4 per year. The discount rate $r$ is assumed to be 5 percent per year, which is standard in the literature. The literature generally assumes a Cobb-Douglas matching function of the form $m(\theta) = \overline{L} \theta^{(1-\eta)}$, where $\overline{L}$ is the matching intensity and the elasticity parameter is $\eta$. Based upon Shimer (2005) and Petrongolo and Pissarides (2001), I set the elasticity parameter $\eta$ as 0.5. Shimer (2005) showed that if $\eta = \beta$, then "Hosios (1990) Rule", the decentralized equilibrium maximizes a well posed social planner’s problem. Therefore, bargaining power of the workers has been set to 0.5. The matching intensity $\overline{L}$ has been set to 7.2. The production function is assumed to be Cobb-Douglas of the form $\phi_I k_t^{0.3}$, where share of capital is 0.3 and the share of labor 0.7.

In the following sections, I have compared the behavior of the two sector search model under random and sector specific search framework. I have also compared the model under standard one sector search framework\footnote{See Appendix 3 for one sector search framework.}.

### 5.1 Productivity Change

In the model with perfectly mobile endogenous flow capital, changes in fully anticipated permanent productivity changes have no impact on the employment and vacancy creation in the economy. All the adjustments come through changes in the flow capital and the wage. Table 2.1(a,b,c) show the effects of permanent productivity change on the model economy. In the two sector model, productivity change only impacts one of the sectors through change in wages and the capital.
There are absolutely no spillover effects across sectors. In the model, the capital is assumed to be linear, e.g. the cost of capital do not increase as a result of increase in demand for capital. Hence, in equilibrium, as a result of productivity change, firms substitute capital for labor. This explains the fact that the productivity change has no impact on vacancies and unemployment. One important thing to note is that there is no difference between the sector specific and the random search models with respect to the long-run effects of permanent productivity changes.

5.2 Preference Change

In the model, it is possible to introduce shifts in consumer preferences across sectors, which is not possible in standard one sector Mortensen-Pissarides models. Changes in consumer preference are introduced through change in parameter $\alpha$, the share of good 1 in the consumption bundle of an employed worker. (Table 2.2a) and (Table 2.2b) shows the results of the preference changes.

In the random search model (Table 2.2a), increase in the relative share of the demand for sector 1 good leads to a fall in $\theta$, which implies falling vacancies and rising unemployment. Though on aggregate, vacancies have fallen and unemployment has gone up, dynamics are very different across the sectors. In sector 1 vacancies increased, whereas in sector 2 it declined. The decrease in sector 2 vacancies are not offset by the increase in sector 1 vacancies. (Table 2.2a) shows that because of shift in preferences towards sector 1 goods, employment increased in sector 1, whereas falling relative demand for sector 2 goods caused decrease in employment in sector 2. Again, increased employment in sector 1 cannot offset falling employment in sector 2 and hence total employment in the
economy declined. As a result total unemployment in the economy increased. In
nominal terms, wages in sector 1 is less than sector 2 but the real wage defined
as \( pw_2 \) is higher in sector 1 as compare to sector 2. This decline in real wage is
due to fall in price of good 2 as a result of preference shift to good 1.

In the sector specific search model (Table 2.2b), increase in \( \alpha \) leads to de-
crease in \( \theta_1 \) and the increase in \( \theta_2 \). Decrease in \( \theta_1 \) is attributable to more than
proportionate increase in unemployment \( u_1 \) compared to vacancies \( v_1 \). Similarly
increase in \( \theta_2 \) is due to relatively larger decrease in unemployment \( u_2 \) compared
to vacancies \( v_2 \). It is counter intuitive that unemployment has gone up in the sec-
tor 1 which experienced favorable shift in demand whereas unemployment has
depaired in sector 2 whose demand for output decreased relatively. The economic
reason is that because of favorable shift in demand for sector 1 goods, wages has
gone up in sector 1 relative to sector 2 and hence proportionately more workers
started searching for job in sector 1 compared to sector 2. This is evident from
the increase in the parameter \( \pi \) which is the pool of workers looking for jobs in
sector 1.

Increase in the matching friction as a result of sectoral reallocation of workers
towards sector 1, lead to increase in unemployment. This result has particular
policy implication. Economies go through structural changes overtime. For
example, in the United States., relative share of the services sector has gone
up compared to the manufacturing sector in the last couple of decades. This
unbalanced shift towards services sector might lead to disproportionately more
reallocation of workers to these sectors, which might lead to increased unemploy-
ment as these booming sectors might not be able to absorb all the workers. As
a result unemployment might go up in the economy.
5.3 Changes in Relative Bargaining Power

In the search literature, wages are determined by Nash bargaining, where bargaining power of the workers and firms are assumed to be constant overtime and typically assumed to be 0.5. There is no reason to believe that both the firms and the workers have the same bargaining power. Technically match will take place for any bargaining power strictly greater than zero e.g. for any $\beta > 0$. Because wages are settled through Nash bargaining, hence any change in the bargaining power of the worker will have influence on the equilibrium wages and hence as a result on vacancies and unemployment.

The impact of changes in bargaining power of workers $\beta$ (Table 2.3a and Table 2.3b) on wages, $\theta$, vacancies, employment and unemployment are exactly same in both the random search and the sector specific search framework. As $\beta$ increases, the wages go up. Increase in wages decrease the value of filled vacancies and hence reduce the number of vacancies created. Therefore overall unemployment in the economy increases. Therefore, the adverse effect of increase in bargaining power of the workers increase the unemployment.

6 Conclusion

An important element in this model is the presence of endogenous flow cost which has been interpreted as job specific capital investment. Typical literature assumes either fixed or flow cost of creating vacancies but in either case costs are exogenous. Introduction of endogenous flow capital makes the vacancies and unemployment completely irresponsible to productivity changes. All adjustments in the economy as a result of productivity changes are due to change in the wages
and the flow capital. Therefore, unlike in standard search models, in this model productivity changes do not generate any kind of fluctuations in vacancy and unemployment.

On the other hand, I have shown that changes in sectoral preference across the economy or changes in the bargaining power of the worker or firms does generate substantial fluctuations in vacancies and unemployment.

References


Appendix 1: Nash Bargaining

Once match between the employer and the worker takes place, it generates a matching surplus. In the model, match surplus is distributed according to Nash bargaining solution. Nash bargaining solution is the $w_i$ that maximizes the
weighted product of the worker’s and the firm’s net return from the job match. Therefore the wage rate for the job satisfies

\[ w_i = \arg \max (J_i^E - J_i^U) \beta (J_i^F - J_i^V)^{(1-\beta)} \]

Using (3) and (8)

\[ w_i = \arg \max \left( \frac{\phi_i - w_i - rJ_i^V}{r + \delta} \right)^\beta \left( \frac{z_i(w_1, w_2) - rJ_i^U}{r + \delta} \right)^{(1-\beta)} \]

Maximizing above expression with respect to \( w_i \) gives

\[ (1 - \beta)(z_i(w_1, w_2) - rJ_i^U) = \beta \left( \phi_i - w_i \right) z_i'(w_1, w_2) \]

Appendix 2: Equilibrium Conditions in the Capital Market

From (1), we get

\[ J_i^V = \frac{\lambda(\theta)(\phi_i f(k_i) - w_i) - \theta(r + \delta)k_i}{r(\theta(r + \delta) + \lambda(\theta))} \]

Hence the firm maximizes the following with respect to \( w_i \) and \( k_i \)

\[ \max_{\{w_i, k_i\}} J_i^V = \frac{\lambda(\theta)(\phi_i f(k_i) - w_i) - \theta(r + \delta)k_i}{r(\theta(r + \delta) + \lambda(\theta))} \]

s.t. \( (1 - \beta)(z_i(w_1, w_2) - rJ_i^U) = \beta z_i'(w_1, w_2) \left( \phi_i f(k_i) - w_i - rJ_i^V \right) \]
The first order conditions are

\[ k_i : \frac{\lambda(\theta)\phi_i f'(k_i) - \theta(r + \delta)}{r(\theta(r + \delta) + \lambda(\theta))} + \gamma(\beta z'_i(w_1, w_2)\phi_i f'(k_i)) = 0 \]

where \( \gamma \) is the Lagrange multiplier.

\[ w_i : \frac{-\lambda(\theta)}{r(\theta(r + \delta) + \lambda(\theta))} + \gamma(\beta z''_i(w_1, w_2)(\phi_i f(k_i) - w_i - r J^V_i) - \beta z'_i(w_1, w_2) - (1 - \beta) z'_i(w_1, w_2) = 0 \]

By solving the above first order conditions in steady state with \( J^V_i = 0 \), we get

\[ f'(k_i) = \frac{\theta(r + \delta) \left( \frac{\beta z''_i(w_1, w_2)}{z'_i(w_1, w_2)}(\phi_i f(k_i) - w_i) - 1 \right)}{\lambda(\theta) \phi_i \left( \frac{\beta z''_i(w_1, w_2)}{z'_i(w_1, w_2)}(\phi_i f(k_i) - w_i) - (1 - \beta) \right)} \]

For the sector specific search, the first order condition is

\[ f'(k_i) = \frac{\theta_i(r + \delta) \left( \frac{\beta z''_i(w_1, w_2)}{z'_i(w_1, w_2)}(\phi_i f(k_i) - w_i) - 1 \right)}{\lambda(\theta_i) \phi_i \left( \frac{\beta z''_i(w_1, w_2)}{z'_i(w_1, w_2)}(\phi_i f(k_i) - w_i) - (1 - \beta) \right)} \]

**Appendix 3: Model Analysis - One Sector Random Search**

The analysis of one sector search models are much simpler compared to two sector search. In one sector search, workers gets what they produce and wages reflects workers’ utility. Workers as consumers do not have to decide which goods to consume to maximize utility, or in other words there is no demand side in the case of one sector model. The logic and rationale for the one sector model follows exactly as in the two sector model I developed in the previous section.
Value of creating new vacancies and the value to fill the existing vacancies are denoted as

\[ r_J^V = \frac{\lambda(\theta)}{\theta}(J^F - J^V) - k \]  
(33)

\[ r_J^F = \phi f(k) - w + \delta(J^V - J^F) \]  
(34)

Similarly workers decision whether to remain unemployed or to take employment is determined by the following sets of equations.

\[ r_J^U = \lambda(\theta)(J^E - J^U) \]  
(35)

\[ r_J^E = w + \delta(J^U - J^E) \]  
(36)

Note in the equation for flow value of being employed, value received by the employed workers is denoted by wage \( w \) and not by the indirect utility function as in the case of two sector model.

We assume that the monopoly rent is shared according to the Nash solution to a bargaining problem. Since workers and firms are risk neutral and have the same discount rate, Nash bargaining implies that \( w \) be chosen so that

\[ (1 - \beta)(J^E - J^U) = \beta(J^F - J^V) \]  
(37)

By substituting \( r_J^E \) and \( r_J^F \) from 33 and 36 into 37, and by imposing the equilibrium condition \( J^V = 0 \), I derive the wage equation

\[ w = r_J^U + \beta(\phi f(k) - r_J^U) \]  
(38)

Workers receive their reservation wage \( r_J^U \) and a fraction \( \beta \) of the net surplus.
that they create by accepting the job. The wage equation can be simplified by making use of 37 and the equilibrium condition for jobs, \( J^F = \frac{tk\theta}{\lambda(\theta)} \) from 34, to substitute \((J^E - J^U)\) out of 35. This gives the following expression for \( rJ^U \):

\[
r J^U = \frac{\beta k\theta}{(1 - \beta)}
\]

which, when substituted in to the wage equation 38 to get the aggregate wage equation which holds in equilibrium.

\[
w = \beta(\phi f(k) + k\theta)
\]  

(39)

By using 33 and 34 and the steady state condition \( J^V = 0 \), in equilibrium

\[
\phi f(k) - w - \frac{(r + \delta)k\theta}{\lambda(\theta)} = 0
\]  

(40)

The steady state equilibrium \( \theta \) and \( w \) is obtained by solving equations (39) and (40).
Table 2.1 a:  
One Sector Random Search Model with Productivity Change

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Table 2.1 b:  
Random Search Model with Productivity Change

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Table 2.1 c:  
Sector Specific Search Model with Productivity Change

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*ur = Unemployment Rate
### Table 2.2 a:
**Random Search Model with Preference Change**

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*ur = Unemployment Rate

### Table 2.2 b:
**Sector Specific Search Model with Preference Change**

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*ur = Unemployment Rate
### Table 2.3 a:

Random Search Model with Changes in Bargaining Power of Workers

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*ur = Unemployment Rate

### Table 2.3 b:

Sector Specific Search Model with Changes in Bargaining Power of Workers

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*ur = Unemployment Rate
Essay 3:

Empirical Matching Function: Estimations Using JOLTS
Abstract

This paper presents evidence on the degree of returns to scale of matching function in the United States labor market using the JOLTS data for the period December 2000 to March 2009. The empirical results find that the matching function shows constant returns to scale (CRS), but CRS is only limited to the non-recessionary time period. This raises serious questions about the stability of the matching function and its use for business cycle analysis. Also, the paper develops a method to estimate job-to-job flows exploiting the information from CPS and the JOLTS.
1 Introduction

The purpose of this paper is to estimate the aggregate matching function for the United States using the Job Openings and Labor Turnover Survey (JOLTS) data. The paper finds that, contrary to prior studies, the constant returns to scale (CRS) assumption for matching function only holds for the non-recessionary period.

Modern economies are characterized by extremely dynamic labor markets. There are large flows of jobs and workers between the states of activity and inactivity every month. For example, in the United States, during the period between December 2000 and March 2009, in a typical month on average 7.95 million people are unemployed and actively looking for jobs. In the same time period, about 3.96 million vacancies are posted every month and about 5.10 million workers hired per month. To complement these labor market flows, average 5.10 million workers have been separated every month. All these flows are significant, when compared with the average 149 million labor force in the United States. Though, the unemployment behavior has caught the interest of the economists for a long time, understanding the important labor market dynamics related to hiring, separations and vacancies are fairly new in the profession.

The negative empirical relationship between vacancies and unemployment is well known since the period after World War II. This relationship is popularly known as U-V curve (or Beveridge curve as it was named in the 1980s after William Beveridge\(^1\)). Beveridge curve originated from the work of two British economists, Dow

\(^1\)Origins of the idea behind the U-V curve can be traced to the work by William H. Beveridge "Unemployment: A Problem of Industry" (1909 and 1930). New ed. London: Longmans, Green,
and Dicks-Mirreaux (1958). They sought to establish a measure for excess demand, as they were primarily concerned about inflation in the goods market. Given the success of the Keynesian revolution during the Great Depression, aggregate demand management emerged as the most important tool in the arsenal of economists during the post-war period. Fluctuations in aggregate demand could easily lead to inflation, therefore, Dow and Dicks-Mirreaux looked for an indicator that could guide the Keynesian policies in such a way that unemployment could be removed if necessary while avoiding inflation.

A series of empirical studies has been done on the Beveridge curve throughout the 1960s and the 1970s. Serious questions has been raised on the stability of the Beveridge curve in the empirical studies done in the 1970s. Studies found supposed ‘breakpoints’ in the Beveridge curve, suggesting shifts of the curve further or closer to the origin corresponding to higher or lower levels of structural unemployment. This raised serious concern about the usefulness of the Beveridge curve for economic analyses.

In the 1980s, Beveridge analysis was completely abandoned, only to be revived later in the decade through labor search theory. Phelps and Holt (1970) argued that Beveridge curve analysis suffered from two flaws: i) it was static in approach ii) it lacked microeconomic foundation. Based upon rational expectations revolution, Phelps and Holt argued that unemployed make rational decisions to accept or reject a job offer. Rejection of job offer is based upon the fact that the unemployed worker

---

2 For a survey of UV or Beveridge curve see (Rodenburg, 2007).
expects higher payoffs from other future job offers. Therefore, unemployment can be viewed as a productive investment from the perspective of individual workers. This branch of thinking, where the labor market is seen as a flow concept, is popularly known as 'search theory'.

In the 1990s, a key branch of search theory, popularly known as 'matching models' became popular. This is primarily based upon the works of Diamond (1982), Diamond and Blanchard (1989, 1994), Pissarides (1990) and Mortensen and Pissarides (1994). The key idea is that unemployed workers are looking for jobs and the firms are looking for workers. This complicated exchange process between the unemployed workers and the firms can be summarized by a well-behaved matching function that gives the number of jobs formed at any moment in time in terms of the inputs of firms and workers into search. Variations in job matches at given inputs reflect changes in the intensity of frictions that characterize labor market trade. With stronger frictions, the labor market becomes less effective in matching unemployed workers to available vacancies and the resulting matching rate is reduced (Pissarides, 2000; Blanchard and Diamond, 1989). The attraction of matching function is that it enables the modeling of frictions in otherwise conventional models, with minimum of added complexity.\(^5\)

Movements along a fixed downward-sloping Beveridge curve are associated with cyclical shocks, while shifts of the curve arise from structural factors that alter the matching efficiency between job vacancies and unemployed workers. One of

\(^5\)Shifts of the Beveridge curve can be explained by the matching process. The flows into and out of unemployment and vacancies, together with the job matching process, determine the outcome of unemployment and vacancies that are summarized in the Beveridge curve. More efficient matching implies higher outflows from unemployment and vacancies and hence inward shift of the Beveridge curve (see Bleakley and Fuhrer, 1997).
the main concerns in studying the aggregate matching function is to identify the level of returns to scale. In particular, Diamond (1982, 1984) and Pissarides (1986) showed that increasing returns are consistent with search externalities and multiple equilibria. Multiple equilibria have important policy implications. More about this in section 2.

Empirically many attempts have been made to estimate the matching function. For the United States, first attempt to estimate a matching function was made by Blanchard and Diamond (1989), then consequently Warren (1996) and Bleakley and Fuhrer (1997). Due to lack of data related to vacancies and hires, all of these studies had to construct the data from different sources for their estimation purpose. This problem of lack of data is partially solved with the introduction of the JOLTS by the Bureau of Labor Statistics (BLS) since December 2000.

The paper is organized as follows: section 2 gives a brief introduction to matching function. Section 3 discusses the empirical literature on estimating matching function. Section 4 discuss the data source and section 5 the empirical implementation. Finally, conclusions are given in section 6.

2 Matching Function

The matching function summarizes the trading technology between the employers and the potential employees. In a decentralized market, workers look for the right job and employers look for the right worker, but each search for varying degree of intensity and success. The primary idea behind matching function is to model this
complex search procedure through a simplified well behaved function that gives the number of jobs formed at any moment in time in terms of the number of workers looking for jobs, the number of firms looking for workers and a small number of other variables.

A matching function generally takes the following functional form.

\[ M = M(U, V) \]

where \( M \) is the number of jobs formed during a given time interval, \( U \) is the number of unemployed workers looking for work and \( V \) the number of vacant jobs. We assume that \( M(U, V) \) is increasing in both arguments and concave, e.g. \( M'(\theta) > 0 \) and \( M''(\theta) < 0 \). Other restrictions usually imposed are \( M(0, V) = M(U, 0) = 0 \).

Based upon above definition of the matching function, \( \frac{M(U, V)}{U} \) is the probability that an unemployed worker finds a job during a unit length of time. Similarly, a vacant job is filled with probability \( \frac{M(U, V)}{V} \). In a stationary environment, the inverse of each probability is the mean duration of unemployment and vacancies respectively.

Critical to our thinking about labor markets is the notion of matching function. Theoretically this function encompasses a complex reality in the ways jobs and vacancies meet. It accounts for all the differences between workers and jobs, as well as the intensity of search on the part of workers and firms. Hence, one legitimate question is whether or not this matching function exists, and even if it exists what are its main characteristics.
As pointed out earlier, returns to scale of the matching function has important policy implications. Pissarides (2000) showed that increasing returns is a necessary condition for multiple equilibria in search framework\textsuperscript{6}. The significance of multiple equilibria in this set up is that if they exist, temporary policy measures may move the economy from lower level equilibrium to higher level equilibrium.

### 3 Empirical Matching Function: A Literature Review

Various attempts made to estimate aggregate matching function for different countries. (Petrongolo and Pissarides, 2001) First attempt to estimate matching function was by Pissarides (1986) for Britain over the period 1967-1983. Results with both linear and log-linear specifications are reported. Pissarides estimated the following log-linear specification

\[
\ln \left( \frac{M}{U} \right)_t = \alpha_0 + \alpha_1 \ln \left( \frac{V}{U} \right)_t + \alpha_2 t + \alpha_3 t^2 + lags + \text{structural variables}
\]

where dependent variable is the average monthly outflow rate from male unemployment during the quarter. The unemployment series used is for registered male unemployment and the series for vacancies is notified vacancies adjusted upwards for incomplete coverage.

\textsuperscript{6}With increasing returns, a rise in the scale of trading for given unemployment and vacancies can increase the probability of matching. This is because the number of job matches increases faster than the inputs.
Both the linear and log-linear specification strongly supports constant returns to scale in U and V. The estimated elasticity of matches with respect to vacancies is 0.3 and with respect to unemployment 0.7. No other variables were found to be significant except for the time trends.

Blanchard and Diamond (1989, 1990) made the first attempt to estimate matching function for the United States. They estimated the following log-linear specification in levels.

\[
\ln M_t = \alpha_0 + \alpha_1 \ln U_t + \alpha_2 \ln V_t + \alpha_3 t
\]

where dependent variable is the new hires. The data on new hires constructed from monthly data in Current Population Survey (CPS) and the data on vacancies are estimated from Conference Board’s adjusted Help Wanted Index\(^7\). New hires are estimated as the sum of the flows into employment from unemployment and from out of the labor force, to which estimated flow from employment to employment is added and from which the estimated flow of workers who are recalled rather than newly hired are subtracted.\(^8,9\) The pool of unemployment is the number of unemployed workers as reported by CPS, from which the workers who are classified as "job losers on layoff" has been subtracted.\(^{10}\) They find clear evidence of aggregate matching function with constant or mildly increasing returns to scale, unit elasticity

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\(^7\)Help Wanted Index is based on the counts of job advertisements in major newspapers. It is collected by the Conference Board on monthly basis (see Abraham, 1987).

\(^8\)Flow of workers who are recalled estimated as 1.5 times manufacturing recalls (see Blanchard and Diamond, 1989).

\(^9\)As is well known in CPS, the reported gross flows are biased upward, as incorrect classification of workers generates spurious transitions and thus increases measured gross flows. Both Abowd and Zellner (1985), and Poterba and Summers (1986) have developed techniques to remove the bias in the raw series. The Poterba and Summers adjustments yield fairly different picture of both the absolute and relative sizes of these flows.

\(^{10}\)Employment to employment flows estimated to be 0.4 times the manufacturing quit rate times employment (see Akerlof et. al., 1988).
of substitution and weight 0.4 and 0.6 on unemployment and vacancies respectively.

Later Warren (1996) and Bleakley and Fuhrer (1997) also estimated the matching function for the United States. Similar matching function has been estimated for other countries. These studies generally accept the assumption of a log-linear function with constant returns to scale. An exception is Warren (1996), who estimates a translog matching function by using the monthly United States manufacturing data and finds increasing returns to scale.

In order to estimate a matching function, typically we need estimates of unemployment, vacancies and hires. In most countries, though fairly good data on unemployment is widely available, but data on hires and vacancies are often nonexistent. For example, in the United States, a good proxy for vacancy is the Help Wanted Index prepared by the Conference Board. Blanchard and Diamond (1990) adjusted the Help Wanted Index following Abraham (1983,1987).

Because of the restrictions imposed by the unavailability of reliable data to estimate matching functions, only a few limited efforts have been made to estimate the matching function for the United States. Since December 2000, Bureau of Labor Statistics introduced a new data series called JOLTS. This is an attempt by BLS to collect data on hirings, separations and job openings using a nationwide sample of establishments. This new series completes the labor market picture by collecting

---


12 Though the vacancies data is collected on regular basis in different countries, but often these are data notified to state employment agencies and often they suffer from under-reporting. Also, the proportion of vacancies-notified varies with general economic conditions, both aggregate and sectoral (see Jackman et. al., 1989).
data from businesses to measure labor demand and job turnover.

4 DATA

In recent years, Bureau of Labor Statistics released a new data product called Job Opening and Labor Turnover Survey (JOLTS). The survey is the only existing data source to measure vacancies, hires and separations at the establishment level at a regular monthly frequency in the United States. Hence it is an ideal data source to estimate matching function.

The JOLTS program publishes monthly estimates of vacancies, hires and separations, with separations broken into quits, layoffs and discharges, and other separations. The data start in December 2000 and are updated monthly. The aggregate estimates are available nationally and for four major regions by 2-digit North American Industry Classification System (NAICS).

The primary unit of observation for the JOLTS survey is the establishment, which covers the operations of firms at a single physical location. It covers nonfarm payrolls, which implies that employment estimates generally exclude self-employed individuals and non-profit organizations not covered under the state unemployment insurance program. A sample of roughly 16,000 establishments surveyed each month. The data is weighted so that its employment estimates match those of the Current Employment Statistics (CES) survey. At the micro level, the data can also be matched with BLS Quarterly Census of Employment and Wages (QCEW) data. For further details on this see Faberman (2005).
According to BLS, the definition the key elements of JOLTS are as follows\textsuperscript{13}:

- **Employment**: JOLTS define Employment as all persons on the payroll who worked during or received pay for the pay period that includes the 12th of the month. This definition is consistent with other BLS establishment-based programs.

- **Job Openings (Vacancies)**: JOLTS define Job Openings as all positions that are open (not filled) on the last business day of the month.

- **Hires**: JOLTS define Hires as all additions to the payroll during the month.

- **Separations**: JOLTS define Separations as all employees separated from the payroll during the calendar month.

The monthly data on unemployment is obtained from the Current Population Survey (CPS).

5 Empirical Implementation

5.1 Estimation of Job-to-Job Transitions

Standard matching function does not account for the fact that there are job to job transitions. The assumption is whenever an employed worker is separated from a job; he joins the pool of unemployed workers. Then matching for new job happens

\textsuperscript{13}See Appendix 1 for details.
from this updated pool of unemployed. According to Fallick and Fleischman (2004) and Nagypal (2005), the rate of job-to-job transitions, moves of workers between employers without an intervening unemployment spell, are twice as large today as the rate at which workers move from employment to unemployment.\textsuperscript{14} Moreover, the relative importance of job-to-job transitions has increased dramatically in recent decades: between 1975 and 2000, the rate of job-to-job transitions increased by 59\% and the rate of employment to unemployment transitions declined by 47\% (Stewart, 2002). This means that job-to-job transitions today are at least as important a means of reallocating labor towards its more productive uses as are the transitions of workers through unemployment.

In order to ensure that estimates are consistent with theory, I have made adjustments for job-to-job transitions in the hiring data. Though, CPS or JOLTS, do not give any standard information regarding the job-to-job flows, but information regarding 'quits' in JOLTS and the number of 'unemployed because of job leavers' in CPS, can be used to estimate the job-to-job transitions. Quits in JOLTS are employees who left voluntarily except for retirements or transfers to other locations. So assumption is that, employees who quit, either going for a new job or joining the pool of unemployed labors. Therefore, by subtracting the number of 'unemployed workers because of job leavers' from the 'quits', we can get a good estimate of job-to-job transitions. Therefore, I define hirings as hirings reported by BLS minus the job-to-job transition.\textsuperscript{15}

\textsuperscript{14}For some earlier estimates of job-job transition see Akerlof et. al., 1986 and Blanchard and Diamond, 1989.

\textsuperscript{15}According to my estimates, job-job transitions accounts for on an average 37.33 \% of the total hirings every month. This figure varies considerably from 22.63 \% to 46.32 \%.
The pool of unemployed workers is adjusted for the unemployed workers who consider themselves as having a job. Hence unemployed is defined as the total number of unemployed workers\textsuperscript{16} minus the job losers on temporary layoff and the job losers who completed some temporary jobs.

### 5.2 Baseline Model

Based upon literature, I assume a Cobb-Douglas matching function of the form

\[ M = AV^\alpha U^\beta, \text{ where } \alpha \geq 0, \beta \geq 0 \]  

(1)

where number of matches \( M \) represents total new hires, \( U \) represents total unemployment and \( V \) represents total vacancies and \( A \) is the technology parameter which accounts for geographical and other differences among workers and the vacancies. The sum of coefficients \( \alpha \) and \( \beta \) gives us the returns to scale. I estimate the following function:

\[ \log M_t = \log A + \alpha \log V_t + \beta \log U_t + \lambda_1 t + \lambda_2 t^2 + \epsilon_t \]  

(2)

where \( \epsilon_t \textsuperscript{17} \) is the random error term which has normal distribution with mean zero and variance \( \sigma^2 \) and \( t \) is the time trend. The time period I am considering is December 2000 to March 2009, which accounts for 100 months in total.

The stationarity test results are shown in Table (3.1). \( M_t, U_t \) and \( V_t \) are all non-stationary with integrated of order 1. Since, at least two series are non-stationary

\textsuperscript{16} Pool of unemployed workers is the usual U3 definition of unemployed as provided by the BLS.

\textsuperscript{17} For AR(1) specification, \( \epsilon_t \) follows \( \epsilon_t = \rho \epsilon_{t-1} + u_t \), where \( u_t \sim N(0, \sigma^2) \). AR(1) specification has been used to improve upon the Durbin-Watson statistic.
of the same order, hence there is possibility of estimating a cointegrating regression. Regression results along with the Augmented Engle-Granger cointegration tests are shown in Table (3.2). Various lead and lag structures have been applied to the variables to account for the different timings of data collection. However, the best results are obtained by using the contemporaneously timed data across all variables. The results are extremely sensitive with respect to time trends, especially the squared time trend. The signs of the coefficients of Log($U_t$) and Log($V_t$) are all positive as expected. The best fit is obtained both in terms of $R^2$ and adjusted $R^2$ for specification 3 which includes a squared time trend term and the errors follow an AR(1) specification.

None of the specifications have constant returns to scale as found in previous studies for the United States. Though decreasing returns to scale is neither theoretically, nor empirically implausible, but it raises question regarding the plausible divergence from prior studies.

---

18 Results of stationarity test are given in Table (1). I tested for stationarity of the data using Augmented Dickey-Fuller (ADF) and the Phillips-Perron (PP) test. All the series are non-stationary at 5% level of significance. At the same time, all the series are integrated of order I(1). Hence I estimate a cointegrating regression (see Engle and Granger (1987), Dickey et al. (1991)).

19 Refer to Table (3.3) for regressions with lagged variables. Explanatory variables Log ($V_t$) and Log ($U_t$) turn out to be insignificant for all the specifications.

20 Since hires are flow variable and vacancies and unemployment are stock variables, the use of contemporaneous values of hires, vacancies and unemployment raises the possibility of downward bias in the least-square estimates of returns to scale (Blanchard and Diamond, 1989, 1990). In recognition to this problem, Blanchard and Diamond used various instruments for unemployment and vacancies. However, Hall in his comments on Blanchard and Diamond expressed considerable skepticism about the validity of these instruments. Therefore instruments have been avoided in the present study.
5.3 Estimates with Recession Dummies

The period under study is marked by substantial ups and downs in the economy within a very short period of time. In this period, the United States experienced two business cycle downturns, first one lasted for eight months between the March 2001 and the November 2001 and the second one started on December 2007 and still continuing. Given that business cycle fluctuations has important implications on the movements of hires, vacancies and unemployment, hence it is important to take in to account business cycle fluctuations while estimating a matching function (see figure 3.1). The Beveridge curves for the recessionary and the non-recessionary periods are plotted separately in figure 3.3 (see figure 3.2 for Beveridge Curve for the whole period). Clearly the curves in two periods have completely different slopes. The Beveridge curve for the recessionary period is steeper than the one during non-recessionary period. To estimate the matching function, it is important to take into account of these differences. One way is to estimate using dummy variable technique. Since both the slope and the constants are significantly different, hence I use the method of interactive dummy to estimate the matching function. The econometric specification is

\[ \log M_t = \log A + \alpha \log V_t + \beta \log U_t + \gamma_1 dum + \gamma_2 dum \times \log V_t + \gamma_3 dum \times \log U_t + \varepsilon_t \]

where dum =1 for recession time periods, otherwise dum = 0. The estimation results are shown in Table 3.4. The signs of the explanatory variables are positive as expected. All the parameters except for the constant term are significant at 5 % level of significance. In the model with interaction dummies e.g. specification 4 and
5, the returns to scale turn out to be between 0.7693 and 0.8487. If recession is taken into account, the aggregate matching function has very close to constant returns to scale. To cross check the above results, I also estimated matching function for the non-recessionary and the recessionary periods separately. The estimated results are shown in Table 3.5 and 3.6. The returns to scale turn out to be 0.8609 for non-recessionary period, which again reinforces the claim that for the non-recessionary time period, matching function does show CRS.

5.4 More Estimations

5.4.1 Estimations Using Not in Labor Force

To check the robustness of the result that CRS only applies to the non-recessionary time periods, I have tested the model using other specifications. Technically hires can be made from the pool of unemployed workers, pool of workers who are looking for jobs while working and the pool of workers from the out of labor force. Though I adjust hires for the job-to-job transitions, but hires are also made from the people out of labor force. According to Blanchard and Diamond (1989), about 40 percent of the hirings in the period between 1970 and 1981 is from the out of labor force. CPS collects information on the number of working age people who are out of labor force and also gives information on the willingness of the people to take a job. In order to account for the hirings from the out of labor force, I estimate the following specification

$$\log M_t = \log A + \alpha \log V_t + \beta \log U_t + \gamma NL_t + \varepsilon_t$$
where $N_L_t$ refer to persons who have searched for work during the prior 12 months and were available to take a job during the reference week of BLS survey. The results of the regression are shown in Table 3.7 for the overall, recessionary and the recessionary and the non-recessionary periods. For the overall time period, returns to scale is 0.5179, whereas for the non-recessionary period, it is 0.8084. Both $U_t$ and $V_t$ for the recessionary period is insignificant and the sign of $V_t$ is negative, which is not expected. Therefore, CRS assumption primarily holds for the non-recessionary periods.

5.4.2 Estimations Using Yield Spread as a Lead Indicator for Recessions

Estimation results clearly show that there is divergence between the recessionary and non-recessionary periods in terms of the stability of the matching function. In order to control for the recessions, estimates are made using yield spread as a lead indicator for the business cycles. Starting from 1968 onwards, before all the recessions, short term interest rates rose above long-term rates, reversing the customary pattern and producing what economists call a yield curve inversion. Estrella and Trubin (2006), using the yield spread created a lead indicator to forecast future recessions 12 months ahead. In order to capture the effects of business cycles, following specification of the matching function estimated

$$
\log M_t = \log A + \alpha \log V_t + \beta \log U_t + \gamma Yiled_t + \varepsilon_t
$$

where $Yiled_t$ is the twelve month ahead forecast for the probability of recession using yield spread as provided by the Federal Reserve Bank, New York. The estimation
results are shown in Table 3.8. For the overall time period, returns to scale is 0.2591, whereas for the non-recessionary period, it is 0.7138. Both $U_t$ and $V_t$ for the overall period and the recessionary period are insignificant and the sign of $V_t$ is negative for the recessionary period, which is not expected. Therefore, CRS assumption primarily holds for the non-recessionary periods. Probability of recession based on yield spread is significant for all the regressions.

5.4.3 Estimations Using Total Employment as a Concurrent Indicator for Recessions

In this section, total employment as provided by BLS is used as a concurrent indicator for business cycles. In order to capture the effects of business cycles, following specification of the matching function estimated

$$\log M_t = \log A + \alpha \log V_t + \beta \log U_t + \gamma T E_t + \varepsilon_t$$

where $T E_t$ is the Total Employment in the non-farm sector. Estimation results are shown in Table 3.9. For the overall time period, returns to scale is 0.4258, whereas for the non-recessionary period, it is 0.8077. $U_t$ for the overall period and the $U_t$ and $V_t$ for the recessionary period is insignificant. Therefore, CRS assumption primarily holds for the non-recessionary periods. The sign of $T E_t$ is negative since more employment means less unemployed and hence fewer matching. Also total employment is significant for all the specifications.
5.5 Estimation of Unemployment to Employment Flows

According to search theory, all the hirings through matching come through unemployment. In this section, I will exploit the information available in the CPS and the JOLTS, to estimate the transition of workers from not in labor force to employment. All the flows to hirings at any time period is either from the pool of unemployed, pool of workers who is currently employed and the workers from out of labor force. In earlier section, using the JOLTS quit data and the CPS information about the number of workers who are unemployed, I estimate the job-to-job transition. In order to estimate the transition from unemployment to employment, I use the method used by Shimer (2001) Shimer infer the job-finding rate from the dynamic behavior of the unemployment level and short-term unemployment level. Let \( u^s_t \) denote the number of workers unemployed for less than one month in month \( t \). Then assuming all unemployed workers find a job with probability \( f_t \) in month \( t \) and no unemployed worker exits the labor force. Hence, unemployment next month is the sum of the number of unemployed workers this month who fail to find a job and the number of newly unemployed workers.

\[
 u_{t+1} = u_t(1 - f_t) + u^s_{t+1}
\]

CPS gives estimates of \( u_t \) and \( u^s_t \), but \( u^s_t \) is defined as the number of workers unemployed for less than 5 weeks. Since, BLS conducts CPS surveys every month, it might lead to double counting of short term unemployment if \( u^s_t \) as defined by BLS is taken. In order to adjust \( u^s_t \) from number of workers unemployed for less than 5 weeks to workers unemployed for less than 4 weeks, I assume that the number of
workers getting unemployed is constant across weeks. Hence, new measure of $u_t^*$ is taken as $0.8u_t^*$.

Using the $u_t$ and $u_t^*$, I estimated the job finding rate $f_t$ for each month separately. Using the $f_t$ and the $u_t$, I estimated the number of people transitioning from unemployment to employment. By subtracting, job-to-job flows and the unemployment to employment flows from the total hirings, we can get an estimate of the not in labor force to employment flows. Figure 3.4 and 3.5 shows the various labor market flows in to hirings. The job-to-job flows are pro-cyclical because during recessions it is much difficult to get a job. This result is consistent with Shimer (2005). On the other hand unemployment to employment flow is counter cyclical because during recessions it is much easier to hire workers from the pool of unemployed. Hence flows from unemployment to employment increases. On the other hand, flows from not in labor force to employment is acyclical.

5.6 Estimation of Matching Function Using Unemployment to Employment Flows

In this section matching function is estimated using only the unemployment to employment flows. Since search theory assumes that all the hirings are made from the pool of unemployed, hence it is expected that the use of the unemployment to employment flows instead of the total hirings should be a better estimation. Hence the following function estimated

$$\log H_t = \log A + \alpha \log V_t + \beta \log U_t + \lambda_1 t + \varepsilon_t$$
where $H_t$ is the estimated hirings from the pool of unemployed workers only. The estimation results are shown in Table 3.10. As expected, the result shows constant returns to scale for the estimations with time trend. If the time trend is not used, then the fit is not good with $R^2$ at 0.0966 and the coefficients of $V_t$ and $U_t$ is insignificant. In the literature, time trend did play an important role (Petrongolo and Pissarides, 2001).

6 Conclusion

This is the first attempt to estimate a matching function for the United States using the JOLTS data. Though empirical results show that Constant Returns to Scale matching function exists for the United States but the CRS only holds for the non-recessionary time periods. In the paper attempt have been made to estimate the flow of workers from unemployment to employment using the CPS data. Estimation results show that when hirings are estimated using CPS, the matching function shows CRS. This raises concerns about the way data is collected for hirings. It seems that there are discrepancies between estimates based upon JOLTS and the estimates based upon CPS. Further studies need to be done to understand the behavior of hirings during the recessionary period.

There are couple of concerns, which are worth mentioning here. First, although the CRS assumption holds only for the non-recessionary time period, but theory predicts that returns to scale do not vary with the business cycle fluctuations. Business cycle fluctuations only cause movements along the Beveridge curve. Second,
though JOLTS is more suited to estimate a matching function for the United States in comparison to other data sources used in previous studies, still the data is limited in terms of showing job-to-job transitions and transitions from unemployment to employment. In the study, attempts have been made to adjust hires for the job-to-job transitions, but no adjustments are made to the vacancies and unemployment. To adjust vacancies and unemployment for job-to-job transitions, we need more disaggregated micro level data.\(^{21}\) Given the growing importance of job-to-job transition (Fallick and Fleischman, 2004 and Nagypal, 2005), any further studies to estimate matching function need to account for the job-to-job transitions more rigorously.

7 Bibliography


\(^{21}\)Recently BLS have started releasing information on micro level labor market dynamics. But still these micro level data are not yet publicly available. (see Boon et. al.,2008).


25. **Nagypal, E. (2005)**. "Job-to-job transitions and labor market fluctuations", *Northwestern University mimeo*


Appendix 1: JOLTS Definitions

7.1 Employment

JOLTS defines Employment as all persons on the payroll who worked during or received pay for the pay period that includes the 12th of the month. This definition is consistent with other BLS establishment-based programs.

INCLUDES:

- Full-time and part-time employees
- Permanent, short-term, and seasonal employees
- Salaried and hourly workers
- Employees on paid vacation or other paid leave

DOES NOT INCLUDE:

- Proprietors and partners of unincorporated businesses
- Unpaid family workers
- Employees on strike for the entire pay period
- Employees on leave without pay for the entire pay period
- Employees of temporary help agencies, employee leasing companies, outside contractors, or consultants. Employees of these types of firms are counted by establishments employing them, not at the site where the work is performed.

### 7.2 Job Openings (Vacancies)

JOLTS defines Job Openings as all positions that are open (not filled) on the last business day of the month. A job is "open" only if it meets all three of the following conditions:

1. A specific position exists and there is work available for that position. The position can be full-time or part-time, and it can be permanent, short-term, or seasonal, and

2. The job could start within 30 days, whether or not the establishment finds a suitable candidate during that time, and

3. There is active recruiting for workers from outside the establishment location that has the opening.

**DOES NOT INCLUDE:**

- Positions open only to internal transfers, promotions or demotions, or recall from layoffs

- Openings for positions with start dates more than 30 days in the future
• Positions for which employees have been hired, but the employees have not yet reported for work

• Positions to be filled by employees of temporary help agencies, employee leasing companies, outside contractors, or consultants. A separate form is used to collect information from temporary help/employee leasing firms for these employees.

7.3 Hires

JOLTS defines Hires as all additions to the payroll during the month.

INCLUDES:

• Newly hired and rehired employees

• Permanent, short-term, and seasonal employees

• Full-time and part-time employees

• On-call or intermittent employees who returned to work after having been formally separated

• Workers who were hired and separated during the month

• Transfers from other locations

• Employees who were recalled to a job at the sampled establishment following a formal layoff lasting more than 7 days
DOES NOT INCLUDE:

- Transfers or promotions within the sampled establishment
- Employees returning from strikes
- Employees of temporary help agencies, employee leasing companies, outside contractors, or consultants working at the sampled establishment. A separate form is used to collect information from temporary help and employee leasing firms for these employees.

7.4 Separations

JOLTS defines Separations as all employees separated from the payroll during the calendar month.

INCLUDES:

- Quits: Employees who left voluntarily. Exception: retirements or transfers to other locations are reported with Other Separations.

- Layoffs & Discharges: Involuntary separations initiated by the employer, including:
  
  - Layoffs with no intent to rehire
  
  - Discharges because positions were eliminated
  
  - Discharges resulting from mergers, downsizing, or plant closings
- Firings or other discharges for cause
- Terminations of seasonal employees (whether or not they are expected to return next season)
- Layoffs (suspensions from pay status) lasting or expected to last more than 7 days. (If the employee was later recalled, they should be reported as a Hire at the time of recall.)

- Other Separations: retirements; transfers to other locations; deaths; or separations due to employee disability

DOES NOT INCLUDE:

- Transfers within the sampled establishment
- Employees on strike
- Employees of temporary help agencies, employee leasing companies, outside contractors, or consultants working at the sampled establishment. These employers are reported by their employer of record.
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<th>Level PP</th>
<th>Level KPSS*</th>
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ADF: Augmented Dickey-Fuller Test (5% level of significance is -3.4563)
PP: Phillips-Perron Test (5% level of significance is -3.4558)
KPSS: Kwiatkowski-Phillips-Schmidt-Shin (5% level of significance is 0.463000)
* Under KPSS test, Null hypothesis = Series is stationary
### Table 3.2

**Estimation of Cobb-Douglas Matching Function for U.S. Non-farm Sector**

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</table>

#D-W: Durbin Watson Statistic (at 5% D-W critical values with 100 observations and 4 explanatory variables are 1.592 for lower level and 1.758 for upper level)

*AEG: Augmented Engle-Granger Test for Cointegration. (AEG critical value at 5% level of significance is -1.9439).

* t-statistic in parenthesis. At 5% level of significance, critical value for t-statistic is 1.96

### Table 3.3

**Estimation of Cobb-Douglas Matching Function for U.S. Non-farm Sector**

<table>
<thead>
<tr>
<th>Log(M&lt;sub&gt;t&lt;/sub&gt;) dependent variable</th>
<th>Const.</th>
<th>Log(V&lt;sub&gt;t-1&lt;/sub&gt;)</th>
<th>Log(U&lt;sub&gt;t-1&lt;/sub&gt;)</th>
<th>Time Trend (t)</th>
<th>Time Trend Square (t^2)</th>
<th>AR(1)</th>
<th>R&lt;sup&gt;2&lt;/sup&gt;</th>
<th>Adj R&lt;sup&gt;2&lt;/sup&gt;</th>
<th>D-W</th>
<th>AE G Test*</th>
<th>Returns to Scale</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5.2172</td>
<td>0.1410</td>
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<td>0.0026</td>
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<tr>
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<td>(1.8097)</td>
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</tr>
</tbody>
</table>

#D-W: Durbin Watson Statistic (at 5% D-W critical values with 100 observations and 4 explanatory variables are 1.592 for lower level and 1.758 for upper level)

*AEG: Augmented Engle-Granger Test for Cointegration. (AEG critical value at 5% level of significance is -1.9439).

* t-statistic in parenthesis. At 5% level of significance, critical value for t-statistic is 1.96
### Table 3.4

**Estimation of Cobb-Douglas Matching Function for U.S. Non-farm Sector using Recession Dummy**

<table>
<thead>
<tr>
<th>Log(M&lt;sub&gt;t&lt;/sub&gt;)</th>
<th>Log(V&lt;sub&gt;t&lt;/sub&gt;)</th>
<th>Log(U&lt;sub&gt;t&lt;/sub&gt;)</th>
<th>Time Trend (t)</th>
<th>Time Trend Square</th>
<th>Const</th>
<th>Dum&lt;sup&gt;*&lt;/sup&gt; Log(V&lt;sub&gt;t&lt;/sub&gt;)</th>
<th>Dum&lt;sup&gt;*&lt;/sup&gt; Log(U&lt;sub&gt;t&lt;/sub&gt;)</th>
<th>AR(1)</th>
<th>R&lt;sup&gt;2&lt;/sup&gt;</th>
<th>Adj R&lt;sup&gt;2&lt;/sup&gt;</th>
<th>D-W&lt;sup&gt;#&lt;/sup&gt;</th>
<th>AEG Test&lt;sup&gt;*&lt;/sup&gt;</th>
<th>Returns to Scale</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>(1.7991)</td>
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<td>(-2.5589)</td>
<td>(-2.1222)</td>
<td>(-2.5589)</td>
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</table>

*<sup>#</sup>D-W: Durbin Watson Statistic (at 5% D-W critical values with 100 observations and 5 explanatory variables are 1.571 for lower level and 1.78 for upper level)

*<sup>*</sup>AEG: Augmented Engle-Granger Test for Cointegration. (AEG critical value at 5% level of significance is -1.9439).

Dum: Recession dummy

t-statistic in parenthesis. At 5% level of significance, critical value for t-statistic is 1.96
## Table 3.5

Estimation of Cobb-Douglas Matching Function for U.S. Non-farm Sector for Non-Recessionary Time Period

<table>
<thead>
<tr>
<th>Log(Mₜ) dependent variable</th>
<th>Const</th>
<th>Log(Vₜ)</th>
<th>Log(Uₜ)</th>
<th>Time Trend (t)</th>
<th>Time Trend Square</th>
<th>AR(1)</th>
<th>R²</th>
<th>Adj R²</th>
<th>D-W #</th>
<th>AEG Test *</th>
<th>Returns to Scale</th>
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<tbody>
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</tr>
</tbody>
</table>

#D-W: Durbin Watson Statistic (at 5% D-W critical values with 100 observations and 4 explanatory variables are 1.592 for lower level and 1.758 for upper level)

*AEG: Augmented Engle-Granger Test for Cointegration. (AEG critical value at 5% level of significance is -1.9439).

*Statistically significant t-statistic in parenthesis. At 5% level of significance, critical value for t-statistic is 1.96.
Table 3.6

Estimation of Cobb-Douglas Matching Function for U.S. Non-farm Sector for Recessionary Time Period

<table>
<thead>
<tr>
<th>Log($M_t$) dependent variable</th>
<th>Const</th>
<th>Log($V_t$)</th>
<th>Log($U_t$)</th>
<th>Time Trend ($t$)</th>
<th>Time Trend Square</th>
<th>AR(1)</th>
<th>$R^2$</th>
<th>Adj $R^2$</th>
<th>D-W#</th>
<th>AEG Test*</th>
<th>Returns to Scale</th>
</tr>
</thead>
<tbody>
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<td>1</td>
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</tr>
</tbody>
</table>

#D-W: Durbin Watson Statistic (at 5% D-W critical values with 100 observations and 4 explanatory variables are 1.592 for lower level and 1.758 for upper level)

*AEG: Augmented Engle-Granger Test for Cointegration. (AEG critical value at 5% level of significance is -1.9439).

t-statistic in parenthesis. At 5% level of significance, critical value for t-statistic is 1.96
Table 3.7

Estimation of Cobb-Douglas Matching Function for U.S. Non-farm Sector with NL_t

<table>
<thead>
<tr>
<th>Log(M_t)</th>
<th>Const</th>
<th>Log(V_t)</th>
<th>Log(U_t)</th>
<th>Log(NL_t)</th>
<th>Time Trend (t)</th>
<th>Time Trend Square</th>
<th>R^2</th>
<th>Adj R^2</th>
<th>D-W^#</th>
<th>AEG Test*</th>
<th>Returns to Scale</th>
</tr>
</thead>
<tbody>
<tr>
<td>All Periods</td>
<td>2.5057</td>
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<td>0.2382</td>
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#D-W: Durbin Watson Statistic (at 5% D-W critical values with 100 observations and 4 explanatory variables are 1.592 for lower level and 1.758 for upper level)

*AEG: Augmented Engle-Granger Test for Cointegration. (AEG critical value at 5% level of significance is -1.9439).

T-statistic in parenthesis. At 5% level of significance, critical value for t-statistic is 1.96
Table 3.8

Estimation of Cobb-Douglas Matching Function for U.S. Non-farm Sector Using Probability of Recession from Yield Spread

<table>
<thead>
<tr>
<th>Log(M_t) dependent variable</th>
<th>Const</th>
<th>Log(V_t)</th>
<th>Log(U_t)</th>
<th>Yield_t</th>
<th>Time Trend (t)</th>
<th>AR(1)</th>
<th>R^2</th>
<th>Adj R^2</th>
<th>D-W#</th>
<th>AEG Test#</th>
<th>Returns to Scale</th>
</tr>
</thead>
<tbody>
<tr>
<td>All Periods</td>
<td>5.8319 (4.4632)</td>
<td>0.1534 (1.8403)</td>
<td>0.1057 (1.3901)</td>
<td>-0.0131 (-4.69)</td>
<td>0.2546 (2.5939)</td>
<td>0.3389</td>
<td>0.3107</td>
<td>2.1056</td>
<td>-10.500</td>
<td>0.2591</td>
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</tr>
<tr>
<td>Non Recessionary Period</td>
<td>1.9732 (0.843)</td>
<td>0.3523 (2.9941)</td>
<td>0.3615 (2.2289)</td>
<td>-0.0108 (-3.93)</td>
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<td>0.2838</td>
<td>0.2522</td>
<td>1.6035</td>
<td>-7.1634</td>
<td>0.7138</td>
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</tr>
<tr>
<td>Recessionary Period</td>
<td>7.5633 (2.6492)</td>
<td>-0.1689 (-1.1251)</td>
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<td>-0.1848 (-2.38)</td>
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<td>1.9417</td>
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<td>0.0669</td>
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</tr>
</tbody>
</table>

#D-W: Durbin Watson Statistic (at 5% D-W critical values with 100 observations and 4 explanatory variables are 1.592 for lower level and 1.758 for upper level)

*AEG: Augmented Engle-Granger Test for Cointegration. (AEG critical value at 5% level of significance is -1.9439).

t-statistic in parenthesis. At 5% level of significance, critical value for t-statistic is 1.96
Table 3.9

Estimation of Cobb-Douglas Matching Function for U.S. Non-farm Sector Using Total Employment

<table>
<thead>
<tr>
<th>Log(M_t) dependent variable</th>
<th>Const</th>
<th>Log(V_t)</th>
<th>Log(U_t)</th>
<th>Log(TEt)</th>
<th>Time Trend (t)</th>
<th>R^2</th>
<th>Adj R^2</th>
<th>D-W#</th>
<th>AEG Test*</th>
<th>Returns to Scale</th>
</tr>
</thead>
<tbody>
<tr>
<td>All Periods</td>
<td>73.396</td>
<td>0.4189</td>
<td>0.0069</td>
<td>-5.8486</td>
<td>0.0033</td>
<td>0.337</td>
<td>0.310</td>
<td>1.739</td>
<td>-8.954</td>
<td>0.4258</td>
</tr>
<tr>
<td>(6.763)</td>
<td>(3.8551)</td>
<td>(0.0684)</td>
<td>(-6.3923)</td>
<td>(5.316)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Non Recessionary Period</td>
<td>43.461</td>
<td>0.4042</td>
<td>0.4035</td>
<td>-3.5926</td>
<td>0.0031</td>
<td>0.287</td>
<td>0.2444</td>
<td>1.6689</td>
<td>-7.379</td>
<td>0.8077</td>
</tr>
<tr>
<td>(2.584)</td>
<td>(3.0589)</td>
<td>(2.0455)</td>
<td>(-2.6224)</td>
<td>(3.880)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Recessionary Period</td>
<td>26.7804</td>
<td>0.1842</td>
<td>0.2016</td>
<td>-1.8649</td>
<td></td>
<td>0.456</td>
<td>0.3785</td>
<td>1.9227</td>
<td>-7.176</td>
<td>0.3858</td>
</tr>
<tr>
<td>(5.663)</td>
<td>(1.0056)</td>
<td>(1.5306)</td>
<td>(-3.7416)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

#D-W: Durbin Watson Statistic (at 5% D-W critical values with 100 observations and 4 explanatory variables are 1.592 for lower level and 1.758 for upper level)

*AEG: Augmented Engle-Granger Test for Cointegration. (AEG critical value at 5% level of significance is -1.9439).

t-statistic in parenthesis. At 5% level of significance, critical value for t-statistic is 1.96
Table 3.10

Estimation of Cobb-Douglas Matching Function for U.S. Non-farm Sector Using Unemployment to Employment Flows

<table>
<thead>
<tr>
<th>Log($H_t$) dependent variable</th>
<th>Const</th>
<th>Log($V_t$)</th>
<th>Log($U_t$)</th>
<th>Time Trend (t)</th>
<th>$R^2$</th>
<th>Adj $R^2$</th>
<th>D-W#</th>
<th>AEG Test*</th>
<th>Returns to Scale</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>9.1659 (4.2259)</td>
<td>-0.1845 (-1.3538)</td>
<td>0.0029 (0.0234)</td>
<td>0.0966</td>
<td>0.0780</td>
<td>2.1709</td>
<td>-12.508</td>
<td>-0.1816</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>-1.7083 (-0.581)</td>
<td>0.4689 (2.6042)</td>
<td>0.6421 (3.7424)</td>
<td>-0.0019 (-4.946)</td>
<td>0.2801</td>
<td>0.2576</td>
<td>2.3320</td>
<td>-12.111</td>
<td>1.111</td>
</tr>
<tr>
<td>3</td>
<td>0.3327 (0.1207)</td>
<td>0.3596 (2.1391)</td>
<td>0.5109 (3.1737)</td>
<td>-0.0018 (-5.276)</td>
<td>0.3118</td>
<td>0.2825</td>
<td>1.9201</td>
<td>-9.5776</td>
<td>0.8705</td>
</tr>
</tbody>
</table>

#D-W: Durbin Watson Statistic (at 5% D-W critical values with 100 observations and 4 explanatory variables are 1.592 for lower level and 1.758 for upper level)
*AEG: Augmented Engle-Granger Test for Cointegration. (AEG critical value at 5% level of significance is -1.9439).

t-statistic in parenthesis. At 5% level of significance, critical value for t-statistic is 1.96
Fig: 3.1
Vacancies, Hirings and Unemployment, Dec 2000 - Mar 2009

Source: Bureau of Labor Statistics
Fig: 3.2
Beveridge Curve Dec 2000 - Mar 2009

Source: Bureau of Labor Statistics
Fig 3.3:
Beveridge Curve Dec 2000 - Mar 2009

Source: Bureau of Labor Statistics
Fig: 3.4

Source: Bureau of Labor Statistics
Fig: 3.5

Source: Bureau of Labor Statistics
Fig: 3.6
Vacancies, Hirings from Unemployment and Unemployment  Dec 2000 - Mar 2009

Source: Bureau of Labor Statistics
Fig: 3.7

Source: Bureau of Labor Statistics