REMEDYING EDUCATIONAL OPPORTUNITY PROGRAM
FRESHMEN’S KNOWLEDGE OF FRACTIONS AND VIEWS OF
MATHEMATICS LEARNING WITH SERIOUS VIDEO GAMES

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REMEDYING EDUCATIONAL OPPORTUNITY PROGRAM FRESHMEN’S KNOWLEDGE OF FRACTIONS AND VIEWS OF MATHEMATICS LEARNING WITH SERIOUS VIDEO GAMES

by

Yan Tian

A Dissertation
Submitted to the University at Albany, State University of New York
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Doctor of Philosophy

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ABSTRACT

Well-designed serious video games have great potential in mathematics education. This study aims to enrich our understanding of Slice Fractions and Slice Fractions 2 as remedy tools for college freshmen in an educational opportunities program. Specifically, drawing on resources-based epistemic cognition theory and scheme theory, the study investigated fourteen participants’ perspectives and experiences of playing these games, the impacts of the gameplay on their fraction schemes/operations and explicit conceptions of fractions, practices in fraction problem-solving, and views of mathematics learning. With a quasi-experimental design, each participant finished a pre-game interview and assessment, played SF1 or SF2, and completed a post-game interview and assessment. The findings reveal the productive game designs suggested by the gameplay and perceived by the players. A preliminary examination of the interaction between prior video game experience and gameplay of SF1 and SF2 shows that the games effectively elicit knowledge of fractions to a similar extent among non-gamers and gamers. The findings also suggest that playing SF1 and SF2 supports the construction of fraction schemes/operations, the development of a better understanding of fractions from multiple perspectives, and improvements in making sense of various operations with fractions. The study contributes to a deepened understanding of how well-designed serious video games support math learning.
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# TABLE OF CONTENTS

LIST OF FIGURES ............................................................................................................. vii

LIST OF TABLES .................................................................................................................. x

1 Introduction ......................................................................................................................... 1

2 Review of Empirical Studies ............................................................................................... 5

3 Research Questions ............................................................................................................. 11

4 Theoretical Framework ......................................................................................................... 12
   4.1 Epistemic Change ........................................................................................................... 12
   4.2 Resources-based Approach to Epistemic Cognition ....................................................... 13
   4.3 Scheme Theory ............................................................................................................. 18

5 Methods .............................................................................................................................. 23
   5.1 Research Design ........................................................................................................... 23
   5.2 Participants ................................................................................................................... 25
   5.3 Slice Fractions and Slice Fractions 2 ........................................................................... 26
   5.4 Data Collection ............................................................................................................. 43
   5.5 Data Analysis ............................................................................................................... 52
       5.5.1 Data Analysis of Assessments ............................................................................. 52
       5.5.2 Data Analysis of Interviews ............................................................................. 54
       5.5.3 Data Analysis of Gameplay ............................................................................. 55

6 Results ............................................................................................................................... 62
   6.1 Experiences and Views of the Games ......................................................................... 62
       6.1.1 Prior Experience with Video Games ................................................................... 62
       6.1.2 Gameplay of SF1 and SF2 ............................................................................... 64
<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.1.3 Experiences and Feedback on SF1 and SF2</td>
<td>71</td>
</tr>
<tr>
<td>6.2 Recollections, Attitudes, and Conceptions of Fractions</td>
<td>76</td>
</tr>
<tr>
<td>6.2.1 Recollections about Fractions Learning</td>
<td>76</td>
</tr>
<tr>
<td>6.2.2 Feelings/Attitudes Towards Fractions</td>
<td>78</td>
</tr>
<tr>
<td>6.2.3 Fractions Schemes/Operations</td>
<td>81</td>
</tr>
<tr>
<td>6.2.4 Fractional Conceptions</td>
<td>85</td>
</tr>
<tr>
<td>6.2.4.1 Initial Explanations of “Fraction”</td>
<td>85</td>
</tr>
<tr>
<td>6.2.4.2 Fractions as Indicators of Multiplicative Relationships</td>
<td>87</td>
</tr>
<tr>
<td>6.2.4.3 Equal Splitting</td>
<td>90</td>
</tr>
<tr>
<td>6.2.4.4 Understanding Improper Fractions</td>
<td>95</td>
</tr>
<tr>
<td>6.2.4.5 Fractions in Everyday Life</td>
<td>100</td>
</tr>
<tr>
<td>6.3 Practices and Knowledge of Fractions in Problem-solving</td>
<td>103</td>
</tr>
<tr>
<td>6.3.1 Pre-game Fraction Multiplications</td>
<td>104</td>
</tr>
<tr>
<td>6.3.2 Post-game Fraction Multiplications</td>
<td>108</td>
</tr>
<tr>
<td>6.3.3 Pre-game Fraction Divisions</td>
<td>112</td>
</tr>
<tr>
<td>6.3.4 Post-game Fraction Divisions</td>
<td>117</td>
</tr>
<tr>
<td>6.3.5 Pre-game Fraction Additions/Subtractions</td>
<td>119</td>
</tr>
<tr>
<td>6.3.6 Post-game Fraction Additions/Subtractions</td>
<td>123</td>
</tr>
<tr>
<td>6.3.7 Summary of Findings and the Gameplay</td>
<td>128</td>
</tr>
<tr>
<td>6.4 Views of Math Learning</td>
<td>133</td>
</tr>
<tr>
<td>7 Discussion</td>
<td>137</td>
</tr>
<tr>
<td>7.1 Discussion of Findings on Experiences and Views of the Games</td>
<td>137</td>
</tr>
<tr>
<td>7.2 Discussion of Findings on Recollections, Attitudes, and Conceptions of Fractions</td>
<td>141</td>
</tr>
</tbody>
</table>
7.3 Discussion of Findings on Practices and Knowledge of Fraction Problem-solving........147
7.4 Discussion of Findings on Views of Math Learning ..........................................................155
7.5 Significance of the Study ....................................................................................................156
7.6 Limitations and Future Research .......................................................................................157
References ................................................................................................................................160
Appendix A. Pre-game and Post-game Interview Protocols ......................................................172
Appendix B. Pre-game and Post-game Assessments ..................................................................174
Appendix C. Comparison of the Assessment Items ....................................................................184
LIST OF FIGURES

Figure 1. Data Collection Process.................................................................24
Figure 2. SF1.I.2: introduction of bubbles in SF1 ........................................29
Figure 3. SF1.I.3: introduction of lava and ice in SF1.................................30
Figure 4. SF2.I.2: introduction of ice, lava, and snail in SF2......................30
Figure 5. SF1.II.2: an example of hints in SF1...........................................31
Figure 6. SF2.II.15: an example of hints in SF2.........................................32
Figure 7. A comparative view of SF1.IV.10, SF1.IV.11, SF1.IV.14, and SF1.IV.7.............34
Figure 8. SF1.IV.9: an example of reward levels ......................................35
Figure 9. SF1.VI.4: an illustration of addition blocks in SF1..........................36
Figure 10. SF2.II.5: an illustration of ghosts in SF2....................................36
Figure 11. SF2.III.2: an illustration of narwhals in SF2..............................37
Figure 12. SF1.V.14: an example of advanced puzzles in SF1.....................38
Figure 13. SF1.VI.23: an example of advanced puzzles in SF1....................39
Figure 14. SF2.II.31: an example of advanced puzzles in SF2......................40
Figure 15. SF2.III.11: an example of advanced puzzles in SF2....................41
Figure 16. An alternative way to solve SF2.III.11......................................42
Figure 17. Two sample items for eliciting the part-whole scheme..................45
Figure 18. Two sample items for eliciting the partitive unit fraction scheme ....46
Figure 19. Two sample items for eliciting the partitive fraction scheme..........47
Figure 20. One sample item for eliciting the splitting operation....................47
Figure 21. Two sample items for eliciting the iterative fraction scheme...........48
Figure 22. One sample item for eliciting the reversible partitive fraction scheme ......48
Figure 46. Tim’s pre-game independent illustration of 4/3 / 2 ..........................................................113
Figure 47. Tim’s and Austin’s pre-game illustrations of 3/4 / 3 ..........................................................114
Figure 48. Zoe’s post-game illustration of 3/4 / 3 ...........................................................................118
Figure 49. Tim’s pre-game illustrations of 1/2 + 1/3 ......................................................................120
Figure 50. Austin’s pre- and post-game illustrations of fraction additions .................................123
Figure 51. Maya’s post-game solution without and with the game in mind .................................126
Figure 52. Merik’s postgame illustration of 1/2 +1/4 ..................................................................127
Figure 53. SF1.IV.14 and SF1.VI.18 setups ..............................................................................130
Figure 54. SF1.VI.23 and SF2.III.11 setups ..............................................................................131
Figure 55. SF2.III.15 setup and Kate’s solutions ..........................................................................132
LIST OF TABLES

Table 1. Structure and content arrangement of Slice Fractions and Slice Fractions 2 ..........27
Table 2. The participants’ completion rates of SF1 ..........................................................51
Table 3. The participants’ completion rates of SF2 ..........................................................52
Table 4. The alignment between research questions and interview and assessment data ..........60
Table 5. The participants’ gameplay of Slice Fractions (SF1) ..............................................66
Table 6. The participants’ gameplay of Slice Fractions (SF2) ..............................................67
Table 7. The participants’ math performance in Slice Fractions (SF1) .................................70
Table 8. The participants’ math performance in Slice Fractions 2 (SF2) ..............................70
Table 9. A summary of the participants’ gameplay by game and their prior experience ..........71
Table 10. The means and standard deviations of the participants' scores by scheme/operation ..83
Introduction

The emergence of Common Core State Standards for Mathematics (CCSSM, National Governors Association Center for Best Practices & Council of Chief State School Officers [NGA & CCSSO], 2010) and consistent nationwide high-stakes assessments called for learning mathematics by focusing on core mathematical ideas, being engaged in mathematical practices, and developing mathematical ways of thinking. Various approaches are available to support students’ learning to be raised to the bars illustrated in the CCSSM, such as enhancing teachers’ pedagogical content knowledge (Zhang & Chen, 2017) and noticing practices (McDuffie et al., 2017), shifting teacher-centered instruction to inquiry-based learning projects (Schoenfeld & Kilpatrick, 2013), and adopting standards-based curriculum (Remillard et al., 2014). Many approaches aim at transforming traditional teacher-centered instruction and largely rely on teachers getting professional development. However, it takes long-term sustained engagement in high-quality professional development programs for math and science teachers’ practices to shift and impact students’ learning (Porter et al., 2002). Hence, while it is important to continue the efforts in furthering teachers’ knowledge and practices, we need more easily accessible and readily available learning tools so that more students, regardless of their teachers’ situations, may have high-quality learning experiences.

I contend that well-designed serious video games (SVGs) can be such learning tools. By well-designed SVGs, I refer to the playful educational video games that incorporate game design and learning design principles (e.g., the environment leaves abundant room for active and critical learning), and that wave together game mechanics and experiences of subject matter (e.g., mathematical or physical concepts are embodied in visualizations and actions). Some examples might be Monument Valley and Monument Valley 2 (Čujdiková, 2020; Shin & Chung, 2017; To
et al., 2016), Rolly’s Adventure (Williams-Pierce, 2016), DragonBox Algebra 12+ (Long & Aleven, 2017; Siew et al., 2016), Crayon Physics Deluxe (Kao et al., 2017), Slice Fractions (Gresalfi, 2018) and Slice Fractions 2, and Motion Math (McKevett et al., 2020; Riconscente, 2013). To further illustrate the general features of these games, I will compare them to merely playful games and to “unplayful” educational video games (or regular SVGs). Playing well-designed SVG does not require familiarity or mastery of formal knowledge, like how playing Angry Birds does not require knowledge of parabolas, acceleration, or the differences of varying materials (glass, wood, stone, etc.). Meanwhile, different from merely playful games such as Angry Birds, well-designed SVGs would purposefully slowly incorporate more complex subject matter to enrich the gameplay and the learning experience, as teaching and learning subject matter remain the core. On the other hand, not all SVGs are well-designed. As an example, a math game might require one to explore a virtual world and talk to different characters to receive instructions and solve math word problems. This game might be labeled as an educational video game or SVG but would not be considered well-designed here, as the subject matter is not organically integrated into the game world. Having made the distinctions, for simplicity, “well-designed SVG” will be referred to as “SVG” in the rest of the writing.

SVGs have many powerful advantages. First, many SVGs are easily accessible. Since SVGs do not require formal knowledge and are playful, they are accessible to most learners regardless of their levels. The relatively less advanced learners may approach them as games. They might not be able to recognize or articulate the subject matter involved. Nonetheless, as they make progress in the game, they are implicitly learning the core ideas or practices. For instance, in DragonBox Algebra 12+, at each puzzle one needs to keep the items or the creatures in two adjacent areas balanced all the time. Some players might not explicitly associate this
feature with the equal symbol, “=”, especially if during formal learning players interpret the equal symbol as a command for answers rather than a sign of an equal relationship (e.g., Carpenter et al., 2003). However, because of the constraints in the game, players are constantly maintaining equivalence relations. Consequently, at least in the context of the game, learners get familiar with the idea of equality and the practice of keeping equivalence relations throughout various operations. Second, incorporating SVGs in teaching and learning is cost-effective and promotes equity. With the ubiquity of Internet access and video game platforms such as personal computers and mobile devices, once published or shared, SVGs are readily available for most people, and playing such games requires no other materials or interventions. Since SVGs provide every learner with rich opportunities to interact with subject matter, if incorporated into teaching and learning, they promote equity. Third, SVGs are flexible to use. Since SVGs are independent of formal curriculum, teachers may directly include them in their teaching, as in-class activities, assignments, or optional activities. More experienced teachers may incorporate them into varying stages of instruction to achieve different goals. For instance, the games can be used as learning hooks to prepare students to learn new concepts, operations, or laws. Or, after teaching the formal knowledge, one could ask students to play games and reflect on the knowledge involved, based on which a teacher may evaluate students’ learning. To sum up, SVGs, flexible to use and with powerful advantages, may play an important role in teaching and learning.

Although SVGs could bring plenty of benefits to education, more understanding of specific SVGs’ affordances and how they support the learning of particular subject matter is needed (Gresalfi, et al., 2017). In a meta-analysis, Clark et al. (2016) found that on average video games are more effective than nongame instructions in enhancing not only students’ knowledge but also their motivation, intellectual openness, and positive core self-evaluation. Yet more
research is needed concerning how video games contribute to these outcomes. More inquiries into specific SVGs may provide insights into the design features that lead to engaging and productive learning, which may inspire novel instructional designs in non-gaming contexts. Moreover, such investigations could help teachers and researchers gain a better understanding of the tools and make more informed decisions about how to incorporate them into their regular classes or professional development programs. This study probes Slice Fractions (SF1) and Slice Fractions 2 (SF2), two SVGs designed for supporting the learning of fractions. The goal is to answer the following questions about these two SVGs.

1. What are the players’ experiences and views of Slice Fractions and Slice Fractions 2?
2. Do Slice Fractions and Slice Fractions 2 affect players’ conceptions of fractions? If so, how?
3. How does the gameplay affect players’ practices and knowledge of fraction operations?
4. Does the gameplay impact players’ views of mathematics learning? If so, how?
2 Review of Empirical Studies

In this section, I will first give a brief overview of Slice Fractions (SF1) and Slice Fractions 2 (SF2) and then review existing empirical studies. A detailed characterization of the two games will be presented later in the methods section. Since there are only three empirical studies on SF1 (and no study on SF2), I will review each study’s design, findings, and implications in detail.

SF1 and SF2 are two serious video games (SVGs) developed in 2014 and 2018 respectively by a partnership between Ululab and Université du Québec à Montréal, and are available in the App Store, Google Play, and Microsoft Store. The games were intended to support K-12 students’ learning of fractions through an intuitive, interactive, and fun gameplay experience, with no text-based instruction. Due to the intuitive and wordless nature of the game, one’s learning and gameplay experience is a distinctive contrast with the language-mediated learning experiences commonly seen in traditional classrooms. This does not imply that the former approach is necessarily better or more effective than the latter approach. After all, ample empirical studies have shown that in conventional classrooms productive mathematics learning can take place, where students develop a conceptual understanding of specific mathematics concepts while being engaged in mathematical practices and inquiries, regardless of the presence or lack of technological tools (e.g., Lampert, 1990; Lehrer, Kobiela & Weinberg, 2012).

However, as mentioned earlier, the delivery of such high-quality mathematics instruction relies on teachers’ insights on how varying mathematics topics are connected and how mathematical concepts can be flexibly represented in different forms and contexts (e.g., Ball, Thames & Phelps, 2008; Ma, 1997), and teachers’ competency in structuring mathematics teaching and learning to be student-driven inquiry or open-ended projects (Boaler, 1998; Kobiela
& Lehrer, 2015), which may take years for a teacher to gain and develop. Thus, before and during teachers’ development of knowledge of mathematics and teaching expertise, ready-to-use technology-enabled learning tools such as SVGs and interactive simulations can fill the gap and engage students in exploratory and active learning. Two such examples are SF1 and SF2. They organically integrate mathematical content and game mechanics, offering a visual and action-based experience of fractions as well as rich opportunities for inquiry, exploration, and strategic thinking.

Overall, researchers found that SF1 supported Grade 3 students’ learning of fractions (Cyr et al., 2019; Gresalfi et al., 2017; Zhang et al., 2020). Cyr et al. (2019) examined students’ learning of fractions in three conditions: regular teaching only, three-hour play of Slice Fractions, and three-hour gameplay accompanied by regular teaching. Significant knowledge gain, evaluated with items from Trends in International Mathematics and Science Study (TIMSS), was observed in the latter two conditions but not in the first one. The researchers made sense of the results by pointing out that the students in the first group outperformed the others in the pre-test and thus had relatively little room for improvement. Even so, we may gain some ideas about the benefits of the game by looking at the two game-related conditions. Compared to the game-only group, the third group had prolonged exposure to fractions via gameplay and regular teaching. Therefore, it is reasonable to expect that students in the third group gained more knowledge than students in the game-only group. However, this was not the case; students in these two groups exhibited similar knowledge gain. Also, on four test items without diagrams, students who played the game but did not receive regular instructions showed the highest gain. Hence, SF1 might be particularly beneficial when it comes to understanding the abstract representations of fractions (e.g., “three-fourth” or “3/4”). Maybe students in the game-only
group learned to visualize and operate with fractions, while traditional instruction did not sufficiently support students to do so (the first condition) and interfered with what students learned from playing Slice Fractions (the third condition). Considering this, playing SF1 might affect players’ conceptions or interpretations of fractions and approaches to solving fractional problems, by helping players “translate” abstract terms into sensible visualizations and specific operations.

In another study, Zhang et al. (2020) included two SVGs (SF1 and Motion Math), since these games focused on fractions in slightly different ways. While SF1 heavily relies on the part-whole interpretation of fractions and represents fractional quantities with area models, Motion Math draws on number lines and the connection between fractions and decimals. In Zhang et al.’s (2020) study design, students in the experimental group spent 2 hours receiving conventional instructions and 2 hours playing one of the two games; those in the control group only received conventional instructions for a total of 4 hours. Two tests were administered, one being more basic and directly connected to the learning content (e.g., comparing fractions with the same numerator but different denominators) and the other requiring transfer and involving more complex fractions ordering (e.g., comparing three fractions with different numerators and denominators). The experimental group and the control group had similar knowledge gain on the basic test, but on the transfer test, the former outperformed the latter. Within the experimental group, students playing different games scored similarly on the transfer test. Based on these findings, playing SF1 and Motion Math might have cultivated certain ways of thinking or doing, which were missing from regular instruction and supported students to analyze and solve novel problems; and how fractions are presented, via part-whole area models or number line models, did not matter as much as the learning contexts, serious video games or traditional instructions.
In both this study and Cyr et al.’s (2019) study, SF1 seemed to have supported students in solving more complex problems. Yet it remained unclear how playing SF1 achieved this.

Besides SF1 and Motion Math, Gresalfi et al. (2017) included three other apps related to learning fractions. All the apps covered roughly the same mathematical content and contained visual representations of fractions. Each app contained a series of puzzles or questions. In their study, SF1 and Motion Math were considered as games due to their playfulness and limited focus on the formal and abstract representations of fractions, and the other three apps as worksheets for their explicit emphasis on performance and formal representations. Students spent a total of 1 hour playing the games or using the worksheet apps. The researchers found a significant effect of time, which suggested that students in both conditions learned and transferred their app using experience to assessment. But there was no effect of condition or condition-by-time interaction. During interviews, students in both groups connected the app experiences to assessment items; when describing the apps, students who played the games referenced game mechanics more frequently and mathematics less frequently. Questionnaire data revealed that students in the game condition reported significantly higher levels of enjoyment, which was consistent with their observation that after the study was over all students spent free time playing SF1. Thus, compared to students who worked on worksheet questions, those who played SVGs gained as much knowledge but enjoyed the experience a lot more.

Considering Gresalfi et al.’s (2017) finding that students who played the games and those who used the worksheet apps had similar test performance, it was plausible that their test was assessing the basic content directly covered by all apps, evidenced by all students easily making connections between the apps and the test. Despite the similar test performances, maybe the students approached the problems in distinct ways, which would not be revealed unless students’
cognition was more closely probed. For two individuals who perform equally well on a conventional math test, one might describe the problem-solving process mostly by recalling standard procedures that bear no personal meaning or understanding, while the other might draw on personal interpretations and previous knowledge. Even though students who played SF1 or Motion Math discussed game elements and mechanics more than mathematics in interviews, their test performance was comparable to others who used worksheet apps (Gresalfi et al., 2017). This suggests that the SVGs managed to intrigue the players with the game designs and provided an impressive experience without sacrificing their learning of fractions and that the fractional knowledge they gained from the gameplay might not be immediately available during reflections. Since playing SVGs left players with a strong impression (Gresalfi et al., 2017) and contributed to solving complex problems (Cyr et al., 2019; Zhang et al., 2020), maybe the gameplay helped the students contextualize and make sense of the abstract problems and operations involved in working out the assessment items. Also, since the experience of mathematics brought by SVGs was distinct from the experience of receiving conventional instructions or using worksheet apps, I wonder whether the gameplay might have some impact on players’ views of mathematics learning and how SVGs can be integrated into formal math teaching and learning.

Concerning the participants, all three studies were carried out with third graders, immediately before or when the topic of fractions was first extensively introduced. I argue that we need more studies where SF1 and SF2 are played by learners at other educational levels, as many more learners might benefit from playing the games. SF1 and SF2 cover varying fractional topics, ranging from equal partitioning and symbols (“m/n”) to additions and multiplications of fractions. According to CCSSM, students are expected to gradually learn fractions and
operations with fractions in Grades 3-5. Hence, the easier levels might be accessible to Grade 1-2 students, if the main goal is to get them familiar with physical operations like partitioning and multiplicatively relationships between two quantities, which form the foundation for formally learning fractions. For Grade 4-5 students, if used before the unit on fractions, playing the games might help review basic ideas about fractions and preview more complicated skills such as the additions and multiplications of fractions. Beyond elementary schools, the games might be valuable as a remedial tool for those with weak knowledge of fractions. Many older learners, including secondary school students, college students, and prospective teachers, etc., carry a flimsy understanding of fractions along their math learning journey, which impedes their learning of other math topics like algebra or limits their capability to teach fractions (e.g., Hackenberg & Lee, 2015; Ma, 1997; Ngo, 2019; Siegler & Pyke, 2013; Son & Crespo, 2009). Since SVGs like SF1 and SF2 bring an alternative experience of learning fractions, they might be helpful for learners who do not grasp fractions well with conventional instructions.

To conclude, previous empirical studies found SF1 to be helpful for Grade 3 students’ learning of fractions, to the same extent as or superior to traditional instructions delivered by teachers or worksheet apps (Cyr et al., 2019; Gresalfi et al. 2017; Zhang et al., 2020). Yet we still lack understanding of how the gameplay contributed to students’ knowledge of fractions and practices in solving relevant problems. Although SF1 and SF2 are about fractions, a basic math concept, they might be beneficial for learners at varying educational levels and in different ways. In this project, I investigated the possibility of using SF1 and SF2 as a remedial tool. Fraction is one of the pre-algebra topics covered in remedial courses designed for college-level Educational Opportunities Program (EOP) freshmen, so it is appropriate to conduct the study with EOP freshmen. These are the specific research questions.
3 Research Questions

1. What are EOP freshmen’s experiences and views of Slice Fractions and Slice Fractions 2?
2. Do Slice Fractions and Slice Fractions 2 affect EOP freshmen’s conceptions of fractions? If so, how?
3. How does the gameplay affect EOP freshmen’s practices and knowledge of fraction operations?
4. Does the gameplay impact EOP freshmen’s views of mathematics learning? If so, how?
4 Theoretical Framework

In this study, I will investigate the impacts of playing Slice Fractions or Slice Fractions 2, with epistemic change being the anchoring term as it broadly captures the object of interest. To approach personal epistemic cognition, I will draw on Hammer and Elby’s (2002; 2003; Hammer et al., 2005; Elby, 2009) resources-based view of knowledge and learning. Lastly, constructs and findings based on cognitive constructivism or scheme theory (Norton et al., 2018; Steffe, 2002; von Glasersfeld, 1995) are used to complement the resources perspective. Below I will review relevant concepts, theories, and models.

4.1 Epistemic Change

According to Muis et al. (2016), epistemic change is a “relatively swift but enduring adaptation in epistemic cognition in response to specific environmental factors” (p. 333). Key phrases or words are unpacked one by one. First, for “relatively swift but enduring”, I concur with Muis and colleagues’ (2016) interpretations to some degree. On the dimension of time, the change considered here differs from epistemic or intellectual development that occurs over a long period and naturally (e.g., Belenky et al., 1986; Kuhn et al., 2000; Perry, 1970). But, different from Muis et al. (2016) and in my assumption, the enduring change may stem from seemingly emergent and context-sensitive experiences during which novel patterns of activation of epistemological resources occur (e.g., Hammer & Elby, 2002; Louca et al., 2004), especially if a subject recognizes the experience as powerful or transformative.

Second, concerning “adaptation”, I adopt Muis et al.’s (2016) assumption and value judgment. Rather than being directionless or having similar values assigned to different epistemic stages, the change or adaptation is considered valid or meaningful if it takes place along the trajectory of epistemic or intellectual development, from a more rudimentary stage to a
more advanced stage, e.g., viewing knowledge from being simple and certain to being complex, conditional or uncertain; from recognizing fact as the only knowledge form to recognizing various forms and the heterogeneity of the realm; from seeing self as a passive receiver to seeing self as an active knower or learner (e.g., Greene et al., 2008; Hofer & Pintrich, 1997; Schommer-Aikins, 2004). In other words, epistemic change involves “changes for the better” and points to cognition, beliefs, and practices that suggest a more advanced epistemic stage. However, this is not to claim that the relatively more basic beliefs and practices are useless, which will be clarified later.

Third, about “epistemic cognition”, while Muis et al. (2016) adopted Bendixen and Rule’s (2004) model of change of epistemic beliefs, I would like to adopt Hammer and Elby’s (2002) framework that takes resource activation as the cognitive foundation for personal epistemic cognition. In the next section, I will elaborate on this framework. Before moving on, I shall first clarify the terms. In Hammer and Elby’s (2002, 2003) original writings, relevant concepts were considered “epistemological”, e.g., epistemological resources. In contrast, in recent years researchers differentiated epistemic from epistemological, the former concerning knowledge and the latter theories of knowledge, and largely agreed upon the appropriateness of adjusting certain constructs from “epistemological” to “epistemic” (Greene et al., 2016). Hence, I will adjust certain constructs from “epistemological” to “epistemic” in the subsequent parts.

4.2 Resources-based Approach to Epistemic Cognition

This theory takes up a bottom-up perspective to view personal epistemology, arguing that a contextualized or situated approach is more productive than a belief-based approach. To begin with, I will give an overview of the “bottom-up perspective”. Rejecting the conventional notion that knowledge pieces or ideas are the most basic cognitive units, Hammer, Elby and colleagues
(2002, 2005) posit that, cognitively and initially, knowledge is emergent and comes from activations of fine-grained resources in respect of contexts. Through repeated occurrences, some resources form internally coherent and stable networks whose activations always co-occur. Such co-activations or frames then serve as the bases for higher-level cognition. In this building-up process, one’s knowledge may gradually crystallize into more and more stable, connected, and complex structures; frequently used frames become established and form the foundation of personal epistemic beliefs. Now I will clarify the key construct involved, resources.

In Hammer and Elby’s (2002) framework, from a young age, we all have a variety of epistemic resources for understanding the nature and process of knowledge, the forms of knowledge, and one’s stance towards knowledge. Concerning the nature of knowledge, examples of resources of this category include knowledge as propagated stuff (knowledge can be transmitted from one person to another), knowledge as free creation (knowledge is invented in a vacuum), knowledge as fabricated stuff (knowledge is constructed or developed from other knowledge), knowledge as direct perception (to know is to perceive), knowledge as inherent (knowledge exists as objects’ inherent property), etc. Resources used to understand the knowing process and epistemic activities include accumulation (gathering or retrieving information), formation (creating, devising, and inventing, etc.), checking (re-evaluating in the same or a novel way), application (using existing knowledge in various situations) and so on. Resources for understanding knowledge forms include stories, lists, facts, statements, theories, categories, rules, etc. Lastly, resources for adopting an epistemic stance include belief, disbelief, doubting, understanding, puzzlement, etc. The relationship among the resources is complex. Some resources are contrasting or exclusive, meaning that their activations cannot co-occur, e.g., knowledge as propagated vs. fabricated stuff. Some resources are consistent with each other.
(e.g., *knowledge as fabricated stuff and formation*), and may form locally coherent sets as they are frequently activated together.

Under different contexts, different (sets of) resources can be activated (Hammer & Elby, 2002). However, what marks a context as distinct might not be some superficial features. For instance, in one case study, Kawasaki et al. (2014) examined a high school student’s epistemic resources activation (or lack of) in four different settings, taking objects, tools, community, rules, and divisions of labor into account. Under the classroom setting and media setting, which seemed different but were closely aligned, consistent epistemic resources were activated; whereas contrasting resources were used in the classroom setting and family interview setting, which shared similarities in all aspects except the people involved (Kawasaki et al., 2014). As another example, Louca et al. (2014) found that in a third-grade science class, a teacher’s instructional intervention could divert students’ thinking from function-oriented to process-oriented. In other words, within the same physical and social context, the teacher’s further guidance triggered students to switch resources for making sense of natural phenomena, from teleological explanations to mechanistic explanations (Louca et al., 2014). Hence, even though resource activation is context-dependent, there is no simple one-to-one correspondence as epistemic cognition may respond to many features in a context, well-defined or emergent, epistemic or non-epistemic.

Although the resources-based framework begins with a situated account of epistemic cognition, it does not claim that students’ thinking is completely emergent without discernible patterns. Rather, the framework provides an account of how personal epistemology or epistemic beliefs are formed. I will elaborate on this, drawing from Elby (2010) and Hammer et al. (2005).
coincidental and dissolve quickly, while others develop stability where the elemental resources
are tightly related to each other and get reliably co-activated under certain circumstances. Frame
or framing, a concept originally from anthropology and linguistics, refers to such activations of
locally coherent sets of epistemic resources. Phenomenologically, a frame encompasses one’s
interpretations and anticipations about a situation. Even though frames are often tacit, they
influentially direct what one sees and how one reacts in various contexts. Over time, if one
consistently applies a particular frame to some situations, one may form solid beliefs about and
approaches to the items involved. As one accumulates and connects various experiences,
epistemic frames may be compiled into personal domain-specific or even domain-general
epistemic beliefs.

Now I will clarify this theoretical framework’s implications on learning and instruction.
Broadly speaking, consistent with the fundamental view that knowledge is emergent, learning
involves forming or entering a cognitive state where certain fine-grained resources are activated
in a particular context (Hammer et al., 2005). Then, based on previous situated experiences, one
may form or display conceptions or ideas as s/he momentarily activates mini generalizations
(Hammer et al., 2005). The conceptions or ideas may serve as cognitive resources for further
knowledge construction or learning.

However, not all resources are of the same value across various contexts. When one
inappropriately draws on certain resources that are more useful in other contexts, misconceptions
may be manifested (Hammer et al., 2005; Smith et al., 1993). For instance, for additions of
fractions, many students may have drawn on the resources appropriate for fractional
multiplication and integer addition to erroneously think that it involves adding up the numerators
and the denominators respectively. In addition, in a specific context, maybe multiple resources
apply to the item at hand but carry different values. As an example, student A may successfully solve a fractional addition problem by citing rules taught by the teacher, thus drawing on the resources that suggest “knowledge is propagated stuff”; student B might achieve the same goal based on one’s prior knowledge of integer addition, fractions as iterations of unit fractions, and fractions as ratios, thus pointing to the constructive nature of knowledge. Both sets of resources are legitimate but have different affordances and limitations. Student A converts the original items to simpler ones and can more efficiently finish the calculation, but as learning continues one’s knowledge can be fragmented and scattered (see Benny’s case in Erlwanger, 1973). Student B’s problem-solving is rooted in making sense, which contributes to learning with understanding and developing productive dispositions (Gresalfi, 2009), but the thinking process takes a much longer time.

Hence, considering ideal learning outcomes, a learner shall be able to explicitly discern the nuances among various situations and flexibly draw on the resources that best fit each situation. If initial learning involves using any applicable resources to a novel context, intermediate learning may involve activating different resources in the same context, which form the foundation for evaluating multiple sets of resources in relation to the context and for flexibly activating appropriate resources in the future. Since the resources-based framework assumes that each learner possesses various epistemic resources from a young age, instruction is to support learners to activate the productive resources they already use in some other contexts to the context of learning (Hammer & Elby, 2003). Further, since resources activation form frames and lay the foundation for higher-level resources, instruction is also about supporting learners to establish productive practices in specific contexts and building up resources. Lastly, considering that ideas and conceptions are mini-generalizations of earlier experiences, cultivating new ideas
requires designing and providing experiential opportunities and guiding learners to generalize. Tackling misconceptions requires parsing and revealing the more fundamental resources, and helping learners refine their understandings about the resources’ potentials and limitations, and the conditions that render certain resources fruitful or fruitless.

Since lower-level resources may be used to build up higher-level resources, clarifications about such resources and their inter-relations are critical as they help diagnose learners’ progress and guide the directions of teaching and instructions. However, the resources-based framework offers an abstract theorization of epistemic cognition but not clarifications on the specific topic-level resources or frames. Consequently, to complement this, I will draw on cognitive constructivism or scheme theory, particularly the constructs and findings about fractions. I argue that scheme theory goes well with the resources-based framework and that fractional schemes and operations shed light on the resources involved in learning fractions.

4.3 Scheme Theory

First, I will clarify how scheme theory goes along with the resources-based framework. As stated earlier, a frame is one’s interpretation of a situation that influences one’s attention and actions (Hammer et al., 2005). This concept is close to “scheme”, which is a three-part structure, including perceived situation, associated activity, and expected result (von Glasersfeld, 1995). Concerning similarities, the triggering of both depends on the situation as experienced and perceived by a subject; both are typically tacit or implicit, and an observer may access them by making inferences based on actions or language use (Hammer et al., 2005; von Glasersfeld, 1995). In scheme theory, learning refers to the cognitive change during which an existing scheme no longer leads to an expected result in a situation and consequently a subject either refines the existing scheme (adding restrictive conditions) or generates a new scheme if the novel result is
desired (von Glasersfeld, 1995). Since the resources-based theory suggests that learning involves refining resources’ activation in various situations, it is fair to claim that these two theories are compatible.

Another major similarity between these two theories concerns assumptions about the basic components’ interrelations and role in development. In scheme theory, action schemes are at the sensorimotor level and help achieve goals in specific contexts; operative schemes result from abstract reflections of the mental processes that involve mental actions or operations, and they serve as epistemic instruments for subjects to form “a coherent conceptual network of structures” (von Glasersfeld, 1995, p. 68). Thus, theoretically speaking, the process of building up and expanding operative schemes comes with frequent identifications of contradictions (or “perturbations” in Piaget’s language), re-evaluations, and negotiations of the viabilities of the schemes involved (or “eliminating perturbations and establishing equilibrium” in Piaget’s words). Hammer and colleagues (2005) posit that sets of locally coherent activations of epistemic resources form frames, cognitive objects that may support further construction. While frames are context-sensitive and may not be mutually consistent, as learning deepens, it is necessary to deliberately reconcile frames from various contexts (e.g., formal school learning and everyday experiences) for the frames to form higher conceptual structures independent of contextual features, “coordination class” in diSessa and Sherin’s (1998) language. To sum up, in both frameworks, long-term learning or development involves constructing a coherent and stable conceptual structure made up of schemes or frames; this process starts from concrete context-sensitive experiences and goes into the abstract cognitive arena.

Having discussed the two theories’ compatibility at the conceptual level, now I will discuss the compatibility at the practical level. When it comes to specific analyses, epistemic
researchers may make inferences about the activated resources or frames based on a subject’s linguistic markers, word choices, and actions (Hammer et al., 2005; Rosenberg et al., 2006; Sandoval, 2005). Similarly, to understand schemes and operations, scheme-focused researchers build models of a subject’s thinking by conducting teaching experiments and eliciting cues of the same types (Steffe & Thompson, 2000). Hence, it is appropriate to rely on existing findings from scheme theory and apply them to investigate epistemic cognition when needed. With the help of existing findings, we can better focus on epistemic cognition and change, which I consider a broader field encompassing constructivist knowing and beliefs. We can better focus on expanding one’s repertoire for approaching fractions, prioritizing and promoting resource activation that points to the construction of personal understanding and a more relativist view of mathematical knowledge while also acknowledging the value of the resources that suggest a more absolute or realist view of mathematical knowledge.

In this study, I intend to draw on the constructs and the instrument from earlier research on individuals' constructive knowing of fractions (e.g., Hackenberg, 2007, 2010; Norton & Wilkins, 2009; Norton et al., 2018; Steffe, 2002, 2003), which provides a detailed framework for examining fractional understanding. Here I will review the fractional schemes and operations in the order that they are developed by learners (Norton & Wilkins, 2009; Norton et al., 2018). The instrument will be introduced in the methods section.

With the part-whole scheme (PWS), a student may view fractions as some parts taken out of an equally partitioned whole, with each part being somewhat unique rather than a recurring unit fraction (Norton et al., 2018; Steffe, 2003). To produce a fractional quantity, say three-fourths, a student would partition a whole into four equal parts and then take out three parts, rather than take out one part and iteratively use it three times. This scheme works fine with
proper fractional quantities but does not support making sense of improper fractions. Use this scheme to consider four-thirds. A whole contains three equal parts in total and thus it is insensible to take out four parts out of it. A student who has constructed the partitive unit fraction scheme (PUFS) can create a unit fraction of an unpartitioned whole and verify the quantity by iterating the unit fraction certain times to recreate the whole (Wilkins et al., 2013; Steffe, 2002, 2003). Implicitly or explicitly, those with this scheme may conceive a unit fraction, like one-seventh, as an iterable basic unit. The emphasis here is the relation between a unit fraction and the corresponding whole, not just any fractional quantities composed of the unit fraction. Fractional language like “one-eighth” may be directly used (Steffe, 2002). Like the PWS, the PUFS does not support the creation or sense-making of improper fractions, as the iteration here does not go beyond unit one. The partitive fraction scheme (PFS) is the more general version of the PUFS. While PUFS involves understanding the relation between a unit fraction and an unpartitioned whole, PFS enables one to create proper fractions by iterating a unit fraction (Hackenberg, 2007). For example, to create four-sevenths, a learner who applies a PFS can first create one-seventh and then iterate this unit fraction four times. At this stage, fractions are limited to proper fractions (Hackenberg, 2007; Norton & Wilkins, 2009).

According to Steffe (2002, 2003), the key difference between the earlier schemes and the splitting operation (SO) is whether a learner regards partitioning and iterating as different or the same operations. Someone who has constructed the PFS but not the SO could partition and iterate sequentially but would not be able to unite these two actions and view them as reverse processes of each other. Constructing the SO is necessary for one to make improper fractions and for developing more complex schemes (Hackenberg, 2010; Norton & Wilkins, 2009). The iterative fraction scheme (IFS) is the more advanced and generalized version of the PFS. It
enables a learner to generate any fraction, proper or improper, by iterating a unit fraction, and to view any fraction as a quantity (Hackenberg, 2007, 2010; Steffe, 2002). With this scheme, a learner may regard a fraction as a multiple of a unit fraction, e.g., four-fifths as a quantity four times as large as one-fifth (Hackenberg, 2010). Compared to previous schemes, the IFS has the merit of supporting a learner to make sense of improper fractions, when the idea of “parts of a whole” no longer works. Also, this marks the establishment of any unit fraction as a distinct unit, whose existence goes beyond a whole or unit one (e.g., one-eighth can make up eight-eighths, nine-eighths, ten-eighths, etc.). The Reversible Partitive Fraction Scheme (RPFS) is built from an integration of the SO and the PFS (Hackenberg, 2010; Norton et al., 2018). With this scheme, one can unite partitioning and iterating and reverse the conventional “partitioning and then iterating” sequence. Given a problem where parts of a whole are known but not the whole, e.g., “find out the length of a bar where 3/5 of it is 2 inches”, a learner who has not constructed the RPFS may try to apply the PFS and calculates the length of three-fifths of two inches (Hackenberg, 2010). On the other hand, someone who has established this scheme may first reverse the iteration process by equally partitioning the two inches into three parts to find out the iterative unit in the original problem, and then reverse the partition process by iterating the iterative unit for five times to find out the original quantity.
5 Methods

5.1 Research Design

In this study, I investigated the potentials of Slice Fractions (SF1) and Slice Fractions 2 (SF2) as remedial tools for learners who were exposed to fractions during previous formal education but did not grasp the topic well. To achieve this goal, I recruited educational opportunities programs (EOP) freshmen at a public university in New York State as participants. Coming from disadvantaged and underrepresented backgrounds, EOP students fall short of the regular criteria for college admission but show promise of academic success (State University of New York [SUNY], n.d.). In EOP programs, students receive pre-freshman services, tutoring, and counseling (SUNY, n.d.). EOP freshmen fit the study for at least two reasons. First, even though they have learned fractions in their earlier education, their knowledge is flimsy and still has substantial room for improvement. Thus, it is appropriate to recruit these students to help explore the impact of playing SF1 and SF2 on their knowledge of fractions. Second, EOP freshmen have a recent experience of learning/reviewing and using their knowledge of fractions in the school context, because basic math topics including fractions are reviewed in EOP summer pre-freshmen programs and because fractions are involved in their freshmen-year courses like pre-calculus. Therefore, they can provide insights into their experience of learning or using fractions in conventional settings and in video games.

To compare EOP students’ conceptions of fractions, fractional problem-solving, and views of mathematics learning before and after playing SF1 and SF2, I employed a quasi-experimental design. Differing from earlier studies that were conducted in whole-class settings (Cyr et al., 2019; Gresalfi et al., 2017; Zhang et al., 2020), I met each participant one-on-one for a few reasons. First, it was easier to arrange a camera above one participant’s screen and record
the gameplay, which could serve as data. Second, in an untimed and one-on-one setting, participants might be less likely to get stressed or rush through gameplay or assessment under peer pressure. And I could better attend to questions or issues that arose during data collection. Overall, for each participant, there was a pre-game session, a gameplay session, and a post-game session.

In pre- and post-game sessions, there were a semi-structured interview and an assessment. The interview questions concerned conceptions of fractions, views of math learning and gameplay experiences (in post-game interviews only). Each assessment had 24 items and was used to measure one’s fractional understanding (for more details, see data collection). During gameplay sessions, each participant played SF1 and/or SF2 for at least one hour (unless one finished a game in less than one hour). There were two games involved, SF1 and SF2. These two games are relatively independent of each other and SF2 is more complex than SF1 in terms of both game mechanics and subject matter (to be explicated later). Therefore, I informally evaluated a participant’s current fractional knowledge based on responses and performance in pre-game sessions and then invited the participant to play SF1 and/or SF2 (Figure 1).

**Figure 1**

*Data Collection Process*
5.2 Participants

Participants were EOP freshmen at a large public university. Upon entering the program, the students took a math test, based on which they were divided into different groups and were taught math by different teachers. The EOP program began with a 5-week pre-freshman summer program, during which pre-college mathematics topics like arithmetic and algebra were reviewed. Many EOP students had relatively weak arithmetic and/or algebraic skills and thus needed remedial measures to get ready for college-level math courses like pre-calculus or calculus. During their freshman year, the students took EOP-specific pre-calculus, calculus, and/or statistics courses depending on the course requirements of their intended majors. The math courses offered by EOP included more academic support, e.g., more frequent classes and quizzes, more teaching assistants, and longer office hours.

A total of fourteen EOP freshmen participated in and completed this study. When the study took place, all students were in their first semester in college with one exception. One student started college in the previous year but the class standing remained freshman due to prior excessive absence. All but one participant were taking EOP-specific Pre-calculus, and one participant was taking EOP-specific Calculus I. The participants took their EOP math courses with three different teachers, depending on their performances on a math placement test at the start of the fall semester. Two participants performed relatively better during the placement test. One was taking Calculus I and the other was taking Pre-calculus taught by the same teacher. Five participants’ performances were ranked in the middle and thus were taking pre-calculus with another instructor. The remaining seven participants were taking pre-calculus with the third instructor. Among them, five participants’ math placement test performances were among the lowest; one participant performed well but was in this class due to conflicts in course scheduling;
one participant was re-taking the course for the third time due to excessive absence in the previous semesters. On average the participants' age was 18 years old. There were seven females, six males, and one non-binary participant. All participants were members of racial minority groups: eight were African Americans, five were Hispanic/Latinos, and three were Asians (the total count here was sixteen since two students identified as mixed races).

5.3 Slice Fractions and Slice Fractions 2

First, I will present the overall structure and the content arrangements within each game. Then I will discuss the games from the videogame perspective, loosely drawing on Gee’s principles that Devlin (2011) perceived to be highly relevant to learning basic mathematics.

A summary of the two games’ levels is presented in Table 1. Each game comprises several sections, each section featuring one topic, e.g., splitting, symbols, fraction parts in SF1 and fractions, multiplication, and common denominator in SF2. Within each section, there are two to four themes. Each theme highlights one specific skill. For example, SF1 Section 1 “Splitting” includes “how to play”, “split groups”, and “slice shapes” three themes. Note that these topics and themes are devised by the game designers. There are several levels on each theme. Since various levels might be frequently mentioned, it is sensible to have a systematic way to name them. Hence, in the rest of the paper, when mentioning specific puzzles, I will follow the “Game.Section.Level” format, such as SF1.IV.13 or SF2.III.6. From the research perspective, not all levels are relevant or “educational”. To distinguish critical levels from the rest, I identify and categorize the levels into the following groups. “Educational” levels (98 or 70% of SF1 levels, 87 or 75% of SF2 levels) contain puzzles to solve, and the puzzles are interconnected with each other. The other levels are considered irrelevant or negligible, including “start”, “award”, “video”, and “transition” levels, for they do not contain fractional puzzles.
Specifically, “start” levels (1 level or 1% in each game) are the grand openings. At “award”
levels (17 or 12% of SF1 levels, 11 or 9% of SF2 levels), a badge and an object (hats in SF1 and
pets in SF2) are awarded. “Video” levels (7 or 5% in SF1, 7 or 6% in SF2) require no actions and
serve the purpose of maintaining the storyline. “Transition” levels (17 or 12% in SF1, 10 or 9%
in SF2) are puzzles unrelated to fractions. They do not impact the storyline or the game
progression but give a player a brief break from fractions.

Table 1

Structure and content arrangement of Slice Fractions and Slice Fractions 2

<table>
<thead>
<tr>
<th>Game</th>
<th>Pre-designed Section</th>
<th>Pre-designed themes and levels within</th>
<th>Irrelevant levels identified by researcher</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slice Fractions (SF1; 140 levels in total)</td>
<td>I. Splitting</td>
<td>How to play (1-5); Split groups (6-10); Split shapes (11-22)</td>
<td>1 (start); 3, 22 (videos); 10, 21 (awards); 16, 20 (transitions)</td>
</tr>
<tr>
<td></td>
<td>II. Symbols</td>
<td>Shape comparison (1-6); Symbolic numerator (7-12); Fractions Reading (13-19)</td>
<td>6, 12, 18 (awards); 9, 17 (videos); 5, 19 (transition)</td>
</tr>
<tr>
<td></td>
<td>III. Fraction Parts</td>
<td>Denominator (1-5); Numerator (6-14); Pie charts (15-22)</td>
<td>5, 14, 21 (awards); 15 (transition), 22 (video)</td>
</tr>
<tr>
<td></td>
<td>IV. Comparison</td>
<td>Size comparison (1-9); Equivalent fractions (10-18); Advanced equivalent fractions (19-27)</td>
<td>1 (video); 6, 15, 23, 25, 27 (transitions); 9, 18, 26 (awards)</td>
</tr>
<tr>
<td></td>
<td>V. Problem Solving</td>
<td>Fractions ordering (1-5); Question marks (6-11); Subtraction (12-18)</td>
<td>4, 10, 16 (transitions); 5, 11, 18 (awards)</td>
</tr>
<tr>
<td></td>
<td>VI. Addition</td>
<td>Addition blocks (1-11); Basic addition (12-21); Advanced addition (22-32)</td>
<td>1, 9, 19, 28 (transitions); 11, 21, 31 (awards); 32 (video)</td>
</tr>
<tr>
<td>Slice Fractions 2 (SF2; 116)</td>
<td>I. Fractions</td>
<td>How to play (1-9); Slice shapes (10-28); Symbols reading (29-37); Fractions reading (39-52)</td>
<td>1 (start); 6, 28, 52 (videos); 9, 21, 27, 37, 51 (awards);</td>
</tr>
</tbody>
</table>
levels in total) 10, 29, 39, 42, 47 (transitions)

II. Multiplication
- Ghosts (1-12)
- Multiplication (13-24)
- Number multiplication (25-35)

III. Common denominator
- Common denominator (1-19)
- Big fractions (20-30)

Note. “Start”, “award”, “video”, and “transition” levels have no fractional puzzles to solve. A “start” level is a game’s grand opening. At “award” levels, a badge and a hat or pet is awarded. “Video” levels require no actions. “Transition” levels include puzzles unrelated to fractions and mainly provide a break from fractions.

With the help of certain design features and game mechanics, these two games support active learning and strategic thinking (Devlin, 2011). From the very start of the games, there are no word-based instructions, and a player is encouraged to observe and to act. Specifically, one is expected to examine the setups, manipulate the environment by tapping or slicing objects, and directly experience the consequences of the interactions. For example, SF1.1.2 (Figure 2) shows that bubbles can hold things up and engage a player in popping selected bubbles. It also shows that the task is to remove obstacles on the mammoth’s path and that gravity exists in the game world.
Figure 2

*SF1.I.2: introduction of bubbles in SF1*

![Images of SF1.I.2](image)

*Note.* In SF1.I.2., the main character, a mammoth, stops in front of a broken path where two bubbles hold a rock up in the air (a). If nothing takes place for a few seconds, a hand shows up and suggests the player to click the bubbles (b). After clicking the two bubbles one by one (the orange circles mark the tapping location at each scene), the rock fills the hole on the ground, thus allowing the mammoth to continue marching (c-d).

SF1.I.3 (Figure 3) introduces lava and ice as game elements, and one may perceive that lava and ice of the same size can resolve each other. Even though no action is required here, the knowledge gained from observation can be directly applied in subsequent puzzles. As another example, SF2.I.2 (Figure 4) instructs one to tap a snail. By acting and seeing the consequences, a player may learn that tapping a snail extends or shrinks its neck along the direction the snail is facing. In addition, SF2.I.2 leaves room for multiple problem-solving (Devlin, 2011) as one may tap the three snails in different orders and thus solve the puzzle in multiple ways. Room for various problem-solving and strategic thinking enhances the games’ playfulness and engagement.
Note. SF1.I.3 requires no action but introduces ice and lava. Initially, four ice hexagons of different sizes lie on the ground and the volcano erupts (a). Three lava hexagons fall from the sky one by one and each of the first two lavas resolves an ice of seemingly equal size (b-c). The third lava is large, and it first hits one ice (d); the remaining lava continues rolling and exactly resolves the last ice (e). After the ices are cleared up, the mammoth continues walking (f).

Figure 4

SF2.I.2: introduction of ice, lava and snail in SF2
Note. In SF2.I.2, on the mammoth’s path there are three obstacles from left to right: a snail’s long neck, a lava on the ground, and a cliff on the way (a). A hand suggests one to tap the snail whose neck blocks the path (a). After tapping, the snail’s neck is shortened and the mammoth marches forward until the lava (b). By tapping the snail who is holding an ice, the snail releases the ice, which resolves the lava and clears up the path (c). Lastly, after tapping, the snail on the cliff edge extends its neck and enables the mammoth to move forward (d).

The games provide a safe environment for making mistakes or failing and give hints when necessary (Devlin, 2011). First, there is no scoring or timer, which can be stressors for some mathematics learners. A player can tackle a puzzle multiple times or spend as much time on a puzzle as one likes. Once a puzzle is successfully solved, the next puzzle would be available. Thus, if one wants to keep track of progress, one may refer to the number of puzzles that have been unlocked. Otherwise, having the puzzles solved is the only goal. Neither approach nor speed matters. If one gets stuck at a puzzle, game-based hints will appear to facilitate problem-solving. For instance, in SF1.II.2 (Figure 5), initially all three ices are available to be released and used to clear up the lavas on the ground. If one fails and restarts the level for two consecutive times, upon the third trial, the setup changes so that only two ices can be released, thus effectively reducing the number of choices and the puzzle’s difficulty.

**Figure 5**

*SF1.II.2: an example of hints in SF1*
Note. In SF1.II.2, there are two lavas on the ground and three ices hanging in the air. In the initial setting, all ices are attached to or supported by bubbles, so a player chooses which two of the ices to be used (a). After failing the level for two times, the setup changes so that the ice in the middle is attached to rocks, thus eliminating one choice (b).

In SF2.II.15 (Figure 6), there are four options for one to start with: slicing the lava horizontally or vertically and tapping the left or right snail. After failing for a few times, the game prompts a player to tap the right snail as the first step (Figure 6.d). If one still cannot solve the puzzle, then the game finishes the first step and carries out a second step, altering the initial setup (Figure 6.i) and leaving the rest for a player. Again, by including such hints, the game simplifies a puzzle. The games step in after several failed trials, thus leaving a player with enough space to independently solve a puzzle and providing help in time to prevent high levels of frustration.

**Figure 6**

*SF2.II.15: an example of hints in SF2*
Note. SF2.II.15 requires one to slice a lava hung in the air and resolve two ices on the ground. Pictures a-c. illustrate one unsuccessful problem-solving, in which one first slices the lava vertically and then drops each lava. If one keeps on failing for a few times, a game prompt shows up at the beginning telling one to tap the snail on the right first (d). Pictures e-f show another unsuccessful problem-solving attempt, where one’s slicing drops a large part of the lava, melts the ice, and even leaves a hole. After some more unsuccessful attempts, more game scaffolds show (g-h), as the game directly carries out the action of tapping the snail on the right and letting the lava swing to the left. Then the game taps the right snail again and reconnects the right snail and the lava (i), leaving a player to work on the puzzle from there.

By arranging many puzzles underneath each section and group, the games provide ample opportunities for one to practice newly gained knowledge or skills (Devlin, 2011). As an example, in SF1, puzzles underneath Section IV all focus on equivalent fractions. The basic setups in SF1.IV.10, SF1.IV.11, SF1.IV.14 and SF1.IV.7 (Figure 7) are very similar. They all require a player to manipulate and arrange certain amounts of ice on the ice bar and then release them to resolve varying amounts of lavas on the ground. Overall, the puzzles are arranged from the easier ones to the more difficult ones (Devlin, 2011). The games begin with perceptible quantities (e.g., Figure 2), and gradually transitions to visual representations with labels (e.g., Figure 5) and then to a mixture of symbols and visual representations (e.g., Figure 7). By incrementing difficulty slowly, the games help build up a player’s skill set and confidence as one plays more and makes more progress. Also, the puzzles are never about abstract calculations, meanings of fractions and fractional operations are rooted in visual representations and concrete actions and outcomes, which may be particularly helpful in establishing understandings (Devlin, 2011).
A comparative view of SF1.IV.10, SF1.IV.11, SF1.IV.14, and SF1.IV.7

Note. A comparative view of SF1.IV.10 (a), SF1.IV.11 (b), SF1.IV.14 (c), and SF1.IV.7 (d) The basic setups in these four puzzles, especially the first three, are similar. To solve the puzzles, one needs to draw on knowledge on equivalent fractions.

Usually there are one or two “milestone” puzzles in each group, where the game presents a badge stating what skill one has learned, a short animation illustrating the skill, and a hat (Figure 8). With these reward levels, a player gets rewarded frequently for every skill gained, which might increase a player’s motivation to continue playing the games (Devlin, 2011). Also, by allowing one to select a hat to wear, a player is given some control of the main character and thus may consequently be more engaged in the gameplay (Devlin, 2011). Lastly, each section features a different color theme. At the end of a section, a puzzle or two might be animations of the mammoth “traveling” to the next world/terrain. Such puzzles help connect different sections and create an attractive overarching storyline about the mammoth going on an adventure.
Figure 8

SF1.IV.9: an example of reward levels

![SF1.IV.9: an example of reward levels](image)

Note. SF1.IV.9 is a reward level. It presents the Badge of Size Comparison (a) and then illustrates “size comparison” with a short animation (b-c). After the animation, the mammoth continues walking and picks up a new hat, adding it to its hat collection (d).

Having introduced both games while considering their similarities, now I will discuss their differences. SF1 is easier than SF2 in terms of both game mechanics and subject matter. SF1 contains fewer game elements, and their functions are more straightforward. Besides the elements shown earlier, SF1 introduces “addition block” in the last section. In SF1.VI.4 (Figure 9), three-fifth is an addition block, which can merge with ice chunks with the same denominator but not others. In SF2, besides snails, there are ghosts and narwhals. Ghosts can transport and duplicate ice/lava chunks. Once a parent ghost to swallow something, a child ghost will spit out the same thing; usually there are one parent ghost and multiple child ghosts so that multiplication is achieved (Figure 10). Narwhals can thrust their horns into ices to indicate where one can slice and what quantities will result from the slicing, and different combinations of narwhals suggest different ways of cutting (Figure 11). While addition block in SF1 extends from an existing element, regular ice, either ghost or narwhal is completely new and unrelated from other elements. Therefore, SF2 is more complex than SF1 as a game.
Figure 9

SF1.VI.4: an illustration of addition blocks in SF1

Note. An addition block can merge with regular ones if they share the same numerator. In SF1.VI.4, three-fifths ice is an addition block. Once one-fifth hits three-fifths, they merge and become four-fifths (b). In contrast, when one-fourth hits four-fifths, the new addition block, the two blocks’ denominators turn red, and one-fourth is bounced away (c-d).

Figure 10

SF2.II.5: an illustration of ghosts in SF2
Notes. In SF2.II.5, Ghosts are present and help duplicating items. SF2.II.5 has one parent ghost and three child ghosts (a). After a parent ghost swallows an ice (b), the ice gets duplicated and transported (c) so that each child ghost spits out a copy (d).

Figure 11

SF2.III.2: an illustration of narwhals in SF2

Note. Throughout SF2.III.2, Narwhals are key tools to help slicing. SF2.III.2 provides three narwhals facing an ice from different angles (a). Each narwhal may thrust its horn into the ice suggesting a way of slicing (b) and multiple narwhals/horns can do so simultaneously (c). After settling on which narwhal(s) to use (c), one can slice the ice and go from there (e-f).

SF1 primarily covers more basic topics and give more problem-solving hints. Equal partitioning, fraction notations (“a/b”) and fraction equivalence are considered Grade 3 and Grade 4 content in CCSSM. The most advanced topics in SF1 are relatively simple fractional subtractions and additions. For example, looking at Figure 12, in SF1.V.14 one is expected to
create five-sixths ice out of partially attached one and four-sixths ices. Each quantity is presented with a background showing the whole and six equal parts. Each ice chunk also has marks on the surface to suggest ways of slicing. There is no need to calculate and figure out five-sixths ice all at once, as one can let ice pieces fall one after another, gradually resolving the lava. Figure 13 shows SF1.VI.23, one of the most difficult puzzles within SF1. Here one needs to create three-sixths ice from partially attached two-thirds addition block and one-half ice. Like SF1.V.14 (Figure 12), the ice chunks have marks to suggest one to cut in certain ways, which simplify the puzzle. In this puzzle, one needs to convert halves and thirds into sixths to add up the ice pieces, which is probably the simplest case in adding fractions with unlike denominators.

**Figure 12**

*SF1.V.14: an example of advanced puzzles in SF1*

*Note.* In SF1.V.14, given one and four-sixths ices, one needs to create five-sixths (a). One solution is to first cut the whole ice into halves or three-sixths horizontally (b). The bottom ice piece falls and resolves three-sixths lava, resulting in two-sixths lava left (c). Then, slice the four-sixths ice in half so that two-sixths ice fall to resolving the remaining lava (d).
Note. The puzzle starts with two-thirds addition block and one-half regular ice and asks us to resolve three-sixths lava hidden behind a cloud (a). One way to solve this is shown here. First cut the two-thirds ice into two one-third chunks (b). Then follow the marks on one-half ice to cut off two pieces, each being one third of the original piece or one-sixth (c-d). Horizontally slice one addition block so that a half of it, weighing one-sixth, falls (e). With the addition block, now the fallen blocks merge into three-sixths ice (f), which can be released to clear up the lava.
SF2 involves multiplying fractions by whole numbers, displaying fractions with unit fractions, using equivalent fractions to add or subtract fractions, and improper fractions, which are Grade 4 and Grade 5 topics in CCSSM. And SF2 is mathematically more complicated because of many extra options. For instance, SF2.II.31 (Figure 14) asks one to create six-fifths. One needs to figure out how much to cut off from one whole ice and the number of child ghosts needed. To solve the puzzle by trial-and-error, there are at least six ways to try out, since the parent ghost may swallow one-fifth to three-fifths ice and there could be three or four child ghosts (3*2 = 6). Also, in this puzzle improper fraction six-fifths is displayed as a single quantity, differing from SF1 puzzles where only proper fractions are used. Throughout SF1, one may make sense of fractions with “part of a whole”. But in SF2, with improper fractions’ presence, a player needs to develop other ideas to understand fractions.

Figure 14

SF2.II.31: an example of advanced puzzles in SF2

Note. In SF2.II.31, given one whole ice and a one-to-four multiplier, one needs to resolve six-fifths hidden lava (a). To solve the puzzle, one needs to first tap the bubble and remove one child ghost (b). With the multiplier being three, two-fifths ice is needed to create six-fifths (c-d).
In SF2.III.11 (Figure 15), ice and lava are displayed as compositions of unit fractions, three one-fourths and two one-thirds respectively. By applying different narwhals, the puzzle visually present equivalent fractions, e.g., one-fourth equals to two one-eighths or three one-twelfths. Using the solution shown in Figure 1-15, one needs to convert both thirds and fourths into twelfths to precisely compare the amounts of lava and ice, which is a slightly more difficult case for adding/subtracting fractions with unlike denominators. If a player does not use any narwhal and cuts off one-fourth or two-fourths ice first, the lava quantity becomes five-twelfths or one-sixth, making the subsequent fraction subtraction harder (Figure 16). Because each narwhal may or may not be used, there are a total of sixteen (2^4) ways to employ the four narwhals. And there are even more ways to slice the ice and the lava. So, resorting to trial-and-error would be quite inefficient. Overall, SF2 is mathematically more complicated.

**Figure 15**

*SF2.III.11: an example of advanced puzzles in SF2*
Notes. SF2.III.11 starts with two one-third lavas surrounded by a three-horn- and a two-horn-narwhals and three one-fourth ices surrounded by a two-horn- and a single-horn-narwhals (a). There are many methods to cut the lavas and the ices (e.g., b-d). One method can convert both quantities into one-twelfth pieces(d). Then a total of eight ice pieces are needed to exactly resolve the lavas (e-f).

Figure 16

An alternative way to solve SF2.III.11

Note. By first cutting one-fourth ice off, there is five-twelfths lava left (a). With the help of the two-horn narwhal, two one-fourth ices turn into six one-twelfth ices (b), enabling precise comparison between the quantities of lava and ice. If two one-fourth ices are dropped first, there is one-sixth lava left. With the two-horn narwhal, one-fourth is converted into three one-twelfth. But no single-horn narwhal is available to convert one-sixth into two one-twelfth. Thus, a player needs to imagine such a narwhal or tap previous knowledge that two one-twelfths is equal to one-sixth to proceed.

To sum up, SF1 and SF2 are well-designed SVGs, weaving together formal knowledge and fun. Considering teaching and learning, the games do not require prior familiarity with
fractions. They support active exploration, multiple problem-solving, and a buildup of understandings and skills, accompanied by plenty of scaffolds and opportunities to practice. As for the playful side, the games engage a player in solving varying puzzles on behalf of a mammoth, with the storyline being gradually revealed as the character makes progress. And the games feature frequent rewards and attractive visual designs. Comparing the two games, SF2 is more challenging than SF1 as for both game mechanics and mathematical content.

5.4 Data Collection

For each participant, data collection included three parts (see Figure 1): a pre-game session (including an interview and an assessment), gameplay on a mobile device, and a post-game session (including an interview and an assessment). Pre- and post-game interviews were audio recorded. During pre- and post-game assessments, each participant’s work or problem-solving processes were video recorded. When playing SF1 or SF2, video recordings were made so that in-game actions (tapping or slicing), gameplay process (e.g., working out a puzzle through analysis or trial-and-error), and game progress (which puzzle a participant reaches by the end of the gameplay session) were collected. Next, details about the tools and procedures used in conducting and collecting interviews, assessments, and gameplay are presented in sequence.

Both pre-game and post-game interviews (Appendix A) were semi-structured. In the pre-game interviews, participants discussed their understanding of fractions and fraction-related rules/procedures, how they learned mathematics especially fractions in their earlier education, and their previous experience with video games. In the post-game interviews, participants discussed their current understanding of fractions and fractional operations, their gameplay experience, and their perceptions of integrating SVGs like SF1 or SF2 in formal math learning. In addition, the participants discussed whether one considers oneself to be a “math person”.
“Math person” was used because its criteria would be much looser than “mathematician”, whom no participant aspired to be and thus was irrelevant to the participants personally. Informed by the situated perspective of epistemic cognition, no interview questions targeted the participants’ epistemic beliefs about mathematics, the domain. It was assumed that their recollections about fractions learning and practices in fraction problem-solving could partially reflect or serve as part of the basis of their mathematical epistemic beliefs.

The pre-game and post-game assessments (Appendix B) were used to elicit and measure a participant’s fraction schemes and splitting operation. Each assessment contained twenty-four items. The two assessments were consistent in the arrangement of items and the types of questions asked. The items in pre-game and post-game assessments were slightly different from each other. The post-game assessment items required partition into more pieces (e.g., “create one-seventh vs. one-fifth”) or more iterations (e.g., “find out five-thirds vs. two-thirds”), and the shapes used were more generic (e.g., a random sector rather than a semi-circle or a quadrant).

The twenty-four items were used to evaluate six schemes/operations, so there were 4 items on each scheme/operation. Twenty items were adapted from Norton et al.’s (2018) study and were intended to measure the part-whole scheme (PWS), the partitive unit fraction scheme (PUFS), the splitting operation (SO), the partitive fraction scheme (PFS), and the reversible partitive fraction scheme (RPFS). Four items were adapted from Hackenberg’s (2007, 2010) and Norton & Wilkin’s (2009) work and were intended to measure the iterative partitive scheme (IPS). Following Norton and colleagues’ (2009, 2018) design and findings, the items were arranged in the order consistent with the development of learners’ fractional knowledge: PWS, PUFS, PFS, SO, IPS, and RPFS.
The items in the pre-game and post-game assessments closely followed Norton and colleagues’ (2018) design. For PWS, PUFS, PFS, and IPS, there were two types of problems evaluating each scheme. For SO and RPFS, there was only one type of problem. In terms of visuals, for items on the PWS, Item 1-4, only bar representations were used; for items targeting other schemes/operations, both bar and pie representations were used. Below are sample items for eliciting and assessing the schemes/operations. On PWS, PUFS, PFS, and IPS, there are two sample items for each, one item for each problem type. On SO and RPFS, there is one sample item for each.

Figure 17 shows two sample items for eliciting the PWS. In the upper item, given a pre-partitioned whole, a learner is asked to identify a proper fractional quantity. In the bottom item, given two quantities each being partitioned by the same unit, a learner names the relationship between the two quantities using a fraction. Figure 18 includes two sample items for eliciting the PUFS. In the first sample item, a learner needs to name the fractional relationship between a part and an unpartitioned whole. In the second sample item, a learner needs to create a unit fraction of an unpartitioned whole.

Figure 17

Two sample items for eliciting the part whole scheme (Norton et al., 2018)
Figure 18

Two sample items for eliciting the partitive unit fraction scheme (Norton et al., 2018)

<table>
<thead>
<tr>
<th>What fraction is the smaller stick out of the longer stick?</th>
</tr>
</thead>
<tbody>
<tr>
<td>![Image of two sticks with different fractions]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Your piece of pie is 1/5 as big as the piece shown below. Draw your piece of pie.</th>
</tr>
</thead>
<tbody>
<tr>
<td>![Image of a circle with 1/5 shaded]</td>
</tr>
</tbody>
</table>

Figure 19 are samples items on the PFS. Like the first PUFS item, the first PFS item also asks a learner to name the fractional relationship of two quantities. Different from the first PFS item where iterations of the smaller part can fit the larger whole squarely, here one needs to partition both quantities with a self-chosen unit fraction, iterate the unit fraction inside each quantity to measure it, and then find out their relationship. In the second PFS item, one needs to create a proper fractional quantity (but not a unit fraction) out of an unpartitioned whole. A semicircle was used as the unpartitioned whole in the sample item. The sample item in Figure 20 is about the SO. Given that a known quantity is certain times as big as an unknown quantity, a learner is asked to find out the unknown quantity.
Figure 19

*Two sample items for eliciting the partitive fraction scheme (Norton et al., 2018)*

<table>
<thead>
<tr>
<th>Question</th>
<th>Image</th>
</tr>
</thead>
<tbody>
<tr>
<td>What fraction is the smaller stick out of the longer stick?</td>
<td><img src="image1.png" alt="Image" /></td>
</tr>
<tr>
<td>Your piece of pie is 3/5 as big as the piece shown below. Draw your piece of pie.</td>
<td><img src="image2.png" alt="Image" /></td>
</tr>
</tbody>
</table>

Figure 20

*One sample item for eliciting the splitting operation (Norton et al., 2018)*

<table>
<thead>
<tr>
<th>Question</th>
</tr>
</thead>
<tbody>
<tr>
<td>The stick shown below is 5 times as long as another stick. Draw the other stick.</td>
</tr>
</tbody>
</table>

Figure 21 includes two sample items for evaluating the IFS. Like the items on the PFS, these items ask one to name a fractional relationship between two quantities or to create a fractional quantity based on a given quantity. Different from the PFS items, the IFS items involve improper fractions. Figure 22 is an item about the RPFS. Given a quantity that is a fraction of an unknown quantity, one is asked to find out the unknown quantity. The RPFS item is the same as the SO item except that the multiplicative relationship between the two quantities is fractional.
If a participant exhibited weak knowledge of fractions in the interview or assessment, sometimes the numbers used in the assessment were revised on the spot to reduce difficulty. For instance, Item 12 in the pre-game assessment was about figuring out three-fifths of a half circle (Figure 19 bottom). Considering that Ada’s earlier performance already consistently showed difficulty in splitting a sector into fifths but not into fourths, the problem was changed into marking three-fourths. Such changes took into account each participant’s existing understanding.
and did not jeopardize an item’s integrity in eliciting and measuring a fractional scheme/operation.

During the assessment, the researcher provided hints and clarifications when noticing mistakes or struggles. Sometimes a mistake was quickly fixed, or a struggle easily overcome. For instance, Merik’s answers to Items 2-3 (Figure 23a) were slightly off. After being suggested to double-check his answers, he realized and corrected his mistakes. As another example, Alana’s initial answers to Items 17-18 (Figure 23b) would have been correct if the problems were about the fraction of the smaller amount out of the bigger amount. The researcher explained this and then re-read the problem to her using pauses and intonations to emphasize the relationship between the two quantities involved. With this assistance, Alana changed her answers. When mistakes or struggles remained after acquiring some hints or support, there was no additional help, and a participant was advised to move on. Throughout the assessments, the participants received no explicit feedback on their final answers’ correctness.

Figure 23

Examples of assisted problem-solving during assessments

Note. Merik first wrote down “1/8” for Item 2 and circled six squares for Item 3 (a). With the researchers’ suggestion to doublecheck, Merik changed the answers to “1/7” and had five squares circled (a). Alana initially thought the answers were “5/7” and “1/4” for Items 17 and 18.
respectively. The researcher pointed out that the problems were not asking for the fraction of the smaller part out of the bigger part. Then Alana changed her answers to “7/5” and “5/4” and she further wrote down the answers in the mixed numbers format.

During the formal data collection, six participants played SF1 and seven played SF2, but one SF2 player’s (Alana) gameplay data was missing; Kate played parts of both games for reasons clarified below. After the formal data collection, three SF1 players played SF2 to provide additional gameplay data. During the pre-game session, Kate answered and solved fraction problems very poorly in the interview, but her performance at the pre-game assessment did not suggest so. It was suspected that Kate knew fractions well but needed some refresher. Since her pre-game problem-solving of all arithmetic operations (e.g., addition, multiplication) of fractions was very poor, she was asked to play SF1.VI, which was about simple fraction addition, and SF2.II-III, which focused on fraction multiplication and common denominators. Alana played SF2 but the gameplay was not appropriately recorded by mistake. So, Alana’s gameplay data is missing from the analysis. After the post-game interviews/assessments (i.e., formal data collection), three participants who played SF1 had extra availabilities and would like to play SF2 on a different day. So, their gameplay of SF2 was collected. Considering that this part was supplemental, these three participants’ gameplay of SF2 is presented separately.

Due to limited time, the participants completed SF1/S2 to different extents. Each participant’s completion rate was calculated by dividing the number of completed levels by the total number of levels within a game. To ensure that there was sufficient gameplay data on the levels approaching the end of the games, some participants were instructed to skip certain levels/sections or to play a later section first once they were familiar with the basics. Table 2 displays the participants’ completion rates of SF1 levels. Six participants completed Sections I
through IV. Two participants completed Sections V and VI and one participant played selective levels in these two sections. As it was clarified earlier, Kate only played Section VI. Considering game progress or completion rate, on average these seven participants finished 74% of SF1. At least two participants played each SF1 level.

Table 2

The participants’ completion rates of SF1

<table>
<thead>
<tr>
<th>Participants</th>
<th>Completion rate in each section</th>
<th>Overall Completion rate</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>I</td>
<td>II</td>
</tr>
<tr>
<td>Nash</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>Ada</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>Harry</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>Zoe</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>Merik</td>
<td>100%</td>
<td>95%</td>
</tr>
<tr>
<td>Maya</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>Kate</td>
<td>n/a</td>
<td>n/a</td>
</tr>
<tr>
<td>Average</td>
<td>86%</td>
<td>85%</td>
</tr>
</tbody>
</table>

Note. If a participant was instructed to skip entire sections, the completion rate was labeled “n/a”.

When calculating average completion rates, “n/a” was treated as “0%”.

Table 3 displays the participants’ completion rates of SF2. Three participants completed Section I. Three participants exhibited solid knowledge of fractions in the pre-game interviews and assessments. So, they were instructed to play only selective levels in Section I to learn the basics of the game. Only one participant played all the levels in Section II. One participant completed Section III and another three completed all but a few levels. As it was explained already, Kate only played Section II-III of SF2. Overall, the participants went through an average of 74% of SF2, and three or more participants played each SF2 level. When collecting additional gameplay data from Nash, Harry, and Zoe, each one of them primarily played two SF2 sections to save time. Harry played Sections II-III but did not have time for Section I.
Table 3

The participants’ completion rates of SF2

<table>
<thead>
<tr>
<th>Participants</th>
<th>Completion rate in each section</th>
<th>Overall Completion rate</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>I</td>
<td>II</td>
</tr>
<tr>
<td>Nora</td>
<td>100%</td>
<td>66%</td>
</tr>
<tr>
<td>Nick</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>Tim</td>
<td>76%</td>
<td>66%</td>
</tr>
<tr>
<td>Hannah</td>
<td>100%</td>
<td>74%</td>
</tr>
<tr>
<td>Austin</td>
<td>57%</td>
<td>91%</td>
</tr>
<tr>
<td>Neo</td>
<td>55%</td>
<td>60%</td>
</tr>
<tr>
<td>Kate</td>
<td>n/a</td>
<td>86%</td>
</tr>
<tr>
<td>Average</td>
<td>70%</td>
<td>78%</td>
</tr>
<tr>
<td>Nash</td>
<td>90%</td>
<td>100%</td>
</tr>
<tr>
<td>Harry</td>
<td>n/a</td>
<td>74%</td>
</tr>
<tr>
<td>Zoe</td>
<td>75%</td>
<td>20%</td>
</tr>
</tbody>
</table>

*Note.* If a participant was instructed to skip entire sections, the completion rate was labeled “n/a”.

When calculating average completion rates, “n/a” was treated as “0%”. Nash, Harry, and Zoe’s gameplay of SF2 was supplemental, so their completion rates were presented separately and not included when calculating the averages.

5.5 Data Analysis

Assessments, interviews, and gameplay as different types of data were examined differently. Consistent with the last section, this section presents analysis of assessments first, followed by analysis of interviews and that of gameplay.

5.5.1 Data Analysis of Assessments

To analyze the assessments, Norton et al.’s (2018) procedures were adapted and applied to evaluate whether a scheme/operation was present. While participants in Norton et al.’s (2018) study solved all the problems independently, participants in this study sometimes received external help during the assessments, as clarified in 5.4. The assistance from the researcher enabled revelation and consideration of “the level of potential development” (Vygotsky, 1978, p.
86), which is determined by problem-solving in collaboration with a more knowledgeable figure, the researcher. Because of the differences in data collection, while the initial item scoring in Norton and colleagues’ (2018) study included two levels (whether the answer suggested the targeted scheme or not), there were three levels in this study (i.e., an independent answer that suggested the scheme’s presence, an assisted answer that indicated the targeted scheme, and an assisted answer that counter-indicated the scheme). In practice, based on a participant’s answer, the participant received 1 (valid and independent problem-solving), .5 (assisted successful problem-solving), or 0 (problematic problem-solving, regardless of assistance) on each item.

A simple sum was calculated for each participant’s overall performance during the pre-game and post-game assessments. The means and the standard deviations of all participants’ overall performances were calculated. To visually display the results, a boxplot of the participants’ overall scores by group (SF1, SF2, and both) and assessment (pre-game and post-game) was created. Since there were four items on each fraction scheme/operation, by taking the sum of the individual scores, each scheme/operation received a score between 0 and 4, which suggested the extent to which a scheme/operation was present (Norton et al., 2018). The means and the standard deviations for all participants’ performances on each scheme/operation were calculated. Using the participants’ overall scores and scores on each fraction scheme/operation, Welch’s two-sample t-tests or Wilcoxon rank sum tests, depending on whether the two groups’ variances were equal, were carried out to statistically evaluate the differences between SF1 vs. SF2 players in the pre-game and post-game assessments respectively; paired t-tests or Wilcoxon signed rank tests, based on whether the paired differences followed a normal distribution, were run to statistically examine the differences between pre-game and post-game assessments among
SF1 and SF2 players respectively. For statistically significant results, Cohen’s $d$ or Wilcoxon effect size $r$ were calculated as the effect sizes for the t-tests or the Wilcoxon tests.

5.5.2 Data Analysis of Interviews

The interviews were analyzed with thematic analysis (Braun & Clarke, 2006). At first, the interviews were manually transcribed semi-verbatim, excluding verbal pauses, filler words, and false starts that did not contribute to the interviewees’ meanings. Mathematical expressions were transcribed abstractly. Namely, “one-third times two” was recorded as “1/3 * 2”. Different fraction readings were differentiated. When a fraction was read as “x-yth”, the fraction was transcribed as “x/y”. Fractions read as “x over y” were transcribed as “x / y”. When “divided by” was mentioned, these exact words were transcribed except when a fraction was divided by an integer or fraction (“one-third divided by two” was recorded as “1/3 / 2”). After transcription, an inductive approach was employed to code five transcripts. Two transcripts were randomly selected from SF1 and SF2 players’ transcripts respectively and the transcript of Kate, who played both SF1 and SF2, was included. Initial codes were generated and refined from open coding as the transcripts were examined iteratively. After refining the initial codes, they were used to code all the transcripts. Similar or related codes were collated into potential themes. Lastly, the themes were reviewed and checked with the data. In the process, patterns among the participants and between the participants’ pre- and post-game responses were sought.

Additional measures were taken when analyzing interview data on fraction problem-solving. Concerning formal procedures, solving fraction additions and subtractions share the first step of converting the fractions to have the same denominator. Then, one needs to add or subtract the numerators and carry over the denominator to get the result. Since the participants were familiar with integer additions/subtractions and the key part of fraction addition/subtraction is
finding a common denominator, the participants’ problem-solving of fraction additions and subtractions was evaluated together. While the conventional formal procedure for fraction multiplications was “multiply straight across” (multiplying the numerators and the denominators respectively), the commonly taught universal rules for fraction divisions were “keep, change, flip” (keep the first fraction, change the sign from division to multiplication, flip the second fraction switching the numerator and the denominator) and then solving the converted equivalent fraction multiplications. The procedures for fraction multiplications and divisions are distinct and thus the participants’ problem-solving concerning these two operations were evaluated separately.

5.5.3 Data Analysis of Gameplay Data

Thirteen participants’ gameplay of educational puzzles (for details, see p.28) was analyzed since Alana’s gameplay data was missing. Each participant’s gameplay was examined in terms of the number of tries it took to solve each educational puzzle. Then the participants’ average number of tries on each puzzle was calculated. For each game, one graph was created to capture how the participants’ average number of tries changed throughout the game. Each participant’s overall problem-solving efficiency was calculated by dividing the total number of tries by the total number of solved puzzles. Also, for simplicity’s sake, each participant’s gameplay speed was roughly calculated by dividing the total time by the total number of completed levels (including educational puzzles and irrelevant levels).

To analyze the gameplay data involving fractions, educational puzzles requiring minimal knowledge of fractions were first identified. Specifically, Section I puzzles, II.3, and VI.2-3 in SF1 (18% of all SF1 levels), and I.1-29, II.3-6, II9, II.20, II.26, III.1-2 and III.13 in SF2 (34% of all SF2 levels) primarily taught game mechanics and new game elements. Thus, these were
excluded from consideration in the subsequent gameplay analysis. The remaining playable puzzles were analyzed. Gameplay of a playable puzzle in SF1 or SF2 was treated as a unit of analysis. Each episode was evaluated concerning whether the problem-solving was self-reliant and reflected an understanding of the fractions and the math operations involved. After examining each episode, a player’s overall self-reliant puzzle-solving rate in the game context was calculated as the percentage of educational puzzles solved by drawing on math understanding in relation to the total number of completed educational puzzles.

Considering that one might understand fractions but fail at solving a puzzle due to game features, verbal expressions showing an understanding of fractions were prioritized over actions when seeking evidence of one’s knowledge of fractions. Given that many participants casually played the games silently, their actions during the gameplay were the primary source to infer their (lack of) knowledge of fractions. If a puzzle was successfully solved on the first try, an understanding of fractions was considered to have been tapped and appropriately applied to the situation. For puzzles solved on the second try or beyond, each episode was examined to see whether the initial unsuccessful problem-solving was caused by math-unrelated issues, e.g., accidents such as mis-slicing or misplacing of ice/lava chunks, and new game features that triggered testing or trial-and-error behavior to help understand the novelties. If the answer was positive and if the mathematical feature of a puzzle was not affected, successful puzzle-solving on the subsequent try was still considered to suggest an understanding of the math involved. On the other hand, if the mathematical situation was altered during one’s first try, the eventual puzzle-solving was considered to have been influenced and supported by the game bound feedback and thus could not reflect a player’s knowledge of fractions.
For instance, Nash released a block on each side on his first try at solving SF1.IV.10 (Figure 24). Without releasing the bar holding the ice blocks and checking his solution, he restarted the level, only released one block on the right side, and then released the horizontal ice bar. Because Nash self-corrected with no feedback on his initial solution, it was considered that he understood the math involved, which was $\frac{1}{4} = \frac{1}{8} + \frac{1}{8}$ or $\frac{1}{8} \times 2$ in this case. In another example, SF1.III.9 was the first level where certain amounts of ice blocks needed to be arranged onto a horizontal ice bar first and then released to resolve lava pieces (Figure 25a). On Nash’s first try, after slicing the ice on the upper right once and leaving two one-halves hanging, he released the horizontal ice bar and saw the result (Figure 25b-c). From this, he realized the function of the ice bar, knowing that he needed to get the correct amounts of ice on the bar first before releasing it. And then he solved the puzzle on his second try (Figure 25d). Considering that Nash’s first try did not trigger game bound feedback that altered the mathematical setting of the puzzle, his problem-solving still suggested that he understood that half of one was one-half and that the sum of one-fourth and two one-eighths was one-half.

Figure 24

*SF1.IV.10 setup (a) and Nash’s first attempt (b)*
Note. Nash’s first attempt is shown in b-c. He solved this puzzle on his second try (d).

In addition, among educational puzzles solved with math understanding, gameplay episodes that revealed one’s knowledge of fractions and fraction operations were selected and qualitatively evaluated. The findings from gameplay were used to triangulate the interview findings on the participants’ improvements in fraction problem-solving. When examining the gameplay data, due to the absence of verbal explications, it was difficult, if not impossible, to know the exact interpretations of and the thinking process in solving a lot of puzzles. When multiple interpretations and ways of thinking were viable, different interpretations were appraised. If a participant solved a puzzle free of game-bound feedback or hints that interfered with the mathematical problem at hand, the most basic knowledge of fractions required for successful puzzle-solving was treated as the knowledge reflected by the participant’s gameplay.
For instance, in SF1.III.4 (Figure 26a), the smallest unit of the hanging lava is one-eighth, and combining different numbers of eighths results in different quantities. One could view the lava as eight one-eighths, four two-eighths (or one-fourths), and one-fourth (two one-eighths) together with three-fourths (six one-eighths), etc. Also, when solving this puzzle, one person might consider the ratio of the ice blocks (1:3) and dissect the lava accordingly. Another person might recognize that one-fourth of lava could resolve the one ice chunk on the left and that the leftover three-fourths of lava could resolve the three ice chunks on the right precisely. After cutting one-fourth off the lava, one person might subtract one-fourth from one to get the remaining amount ($? = 1 - 1/4$), while someone else might count three one-fourths in the remaining chunk ($? = 1/4 + 1/4 + 1/4$ or $1/4 * 3$). Therefore, without verbalization, it was unclear how a participant perceived the lava or solved the puzzle exactly. On the other hand, because the puzzle had to do with one-fourths or two-eighths, it was unlikely that a person started by considering the lava as three-eighths together with five-eighths (Figure 26b) or resolved the three ice chunks (each being one-fourth) by thinking that the total amount was equal to two three-eighths ($3 * 1/4 = 2 * 3/8$, Figure 26c), which were less straightforward interpretations and thus were less likely the case.

**Figure 26**

*SF1.III.4 setup (a) and two examples of less likely interpretations (b-c)*
Looking at the participants’ gameplay, Zoe solved SF1.III.4 on her first try while others used at least three tries. Zoe’s gameplay suggested her knowledge that one-fourth could be created by splitting one into four equal parts and taking one part and that three one-fourths were left after cutting one-fourth off one. Among the five participants who had multiple tries, Ada cut the hanging lava piece in different ways in her first few tries but did not release any lava. Consequently, she did not receive feedback that interfered with the math problem-solving (i.e., the consequence of merging some released lava with some ice and whether the dropped lava was bigger/smaller than the ice) and her puzzle-solving of SF1.III.4 was considered to have solely relied on her knowledge of fractions.

Addressing different research questions requires different parts of data. Table 4 presents the alignment between the research questions and the collected data. In the next chapter, data analysis results are presented in the same sequence of the research questions.

Table 4

The alignment between the research questions and the collected data

<table>
<thead>
<tr>
<th>RQ</th>
<th>Interview</th>
<th>Assessment</th>
<th>Gameplay</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Pre-game Interview</td>
<td>Post-game Interview</td>
<td></td>
</tr>
<tr>
<td>1. What are EOP students’ experiences and views of Slice Fractions and Slice Fractions 2?</td>
<td>1. What do you think of this game, as a game? As a math-learning tool?</td>
<td></td>
<td>Gameplay was examined and summarized, with all educational puzzles included in the analysis</td>
</tr>
<tr>
<td>2. Do SF1 and SF2 affect EOP students’ conceptions of fractions? If so, how?</td>
<td>1. What does the word “fraction” make you feel? 2. What do you know or remember about fractions?</td>
<td>2. About fractions, what do you think you have learned from the game if anything? 3. After playing the game, how do you</td>
<td>Assessment findings indicate schemes/operations before/after the gameplay. Changes suggest the impact of playing SF1/SF2</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4. In your opinion, why do we need to learn this concept?</td>
<td>feel when hearing “fractions”? on fraction schemes/operations.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. How does the gameplay affect EOP students’ practices and knowledge of fraction problem-solving?</td>
<td>How do you solve fraction operations, e.g., 1/3 * 2, 1/3 * 1/2, 1/3 + 1/3, 1/3 + 1/2, 2/3 – 1/2, 3/4 / 3? How would you explain it to someone who hasn’t learned fractions? Could you draw a representation of the problem-solving?</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4. Does the gameplay impact EOP students’ views of mathematics learning? If so, how?</td>
<td>3. What do you remember about learning math, especially fractions, at school? 5. c. Would you consider yourself a math person and or a gamer? Why? 4. Would you like to see more games like SF to be used in math learning? 5. After playing SF, how would you identify or perceive yourself? Any change?</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Gameplay episodes were selectively qualitatively examined.
6 Results

6.1 Experiences and Views of the Games

This section will present the participants’ prior experience with video games and serious video games first, followed by their gameplay of Slice Fraction (SF1) and Slice Fraction2 (SF2) and their experiences and perspectives of SF1 and SF2.

6.1.1 Prior Experience with Video Games

All six SF1 players and four SF2 players were very familiar with video games. Among them, four SF1 players (Nash, Ada, Merik, and Maya) and two SF2 players (Nick and Tim) were avid gamers who frequently played multiple video games at least a few times per week; the others (including two SF1 players, Harry and Zoe, and two SF2 players, Hannah and Neo) played video games regularly until college. Three SF2 players (Nora, Austin, and Alana) and Kate were non-gamers, relatively inexperienced in video games, and only played casual mobile games sometimes. When describing the participants’ prior experiences with video games, no differentiation is made between SF1 and SF2 players, or between currently active players and previously active players, because their answers were highly similar. The distinction is only made between the ten participants familiar with video games and the four participants unfamiliar with video games except for casual mobile games.

Considering the answers of the ten participants who (used to) regularly played video games, commonly mentioned game genres and games were team-based first-person or third-person shooting (e.g., Fortnite, Overwatch, and Call of Duty) or fighting (e.g., BedWars), simulation and sports (e.g., NBA 2K, The Sims, and Rocket League), sandbox (e.g., Minecraft and Terraria), action-adventure (e.g., Hollow Knight and Grand Theft Auto), multiplayer online battle arena (e.g., League of Legend), and action-fighting (e.g., MultiVersus and Mortal
Kombat). These games were played on personal computers or game consoles. Although none of these participants dedicated themselves to games like professional gamers, they all expressed enjoyment in playing video games. In addition, two participants mentioned that it helped them gain confidence. Two participants expressed that video games brought social opportunities, as they played team-based games or participated in game-specific communities. Four participants (Nora, Austin, Alana, and Kate) did not play or enjoy video games much. Nora sometimes played The Sims and Alana played NBA 2K, but they did not enjoy them to the extent of regularly playing them. These participants only played casual puzzle games or strategy games on their phones (e.g., Block Puzzle, Toy Blast, and Candy Crush) to kill time.

Regarding math educational games, three SF1 players (Nash, Zoe, and Merik) and one SF2 player (Nora) who were taking EOP pre-calculus with the same teacher mentioned King of Math. This mobile game was used in their EOP pre-calculus class as an auxiliary tool to help students develop arithmetic proficiency. King of Math included plenty of practice problems on various K-12 math topics, such as addition, subtraction, mixed operations, fractions, powers, geometry, etc. The practice problems were the same as the multiple-choice problems commonly seen in math worksheets (Figure 27). Speed and correctness are rewarded in the game by acquiring higher scores. Kate mentioned ST Math and Khan Academy. Three participants described games where the game mechanics and the math content were separate. For example, Nick once played a game where he was given some arithmetic problems and needed to shoot the asteroids labeled with the correct answers. To conclude, most participants had some experience with educational games where game mechanics were added to abstract arithmetic practices to enhance playfulness. Their prior experience with well-designed serious video games that integrated operations of quantities was limited or not striking enough for them to recall easily.
Figure 27

*Sample problems in King of Math*

![Sample problems in King of Math](image)

*Note.* The score on the top right decreases as a player takes time to select an answer. Practices of fractions came after “addition”, “subtraction”, “mixed 1”, “multiplication”, “division”, “arithmetic”, and “geometry”. Only King of Math was used. King of Math 2, which had better graphics and more variety of problems, was not used in the class at the time.

6.1.2 Gameplay of SF1 and SF2

As it was described earlier, both SF1 and SF2 integrated game design principles and instructions on fractions, but SF2 seemed harder due to more challenging game elements and math content. Since the two games differed in their structures and content, the participants’ gameplay of these two games is presented separately. Three SF1 players also played SF2 after the formal data collection, comparison results between these three players’ SF1 gameplay versus their SF2 gameplay and others’ SF2 gameplay are described at the end.

Seven participants, including Kate, played Slice Fractions 1. Figure 28 shows the average number of tries it took the participants to complete each educational puzzle in SF1. In general, the difficulty level of Section I puzzles was stable since each puzzle was solved with an average of three or fewer tries. In Sections II-VI, the difficulty level was less stable, as the average
number of tries used to solve the puzzles fluctuated between 1 and 5. In the graph, the spikes suggested that the corresponding puzzles (e.g., II.2, III.7, III.17, IV.2, etc.) were relatively more challenging, probably because the game mechanics and subject matter got more difficult. Since a sharp drop in the average number of tries was usually observed right after a spike, this empirical finding supported the claim that the puzzles were interconnected and that a player could immediately apply what one learned from an earlier puzzle to the subsequent one. To summarize, 54% of educational puzzles were solved with an average of 2 or fewer tries, 30% of puzzles with 2-3 tries, and the remaining 16% of puzzles with more than 3 tries. Table 5 displays a summary of each participant’s gameplay of SF1. Similar to what Figure 28 showed, the average try/puzzle by section in Table 5 indicated that Sections II, III, IV, and VI were relatively more difficult, while Sections I and V were easier. The mean try/puzzle across all puzzles was 2.31 (SD = .35), and it took an average of 40.62 seconds (SD = 6.88) to finish each level. Considering each participant’s data, Zoe’s gameplay was remarkable. On average, it took her the least number of tries (1.64) to solve the puzzles in each SF1 section, almost two standard deviations lower than the mean.
Figure 28

The participants’ average number of tries to solve educational puzzles in SF1

![Graph](Image)

Table 5

The participants’ gameplay of Slice Fractions (SF1)

<table>
<thead>
<tr>
<th>Participants</th>
<th>Try/puzzle in each section</th>
<th>Total tries/total puzzles</th>
<th>Second/level</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>I</td>
<td>II</td>
<td>III</td>
</tr>
<tr>
<td>Nash</td>
<td>2.13</td>
<td>2.75</td>
<td>2.71</td>
</tr>
<tr>
<td>Ada</td>
<td>1.93</td>
<td>2.92</td>
<td>3.00</td>
</tr>
<tr>
<td>Harry</td>
<td>1.60</td>
<td>3.08</td>
<td>2.59</td>
</tr>
<tr>
<td>Zoe</td>
<td>1.20</td>
<td>1.67</td>
<td>1.65</td>
</tr>
<tr>
<td>Merik</td>
<td>1.73</td>
<td>2.00</td>
<td>2.94</td>
</tr>
<tr>
<td>Maya</td>
<td>1.73</td>
<td>3.17</td>
<td>1.71</td>
</tr>
<tr>
<td>Kate</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>Mean</td>
<td>1.72</td>
<td>2.60</td>
<td>2.43</td>
</tr>
<tr>
<td>(sd)</td>
<td>(.32)</td>
<td>(.62)</td>
<td>(.60)</td>
</tr>
</tbody>
</table>

The remaining six participants and Kate played SF2 as part of data collection. Based on these seven participants’ data, Figure 29 presents the average number of tries it took them to pass SF2 educational puzzles. The first several puzzles in Sections I and II were relatively easy and took at most two tries to solve. In other parts of SF2, the difficulty level changed constantly. Like
Figure 28, in Figure 29, after spikes where the challenging puzzles were located, drops took place and suggested that the participants adopted and applied what they learned from the earlier puzzles to the subsequent ones. Of all the SF2 educational puzzles, 57% were solved with an average of 2 or fewer tries, 28% with 2-3 tries, and 15% with more than 3 tries. Table 6 presents these participants’ gameplay of SF2. The participants’ average try/puzzle across the three sections was similar. It took them 2.10 tries (SD = .38) and 49.30 seconds (SD = 9.11) on average to solve an SF2 educational puzzle. Nora, Tim, and Neo played SF2 relatively better and spent fewer tries to solve the educational puzzles in SF2.

**Figure 29**

The participants’ average number of tries to solve educational puzzles in SF2

![Graph showing average number of tries across sections](image)

**Table 6**

The participants’ gameplay of Slice Fractions 2

<table>
<thead>
<tr>
<th>Participants</th>
<th>Try/puzzle in each section</th>
<th>Total tries/total puzzles</th>
<th>Second/level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nora</td>
<td>1.81</td>
<td>1.78</td>
<td>62.25</td>
</tr>
<tr>
<td>Nick</td>
<td>1.70</td>
<td>2.08</td>
<td>55.84</td>
</tr>
<tr>
<td>Tim</td>
<td>1.63</td>
<td>1.64</td>
<td>35.87</td>
</tr>
<tr>
<td>Hannah</td>
<td>2.51</td>
<td>2.35</td>
<td>45.49</td>
</tr>
<tr>
<td></td>
<td>Austin</td>
<td>Neo</td>
<td>Kate</td>
</tr>
<tr>
<td>----------</td>
<td>--------</td>
<td>------</td>
<td>-------</td>
</tr>
<tr>
<td></td>
<td>3.04</td>
<td>1.85</td>
<td>N/A</td>
</tr>
<tr>
<td>2.21</td>
<td>1.80</td>
<td>2.91</td>
<td>2.07 (.48)</td>
</tr>
<tr>
<td>2.67</td>
<td>1.78</td>
<td>1.96</td>
<td>2.14 (.44)</td>
</tr>
<tr>
<td>2.64</td>
<td>1.81</td>
<td>2.41</td>
<td>2.10 (.38)</td>
</tr>
<tr>
<td>46.81</td>
<td>42.86</td>
<td>56.00</td>
<td>49.30 (9.11)</td>
</tr>
</tbody>
</table>

**Additional gameplay of SF2**

<table>
<thead>
<tr>
<th></th>
<th>Nash</th>
<th>Harry</th>
<th>Zoe</th>
<th>Mean (sd)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.94</td>
<td>N/A</td>
<td>1.29</td>
<td>1.62 (.46)</td>
</tr>
<tr>
<td></td>
<td>2.64</td>
<td>2.28</td>
<td>1.00</td>
<td>1.97 (.86)</td>
</tr>
<tr>
<td></td>
<td>2.00</td>
<td>2.38</td>
<td>2.00</td>
<td>2.13 (.22)</td>
</tr>
<tr>
<td></td>
<td>2.21</td>
<td>2.33</td>
<td>1.54</td>
<td>2.03 (.43)</td>
</tr>
<tr>
<td></td>
<td>34.74</td>
<td>38.03</td>
<td>34.81</td>
<td>35.86 (1.88)</td>
</tr>
</tbody>
</table>

**Note.** Nash, Harry, and Zoe played SF2 as extra data collection, so their data was presented separately and their means and standard deviations were separately calculated.

Three SF1 players tried out SF2 after their post-game interviews/assessments. Considering their gameplay of SF1 and SF2, on average they used 2.26 tries (SD = .55) to solve an SF1 puzzle, versus 2.03 tries (SD = .43) for an SF2 puzzle. Comparing the SF2 gameplay of these three participants and the previously mentioned seven participants, Nash, Harry, and Zoe’s mean try/puzzle in Section I (M = 1.62, SD = .46) and II (M = 1.97, SD = .86) were lower than the other seven participants’ mean try/puzzle in Section I (M = 2.09, SD = .56) and II (M = 2.07, SD = .48). But these two groups’ try/puzzle in Section III were similar (M = 2.13, SD = .22 for the three participants; M = 2.14, SD = .44 for the seven participants). Overall, the three players’ mean try/puzzle in SF2 was 2.03 (SD = .43), only slightly lower than the seven participants’ (M = 2.10; SD = .38). Additionally, the three SF1 players spent 35.86s on average (SD = 1.88) to finish a SF2 level. This speed was similar to their speed when playing SF1 (M = 35.11, SD = 1.49) and faster than the seven SF2 players (M = 49.30, SD = 9.11). Considering each individual’s SF2 gameplay, Zoe’s game performance stood out as it took her the least number of tries to solve Section I-II puzzles.

As discussed earlier in 5.5, a participant’s self-reliant puzzle-solving during the gameplay was evaluated as the extent to which one’s solving of fraction-heavy educational puzzles implied
active tapping of knowledge of fractions. Tables 7 and 8 present the participants’ self-reliant puzzle-solving rates in SF1 and SF2 respectively. A higher percentage suggested that a participant more frequently drew on one’s understanding of fractions when solving the puzzles, while a lower percentage suggested that trial-and-error and visual feedback within the game supported puzzle-solving to a greater degree. Among SF1 players, two participants solved more than 60% of fraction puzzles on their own; three participants drew on their math knowledge to solve around half of the puzzles; and the remaining two participants did so to solve around 40% of the puzzles. On average 53% of the fraction puzzles in SF1 were solved by the participants drawing on one’s math understanding. Based on the results, puzzles in SF1.III, compared to puzzles in other sections, were more often solved by the participants by themselves (59%).

Generally, SF2 players (M = 67%) tapped their math knowledge more frequently than SF1 players (M = 53%). Specifically, Tim, Hannah, and Neo worked out more than 70% of SF2 puzzles by utilizing their understanding of fractions; Nora, Austin, and Kate solved 60% or more puzzles by themselves; Nick solved half of SF2 fraction puzzles with his math knowledge. In SF2, higher percentages of puzzles in Sections I and III were independently solved by the players, as the average self-reliant puzzle-solving rates were 75% and 72% respectively.

Considering the supplemental SF2 gameplay from the three SF1 players, their average self-reliant puzzle-solving rate in SF2 was similar to the rate of the seven SF2 players. Each of the three SF2 players’ self-reliant puzzle-solving rate in SF2 was much higher than that in SF1, and this might be caused by the differences between the two games.
### Table 7

**The participants’ self-reliant puzzle-solving rates in Slice Fractions (SF1)**

<table>
<thead>
<tr>
<th>Participants</th>
<th>% of puzzles solved with math understanding in each section</th>
<th>Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>II</td>
<td>III</td>
</tr>
<tr>
<td>Nash</td>
<td>48%</td>
<td>53%</td>
</tr>
<tr>
<td>Ada</td>
<td>36%</td>
<td>41%</td>
</tr>
<tr>
<td>Harry</td>
<td>41%</td>
<td>47%</td>
</tr>
<tr>
<td>Zoe</td>
<td>57%</td>
<td>88%</td>
</tr>
<tr>
<td>Merik</td>
<td>60%</td>
<td>41%</td>
</tr>
<tr>
<td>Maya</td>
<td>32%</td>
<td>82%</td>
</tr>
<tr>
<td>Kate</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>Mean</td>
<td>46%</td>
<td>59%</td>
</tr>
</tbody>
</table>

*Note. Cells without data (“N/A”) were excluded when calculating the means.*

### Table 8

**The participants’ self-reliant puzzle-solving rates in Slice Fractions 2 (SF2)**

<table>
<thead>
<tr>
<th>Participants</th>
<th>% of puzzles solved with math understanding</th>
<th>Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>I</td>
<td>II</td>
</tr>
<tr>
<td>Nora</td>
<td>75%</td>
<td>55%</td>
</tr>
<tr>
<td>Nick</td>
<td>81%</td>
<td>33%</td>
</tr>
<tr>
<td>Tim</td>
<td>75%</td>
<td>73%</td>
</tr>
<tr>
<td>Hannah</td>
<td>69%</td>
<td>73%</td>
</tr>
<tr>
<td>Austin</td>
<td>64%</td>
<td>71%</td>
</tr>
<tr>
<td>Neo</td>
<td>83%</td>
<td>67%</td>
</tr>
<tr>
<td>Kate</td>
<td>N/A</td>
<td>41%</td>
</tr>
<tr>
<td>Mean</td>
<td>75%</td>
<td>58%</td>
</tr>
</tbody>
</table>

| Nash | 69% | 44% | 80% | 64% |
| Harry| N/A | 55% | 64% | 59% |
| Zoe  | 100%| N/A | 67% | 83% |

| Mean | 84% | 49% | 70% | 69% |

*Note. Nash, Harry, and Zoe played SF2 as extra data collection, so their data was presented separately. Cells without data (“N/A”) were excluded when calculating the means.*

Having presented each participant’s and group’s (SF1 and SF2) gameplay, Table 9 summarizes the participants’ gameplay (only using data from the formal data collection) while
taking into account their prior video game experience. Comparing experienced and inexperienced players’ gameplay of SF1, their try/puzzle and self-reliant puzzle-solving rates were similar but experienced gamers were faster in going through each level. Among those who played SF2, the participants experienced in video games solved the puzzles with fewer tries, played the levels faster and drew on slightly more math understanding than those inexperienced. Considering that the number of participants in each group was small, no statistical tests were carried out.

Table 9

<table>
<thead>
<tr>
<th>Group (N)</th>
<th>Mean try/puzzle (SD)</th>
<th>Mean second/level (SD)</th>
<th>Mean self-reliant puzzle-solving rate (SD)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exp gamers_SF1 (6)</td>
<td>2.32 (.39)</td>
<td>38.24 (3.62)</td>
<td>53% (11%)</td>
</tr>
<tr>
<td>Non-gamers_SF1 (1)</td>
<td>2.26 (N/A)</td>
<td>54.19 (N/A)</td>
<td>53% (N/A)</td>
</tr>
<tr>
<td>Exp gamers_SF2 (4)</td>
<td>1.97 (.31)</td>
<td>45.02 (8.28)</td>
<td>69% (10%)</td>
</tr>
<tr>
<td>Non-gamers_SF2 (3)</td>
<td>2.28 (.44)</td>
<td>55.02 (7.76)</td>
<td>66% (4%)</td>
</tr>
</tbody>
</table>

Note. Kate double-counted as a SF1 non-gamer and SF2 non-gamer. Since there was only one non-gamer who played SF1, the standard deviations were not available. Data of Alana, a non-gamer who played SF2, was missing from this table since her gameplay data was not available.

6.1.3 Experiences of and Feedback on SF1 and SF2

Based on the interviews, the participants' perceptions of SF1 and SF2 as games and as learning tools were highly alike. Distinctions were only observed when the participants discussed SF1 and SF2 as learning tools. The experienced gamers and the non-gamers shared similar views and feedback. Since the participants’ responses shared more similarity than differences, in this section the fourteen participants’ reflections on these two games are presented together except
when differences in their responses were observed. The participants’ perspectives on SF1 and SF2 as games are presented first, followed by their thoughts on these two games as learning tools. Both the positives and the negatives or concerns are described.

Overall, the participants' experiences of SF1 and SF2 as games were positive. The participants usually characterized the games as being “fun” and “engaging”. Considering that their gameplay experiences were highly similar, a summarized story capturing most participants’ shared experiences will be presented. At first, a player might feel confused, for it took time and exploration to figure out what game elements were manipulable (e.g., bubbles) and the purpose of the game (i.e., clearing up paths to allow a mammoth to move forward). As one got familiar with the game, comfort level and confidence level increased. The puzzles in the game were interconnected, for what one learned at an earlier level was immediately practiced in subsequent levels. Across different puzzles, the difficulty level gradually increased, and thus one’s skills were slowly built up. Sometimes new game elements (e.g., clouds or addition blocks in SF1, ghosts or narwhals in SF2) or game mechanics (e.g., addition blocks selectively merge with some ice blocks; after a parent ghost swallows a block, each of its children ghosts spit out one block) were introduced. Occasionally the puzzles felt challenging. Hints were helpful and kept the challenges manageable. So was the design that one could re-do a puzzle as many times as one wanted with no penalty or time limit. To most participants, the increasing but manageable difficulties kept the game fun. As it was discussed in an earlier chapter, planning and strategies were often required when solving the puzzles. Neo commented, “It was interesting to try to figure out exactly which fractions equate to which, and some of the alternative ways to solve the problem. One time I solved the problem in a way that was not intended.”
The games reminded the participants of other mobile games they played before, e.g., Fun Run, Cut the Rope, or Where’s My Water. The graphics and sound effects were enjoyable and bring a childhood sense to several players. On the other hand, Neo was concerned that the graphics might be too childish for college learners. Most participants felt that the games were fun and engaging overall and challenging occasionally. According to Austin, who was a non-gamer and only played time-killer mobile games like Candy Crush, SF2 was not relaxing enough to be a playful game for him, but he preferred to learn a math topic in this way. Concerning criticisms, two SF1 players and three SF2 players, all experienced with video games, thought the games were not challenging enough. They suggested adding penalties, timers, or leaderboard, monsters, etc. to enhance players’ motivation to figure out and work through the puzzles faster or more efficiently. Two SF1 players thought the game was too slow-paced, as the animations were slow.

The participants’ feedback about SF1 and SF2 as math learning tools was positive as well. The most helpful characteristic was the visualization of various fractions and processes. SF1 players, participants who had relatively weak knowledge of fractions, felt that playing SF1 improved their understanding of fractions and confidence in dealing with fractions. SF2 players, participants who knew fractions relatively well, thought that SF2 gave a different perspective of math and that playing it would benefit visual learners. Three participants thought that playing SF1 or SF2 refreshed their memory about fractions. Kate mentioned several things she learned from playing SF1 and SF2. For example, different fractions might add up to the same fraction; a total sum could be cut into combinations of different fractions; certain fractions could be cut into smaller numbers. All participants felt that the games would be friendly to learners new to fractions, since they introduced fractions gradually. Maya thought the introduction was so gradual and subtle that a player might not recognize the use of fractions immediately. To Nash
and Merik, SF1 showed how fractions were created by splitting up one whole or relatively larger fractions. In Hannah’s opinion, SF2 would enable a beginner to learn fractions more easily, for a player could see different ways to break up a quantity and how different resulting fractions are created.

Another commonly praised feature of SF1 and SF2 as math learning tools was their interactivity. As hands-on games, they did not directly teach a learner what fractions were or how to operate them. The participants appreciated the opportunities to explore and figure things out on their own. In Ada’s words, “It doesn’t just throw work at you. It slowly brings you into fractions.” Because of the interactivity, the games allowed a player to solve a problem bit by bit and not in one shot. Therefore, one’s gameplay experience was close to problem-solving in real life. Zoe felt that this made the content knowledge more memorable and allowed her to more readily apply what she learned in everyday life. Since successful problem-solving was necessary to move forward, the interactivity also allowed a player to receive instant feedback during problem-solving. Such instant feedback was helpful and drove Kate to make sense of things and to keep going. She said, “In the classroom, I would just wait for the teacher to come and to correct me. With this game, you’re gonna have to figure it out on your own.” As fraction was seamlessly integrated into these interactive games, five participants thought that learners who were not motivated by lecture-based learning might be more interested and motivated in learning fractions by playing SF1 and SF2. Neo’s expression captured how well content knowledge and playfulness were merged in these games,

It's more like a problem that needs to be solved…It's different from what we experience and what we call math. It's less (about) procedure and more like a game. It's more
engaging. It's still math problem-solving but it's problem-solving on something that doesn't feel like it's math.

While most feedback about the games as math learning tools was positive, there were some dislikes or concerns at the same time. Two SF2 players explicitly mentioned they preferred to learn fractions in the traditional symbolic and procedural way. As shapes and symbols were presented together, Alana felt challenged to coordinate the two types of information. Four participants, three SF1 and one SF2 players, felt that learners might focus on making progress and finishing the games but not on understanding or retaining the content knowledge involved. Because the games did not contain explicit formal teaching, one SF1 player and two SF2 players were concerned that learners might not be able to explicate the content knowledge in the games and make connections and that what one learned in the games would not help conventional paper-and-pencil tests. After all, during conventional tests, no narwhal would be there to help preview different ways of splitting something. Two participants (one SF1 player and one SF2 player) felt that the math involved could be harder. It was reasonable that Nora thought so, as her knowledge of integers, fractions, and math operations was relatively good during the pre-game assessment/interviews. In contrast, it was surprising that Merik thought so, for his performance in the pre-game session suggested a fragile mastery of fractions. Additional criticisms or concerns about using SF1 and SF2 as math learning tools are the following. Nash mentioned the lack of opportunities to socially interact with others in these games. Zoe was worried about accessibility because not all learners had a mobile device or a tablet. Tim had concerns about learners having too much screen time and about learners being easily distracted by other apps on the same device. According to Neo, how the puzzles could be solved was a little bit restrictive, due to the nature of the subject, mathematics.
6.2 Recollections, Attitudes, and Conceptions of Fractions

This section concerns the impact of playing SF1 and SF2 on the participants’ attitudes and conceptions of fractions. At the beginning, the participants’ recollections about how they learned fractions at schools are presented. Following this, their feelings or attitudes towards fractions before and after the gameplay are summarized. Then the participants’ fraction schemes/operations as reflected in the pre-game and post-game assessments are displayed. Lastly, fractional conceptions explicitly suggested in the participants’ interviews and implied in their assessment performances are described.

6.2.1 Recollections about Fractions Learning

Since fractions were taught in elementary and middle schools, the participants’ memory of how fractions were taught in school was not fresh. Nonetheless, their limited recollections shed light on what it was like. Regardless of the participants’ pre-game performance and knowledge of fractions, their recollections were highly similar. Hence, the responses of SF1 and SF2 participants are discussed together. When individual fractions (e.g., two-thirds, five-fourths) were first introduced, diagrams of pies or other everyday objects were used. After the initial introduction, pictures or diagrams were no longer involved. Arithmetic operations of fractions were taught solely in the abstract way as a matter of following rules or procedures. There was no explanation of the procedures, which were only meant to be memorized and applied. Neo (SF2 player) said, “I just remember that these are the rules that we have to follow, or else we would do the steps incorrectly.” Maya (SF1 player) said, “No matter what type of math (courses) you take, there are just so many steps and a lot of rules you have to memorize.”

None of the participants recalled learning the meanings of the rules or why they work. After clarifying the steps, teachers would show a few examples and then students independently
practiced the procedures by working on problem worksheets. Once math learning was about manipulating numbers and following rules, the learners needed to make sense of things on their own, if they ever tried to understand them. Furthermore, the learning environment was not friendly to those who did not instantly “get it”. Kate (who played both games) mentioned that it felt embarrassing to admit not knowing or understanding. Consequently, many participants who fell behind at some point never had the opportunity to catch up. From the pre-freshmen summer program, Neo (SF2 player) observed that fractions were “one of what the class struggled with the most”. As Maya (SF1 player) reflected on her math learning experience, she realized the connection between her not knowing how to put addition or other operation signs into drawings during the interviews and her never practicing arithmetic operations of fractions in visual ways during formal learning.

When describing past experiences with fractions, only Nora’s and Hannah’s (both SF2 players) answers featured manipulatives like stackable cube blocks and diagrams. Nora also mentioned being instructed to draw pictures. Alana (SF2 player), who transferred to the US in the 7th Grade and never learned fractions properly at school, clearly remembered that one EOP instructor taught fractions by manipulating several pens during the pre-freshmen summer program. It was eye-opening and impressive to Alana when she learned that fractions could represent multiplicative relationships (e.g., one-fourth could represent one pen out of four pens) and that “the whole (in “part of a whole”) could be a group and not just one single item”.

Zoe and Ada, both females and SF1 players, remembered the salient negative feeling of being forced to work on math problems and not “really learning” or understanding math during their K-12 education. Zoe felt that in school fractions were not taught well for her to understand and to be confident enough to solve the problems. The learning experience was not memorable
for her. She only remembered teaching herself the procedures using worked-out examples. Zoe said, “It wasn’t necessarily anything that we learned. It was more like ‘Here is the problem. Do it.’” Ada’s description reflected similar sentiments:

I feel like the work was just thrown at me in the way where I wasn't really learning.

Because I'm not the type of learner. Like, if you throw textbooks and read at me, I'm not gonna learn that way. And the teachers I had never really sat down to actually ask me what is hard and what I understand. We just moved on and I just failed.

Not all participants had issues with learning operations with fractions primarily through rules or procedures. None of the four male participants who could not explain faction problem-solving well (Nash, Harry, Merik, and Nick; to be presented later) expressed frustrations about not understanding the rules/procedures during their formal math education. As will be presented in later sections, Tim, Austin, and Neo solved fraction operations well by following formal procedures and could make sense of most operations even before the gameplay. Austin and Neo explicitly considered following standard algorithms to be less cumbersome than visually representing and explaining the problem-solving. During the interview, at first, Austin said “You can’t really do it (arithmetic operation of fractions) visually.” Later he revised the statement, “You can do it visually, but there are more steps.”

6.2.2 Feelings/Attitudes Towards Fractions

Before the gameplay, based on their self-report, only two SF1 players felt comfortable and confident when dealing with fractions, while six SF2 players felt so. Among them, Alana was a peculiar case, as her comfort level and confidence with fractions developed mostly during the EOP pre-freshmen summer program. In her case, she did not learn about fractions before immigrating to the continental U.S. Nor did she receive extra assistance on math, including
fractions, at her new school. So, Alana never learned fractions properly during her K-12 education. It was during the EOP pre-freshmen summer program that she gained an understanding of fractions and how to do certain calculations. Before the summer program, she was confused about fractions. At the time of data collection, she felt comfortable working with fractions.

Six participants, including four SF1 players, one SF2 player, and Kate, were uncomfortable and unconfident with fractions. For example, Ada said she would get stressed and not know what to do when seeing fractions. Harry often got confused or felt weird about fractions. In his words, "It's terrible. Fractions are hard." However, he would feel more comfortable when the problems were proceeded by a worked-out example or clear instructions on what to do step-by-step. Kate had an avoidant attitude toward fractions. She admitted not knowing how to convert a mixed number to a fraction or to convert a fraction to a decimal. Even though she felt fine with math in general and even felt positive about the pre-calculus I she was taking at the time of this study, she still disliked fractions and got nervous when fractions were involved in pre-calculus problems. To Kate, this was because she was left behind when fractions were first taught at school, and she never caught up. She said, “Ever since I first learned fractions, they didn't click for me. So then when I went to my next grade and the next grade after that, I went (through) my whole education system not knowing fractions.”

Besides the self-reports, two SF1 players’ stress/anxiety was directly observable. During the pre-game session, Merik was so anxious that his hand shook involuntarily, and the assessment/interview had to be paused for some time. This was surprising since Merik was one of the two SF1 players who claimed to be comfortable with fractions. Stressed about not being able to verbally explain fractions well, Zoe shed tears during the pre-game assessment/interview.
Although Zoe did not feel comfortable or confident with fractions, she performed remarkably well when playing SF1 and SF2 (see 6.1.2), and her performances and responses in the post-game assessment/interview were decent (to be presented in 6.2.2 and 6.2.3).

After playing SF1 or SF2 for one hour, all but one SF1 player, three SF2 players, and Kate felt slightly more comfortable or confident about fractions. For example, in the pre-game interview, Zoe felt "not that comfortable or confident with fractions". After the gameplay, her answer became "It is actually not that hard." Two SF1 players and one SF2 player explicitly attributed the slight increase in comfort/confidence level to playing SF1/SF2, claiming that the gameplay helped refresh their knowledge of fractions. Kate mentioned that the researcher’s explanations after her post-game problem-solving helped her become more comfortable and confident with fractions. The remaining participants did not discuss what contributed to their increased positivity towards fractions.

One SF1 player and four SF2 players reported no change in their feelings or attitudes toward fractions after the gameplay. Nora considered the gameplay to be too short to make a substantial change in how she felt toward fractions. Harry still considered that he needed a worked-out example to feel more comfortable or confident. Austin was comfortable with solving fractional problems using algorithms but not with self-made drawings, even after the gameplay. Neo, who was already very comfortable with fractions before playing SF2, reported that his confidence levels in explaining or teaching fractions to others increased, for the game provided him with examples of how to visually represent fractional quantities and processes. To Alana, playing SF2 would have helped her better understand fractions if the study took place before the EOP pre-freshmen summer program, where her understanding of fractions developed from non-existent to indicators of multiplicative relationships (to be specified in 6.2.3).
6.2.3 Fraction Schemes/Operations

Both between-group (e.g., SF1 vs. SF2) and within-groups (e.g., SF1 pre-game vs. post-game) comparisons were made. In this section, between-group comparison results are presented first, followed by within-group comparison results. Figure 30 displays the boxplots of the participants’ overall performances on the pre- and post-game assessments of fraction schemes/operations, each dot representing a data point. Whether SF1 vs. SF2 players performed differently in the two assessments was examined. A Welch two-sample t-test revealed that the overall performances of SF2 players (M = 21.43, SD = 2.79) were significantly higher than those of SF1 players (M = 12.75, SD = 5.20) during the pre-game assessments \[t(7.39) = 3.66, p < .01; \text{d} = 2.08\]. Also, the SF2 players’ overall performances (M = 22.00, SD = 2.69) were still significantly higher than the SF1 players’ (M = 15.25, SD = 5.09) during the post-game assessments \[t(7.34) = 2.92, p < .05; \text{d} = 1.66\]. In both assessments, Kate’s overall performances (21.5 pre-game and 23.00 post-game) were similar to SF2 players’. These findings showed that, as expected, SF1 players’ knowledge of fractions was weaker than SF2 players’, for SF1 players had constructed fewer fraction schemes/operations than SF2 players either before or after the gameplay.

Within each group, whether there were differences between a group’s post- and pre-game assessment performances were investigated. Kate had a 7% increase in her overall assessment performance after playing SF1 and SF2. SF1 players on average had a 19.6% increase in their overall performances after playing SF1, but the difference between SF1 players’ post- and pre-game overall performances was not statistically significant \(p > .5\). SF2 players had a mean of 2.66% increase in their overall scores. Because the paired differences of SF2 players’ post- and pre-game overall performances were not normally distributed, a Wilcoxon signed-rank test was
carried out and a marginally significant difference was detected \([V = 24.5, p = .07; r = .72]\). To conclude, within each group, the participants’ performances on the assessments of fraction schemes/operations improved after playing SF1 and/or SF2 for one hour, but the improvement was marginal or not statistically significant, likely due to the small sample size (\(N =< 7\)) and the large variance within the data.

**Figure 30**

*The participants' overall performances on the assessment of fraction schemes/operations*

A breakdown of the participants’ scores by fraction scheme/operation is presented in Table 10. It shows that most participants had constructed the most basic part-whole scheme (PWS) before the study, as their average scores were high \((M_{SF1} = 3.58, M_{SF2} = 4; \text{Kate scored 4})\) and the standard deviations were minimal \((SD_{SF1} = .38)\) or non-existent \((SD_{SF2} = 0)\). On the more complex schemes/operations except the reversible partitive fraction scheme (RPFS), during the pre- or post-game assessments, the average scores of SF1 players were at least two standard deviations lower than the average scores of SF2 players. SF1 players’ average RPFS scores \((M_{pre} = 1.17; M_{post} = .92)\) were still lower than SF2 players’ \((M_{pre} = 2.64; M_{post} = 2.86)\), but the
standard deviations in both groups were relatively high ranging between 1.50 and 1.95. In contrast, Kate’s scores were all within the one standard deviation of SF2 players’ average scores with only two exceptions. During the pre-game assessment, on the partitive unit fraction scheme (PUFS) and the iterative fraction scheme (IFS), Kate’s scores (3 on both schemes) were much lower than SF2 players’ average score (M_{PUFS} = 3.93, SD_{PUFS} = .19; M_{IFS} = 3.57, SD_{IFS} = .61).

Kate achieved full scores on the RPFS both before and after the gameplay. In the pre-game assessment, SF2 players scored significantly higher than SF1 players in the PWS \([W = 7, p < .05; r = .68]\), the PUFS \([W = 5, p < .05; r = .69]\), the SO \([W = 7.5, p < .05; r = .57]\), and the IFS \([t(5.29) = 7.12, p < .001; d = 4.10]\). In the post-game assessment, the significant difference was no longer present on the PWS or the SO \((p > .5)\), SF2 players still scored significantly higher than SF1 players in the PUFS \([W = 8, p < .05; r = .59]\) and the IFS \([W = 8, p = .06; r = .53]\).

Additionally, while no statistically significant difference in the PFS or the RPFS \((p > .05)\) was detected in their pre-game assessment scores, the differences of SF1 and SF2 players’ post-game scores in the PFS \([t(5.92) = 2.35, p = .058; d = 1.35]\) and the RPFS \([t(10.90) = 2.03, p = .068; d = 1.12]\) were marginally significant.

**Table 10**

The means and standard deviations of the participants’ scores by scheme/operation

<table>
<thead>
<tr>
<th>Game played_Test</th>
<th>Part-Whole</th>
<th>Partitive Unit</th>
<th>Partitive Fraction</th>
<th>Splitting Operation</th>
<th>Iterative Fraction Scheme</th>
<th>Reversible Partitive Scheme</th>
</tr>
</thead>
<tbody>
<tr>
<td>(N) SF1 Pre (6)</td>
<td>3.58 (.38)</td>
<td>3.00 (.89)</td>
<td>2.42 (1.36)</td>
<td>2.08 (1.74)</td>
<td>.42 (1.02)</td>
<td>1.17 (1.60)</td>
</tr>
<tr>
<td>SF1 Post (6)</td>
<td>3.75 (.42)</td>
<td>3.00 (.84)</td>
<td>2.58 (1.20)</td>
<td>3.08 (1.63)</td>
<td>1.92 (1.63)</td>
<td>.92 (1.50)</td>
</tr>
<tr>
<td>SF2 Pre (7)</td>
<td>4.00 (0)</td>
<td>3.93 (.19)</td>
<td>3.79 (.27)</td>
<td>3.64 (.75)</td>
<td>3.43 (.19)</td>
<td>2.64 (1.89)</td>
</tr>
<tr>
<td>SF2 Post (7)</td>
<td>4.00 (0)</td>
<td>3.93 (.19)</td>
<td>3.79 (.39)</td>
<td>3.93 (.19)</td>
<td>3.57 (.61)</td>
<td>2.86 (1.95)</td>
</tr>
</tbody>
</table>
Comparing the participants’ post- versus pre-game assessments, SF1 players’ scores were increased on four out of the six fraction schemes/operations examined, with the two highest percentage increases being observed on the splitting operation (SO, 48%) and the iterative fraction scheme (IFS, 357%). SF1 players’ average RPFS scores decreased by 21%. Results from paired t-tests or Wilcoxon signed rank tests showed that only the difference between SF1 players’ pre-game and post-game IFS was marginally statistically significant \([t(5) = 2.28, p = .07; \ d = .93]\), while the differences were insignificant on the other five schemes/operations \((p > .5)\). Many SF2 players and Kate already received full scores on various fraction schemes/operations before the gameplay. Even so, after gameplay, SF2 players improved their average performances on three fraction schemes/operations, and the highest percentage increase occurred in their performances on the RPFS (8.3%) and the SO (8.0%). There were no statistically significant differences between SF2 players’ pre- and post-game performances on any fraction scheme/operation \((p > .05)\). Kate’s scores on the PUFS and the SO increased by 33% and 14% respectively. To conclude, on the individual fraction scheme/operation, no statistically significant difference was detected except in SF1 players’ pre- and post-game performances on the IFS. The lack of significant difference was probably related to the limited number of participants and the relatively large standard deviations within each group.

Comparing each individual’s post- and pre-game assessment performances, three SF1 players had a 45% or more increase; one SF1 player had a 24% increase; one SF2 player had a 12% increase; one SF1 player and Kate had a 6-7% increase. Five SF2 players had a 2% increase, among whom four received almost full scores before the gameplay and thus had little
room for improvement. One SF2 player had a 3% decrease and one SF1 player had a 21% decrease. To sum up, most individuals improved in their performances at the test of fraction schemes/operations after playing SF1 or SF2, but the extent of the improvement varied.

6.2.4 Fractional Conceptions

Considering that changes in conceptual understanding might be implicit, apart from the self-reports in the interviews, the participants’ post-game assessment performances were also examined focusing on their understanding of fractions as reflected in their solutions. All participants thought that playing SF1 and/or SF2 was a refresher and that they learned little, if anything, about fractions from playing the games. However, overall, the participants provided richer and more detailed explanations of the meanings of “fraction” along with better illustrations after the gameplay. Improvement was also observed in equal partitioning, making sense of improper fractions, and generating examples of fractions in every life. The findings in each area were presented one by one.

6.2.4.1 Initial Explanations of “Fraction”. In the pre-game interview, when asked to explain the meaning of "fraction", half of the participants (four SF1 players, two SF2 players, and Kate) struggled to independently clarify what it meant or to put their understanding into words. This suggested that they were unfamiliar with verbally making sense of mathematical terms. Among the four SF1 players, Nash and Ada gave examples of fractions saying “three-fourths of a pizza” or “one-half of something” but could not be more general; Zoe could merely come up with the answer that fractions were decimals in another form; Harry struggled the most with this question. He said, “Fractions are in only certain math equations, but I don’t know the specifics…It is a term in math…No (concrete meanings I could think of), not off the top of my head. It is kind of dead.” When asked to identify the fraction part in the math expression $3/5 \times 2,$
at first Harry hesitated between “three over five” and “the multiplication part”. Then he remembered and confirmed that the $3/5$ (“three over five”) was the fraction, because “a fraction is a number, a conjunction of numbers”. Looking at the two SF2 players’ and Kate’s responses, Nick’s answer was similar to Zoe’s as he said, “A fraction is almost like a percentage”; Kate and Hannah described the symbolic setup, e.g., “a numerator over a denominator and you try to simplify them”, “the number at the top divided by the number at the bottom”.

The remaining seven participants could easily come up with an answer about the meaning of fractions. Two SF1 players (Merik and Maya) and three SF2 players (Nora, Tim, and Alana) discussed that a fraction meant “part of a whole” or “taking away the numerator number of pieces from a total of the denominator number of pieces”. This type of answer stressed that a fraction resulted from partitioning a whole and taking away some parts and a fraction was smaller than a whole. Nora’s explanation was representative, “When you do a fraction, the whole or what everything would be is the denominator and the part would be the top.” Two SF2 players, Austin and Neo, explained that fractions were made up of pieces from split-up wholes and that fractions could be whole numbers. Both of them clarified that no taking away needed to occur and that fractions were not always smaller than whole numbers. Neo explained six-thirds verbally and visually (Figure 31) in this way,

That will be six pieces of three…the denominator is denoting how much to create one whole, and the numerator being how many pieces there are. Since there are three to make one whole…It would only make sense if it would make a secondary circle.

**Figure 31**

*Neo’s pre-game illustration of six-thirds*
After the gameplay, none of the seven participants who previously had trouble explaining the meaning of “fraction” had such struggles anymore, and most provided more complex responses in the post-game interviews. “Part of a whole” where the whole referred to unit one remained the most common explanation. Besides this, Nash discussed that fractions were divisions and could represent multiplicative relationships of two quantities. Four participants (two SF1 players, one SF2 player, and Kate) discussed in the post-game interview that fractions could be further broken down into smaller fractions. This signified a preliminary understanding that the whole in “part of a whole” did not need to be unit one and provided the foundation for making sense of more complex fraction multiplications/divisions and considering fractions as indicators of multiplicative relationships (e.g., one-sixth could be generated by creating one-half of a third, and the “one-half” is a multiplicative relationship indicator in this case).

6.2.4.2 Fractions as Indicators of Multiplicative Relationships. In the pre-game interview, responses from three SF2 players (Tim, Neo, and Alana) strongly suggested an understanding of fractions as indicators of multiplicative relationships and ratios/proportions between two quantities. When asked under what circumstances one-half might be smaller than one-third, Tim immediately answered that one-third of a much larger object could be bigger than one-half of a smaller object. The quick response suggested his familiarity with using fractions to describe multiplicative relationships between two quantities. Like others, when generating examples of fractions, Neo began with a shape that represented one. After creating one-fourth, he proceeded to split one-fourth into halves and then proceeded to split it into quarters (Figure 32).
His drawing straightforwardly showed the relationship between one and one-fourth. But, for one-eighth and one-sixteenth, he created these quantities not by splitting the whole but by splitting one-fourth. His answer went along with his drawing, "If you get one quarter and split that even further, you get one-eighth. Then it's one-sixteenth...I think about the quarter here, that (one-sixteenth) would be a quarter of a quarter, one-fourth of one-fourth." Soon after he explicitly connected fractions to ratios/proportions, "It doesn't matter how many pieces. It's just that these pieces altogether create one whole. And however many pieces are part of it or however many you take out would be the portion." Alana explained one-fourth by holding four pens together and putting away one pen, describing the four pens as a whole and the missing pen as "losing one-fourth of all four pens". Later, Alana said,

It is easier to see groups losing or adding things rather than just individual things...in fractions, you are working with groups...Even though all these pens are different, they are still writing utensils, which puts them in one group as a whole, instead of having to individualize every single one of them...Fractions just help make everything seem simpler and smaller...like group them up in different categories.
Kate and one SF2 player, Nick, showed some understanding of fractions as ratios (percentages/proportions). Compared to the evidence presented in the last paragraph, the evidence for Kate’s and Nick’s understanding was not as strong. When giving examples of the use of fractions in real life, Kate talked about keeping track of the number of cups arranged out of the total number needed during event organization. To explain the meanings of fractions, Nick first thought of percentages/ratios as he said, "Fifty percent in fractional terms can be five-tenths or two-fourths". Later in the interview, he brought up "free throw percentage", the percentage of a player's successful free throws out of total attempts, as an example of fractions' presence in real life. Lastly, the responses of two SF1 players, Nash and Ada, weakly suggested an understanding of fractions as indicators of multiplicative relationships. Besides "part of a whole (one)", Nash mentioned "half of an angle" as an example of the use of fractions in geometry; Ada mentioned "half of something" and "a fraction is a part of something, like a part of a whole". But neither made further explanations.

In the post-game interview, six additional participants (three SF1 players and SF2 players) made comments that strongly suggested an understanding of fractions as indicators of multiplicative relationships between two quantities, besides the three SF2 players (Tim, Neo, and Alana) who already did so before the gameplay. Hannah (SF2) explained fractions as “parts of a whole”. After being prompted to clarify fractions bigger than one, Hannah added that “one is not
always the whole”. Zoe (SF1) and Nora (SF2) both independently came up with the answer that one-half of something might be smaller than one-third of a different thing. Hannah and Austin (both SF2 players) also easily recognized this, although Austin convincingly argued that directly comparing two abstract fractions from two different things was insensible. When discussing fractions in everyday life, both Harry and Nash noticed that there were four tables in the room. They casually mentioned that we could use fractions like one-fourth or three-thirds to describe one or three tables out of the total four tables. Same as before, Kate and Nick discussed percentages as examples of fractions, but there was no stronger evidence suggesting their understanding of the use of fractions in describing multiplicative relationships.

6.2.4.3 Equal Splitting. Further looking into their interview responses and assessment performances, four SF1 players (Nash, Ada, Harry, and Merik) and one SF2 player (Nick) did not consider equal splitting to be always necessary to create a fraction, especially when cutting up a circle or sector. All four SF1 players applied equal splitting when drawing one-half or one-fourth out of a circle that represented one (Figure 33g), but when creating other fractions in the assessment or interview, they cut up circles unevenly and relied on the unevenly split circles to figure out one-fifth (Figure 33a, d), three-fifths (Figure 33c, h), and one-sixth (Figure 33e). In the interview, when Harry’s inconsistent splitting patterns (Figure 33g-h) were pointed out, he argued that equal splitting was necessary only if the denominator was an even number. Item 9 of the pre-game assessment asked the fraction of a given piece of pie out of an unshown whole pie (Figure 33b, f, i). All four SF1 players considered the given sector to be equivalent to one-half (one-half, two-fourths, or three-sixths) based on their uneven splitting when the sector was visibly bigger than half of a circle.
**Figure 33**

*Four SF1 players’ unequal splitting in the pre-game interview/assessment*

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*Note.* Nash was showing one-fifth(a) and three-fifths (c) in the pre-game assessment. In b, both Nash and Merik partitioned and supplemented the given piece in the same way and concluded that the sector was three-sixths or one-half of a whole pie. In d, Ada was marking one-fifth. Both Ada and Merik illustrated one-sixth as shown in e. Ada considered the sector two-fourths based on her portioning in f. Harry drew two-fourths (g) and three-fifths (h) in the pre-game interview. He considered the sector one-half (i) at first. With the interviewer’s assistance, Harry knew that the sector was larger than one-half and his answer changed to one-third.

Besides circles, Harry and Merik occasionally relied on unequally split bars in the pre-game assessment. Harry’s answer to Item 19 (see Figure 19 top, p. 48) was one-third. He knew that repeating the given bar twice would not be enough to create the longer bar, so he considered the denominator of the answer to be bigger than two and chose three. Since the smaller stick was obviously longer than one-third of the longer stick, one-third as an answer was sensible only if
the longer stick was unequally split. Similarly, when Merik worked on Item 3 (see Figure 23a, p. 50), before receiving hints from the interviewer, he marked five-eighths by circling out five pieces among which one piece was twice as big as each of the other four pieces.

In the beginning, Nick considered equal splitting necessary and did so when creating an illustration of three-fourths. When asked to show five-tenths, Nick first cut a rectangle into ten unequal pieces (Figure 34a), but he immediately decided to re-draw the picture, using a circle and cutting it into ten sectors (Figure 34b). Although the drawing was flawed, Nick intended the circle to be split up evenly. However, a few moments later, during problem-solving, he drew two-thirds by cutting a rectangle into three unequal pieces and highlighting two pieces that totaled one-half of the rectangle (Figure 34c). When prompted to explain, he took back his earlier words and justified, "Because it's still three pieces overall. If I shade all three, it will be three over three, right? But I shade two parts, so it's only two-thirds." When asked whether the amount would be two-thirds if one smaller part and one longer part were shaded, Nick became undecided, although he inclined toward a negative answer. During the assessment, Nick always intended to equal split bars or circles, although his drawings were inaccurate sometimes.

**Figure 34**

*Nick’s pre-game inconsistent splitting*

Two SF1 players (Zoe and Maya) and all SF2 players except Nick were certain that a whole needed to be split equally to create a fraction, and they were consistent about creating fractions by equally splitting wholes. Rarely did they make the mistake of unequally splitting a
quantity during the pre-game interview and assessment. Initially, Kate’s illustration of two-thirds was surprisingly odd for it had nothing to do with splitting (Figure 35 top). In the drawing, the individual circles on the different sides of the bar added up to the numerator and the denominator respectively. Then she simplified the fraction and said, “You would cancel out the two circles on the top and (on the bottom) then you are left with one on the bottom.” This simplification would have been sensible and correct if the individual circles on either side of the bar were in a multiplicative relationship. Unfortunately, this was not the case. When prompted to illustrate two-thirds in a given square that represented one, Kate first shaded one-half and then shaded another one-fourth (Figure 35 bottom), because she thought that two-thirds was equal to .75 in decimal. Although the equivalency between two-thirds and three-fourths was incorrect, in this drawing and Kate’s other drawings, equal splitting was consistently applied.

Figure 35

Kate’s pre-game illustrations of two-thirds

Among the five participants who did not always equally partition a whole, Ada and Merik showed improvements in equal partitioning in the post-game assessment and Nick did so in the post-game interview. Although the way Ada cut up a circle and selected one-sixth remained unchanged (Figure 33e), she verbally expressed the intention of splitting the circle evenly. With a hexagon, Ada easily partitioned it evenly and marked one-sixth (Figure 36a). Ada
previously solved Items 7 and 9 (Figure 33f) based on unequal splitting. In the post-game assessment and after playing SF1, she applied equal splitting to solve the corresponding items (Figure 36b-c). However, the idea of equal splitting to create fractions was unstable, since occasionally the participants still unevenly split wholes. In the interview, when illustrating \(2 \times \frac{1}{3}\), Ada split two circles unevenly and marked a quarter of each circle as one-third (Figure 36d).

Merik’s improvement was seen in his solutions to Items 3, 7, and 9. In the post-game assessment, he independently complemented the given diagram where a whole was unequally partitioned and then figured out the correct answer. Merik’s post-game problem-solving of Items 7 and 9 was highly similar to Ada’s (Figure 36b-c). The only difference was that Merik’s answer to Item 9 was four-sevenths while Ada’s was six-tenths. Nick’s splitting in the assessments was consistently correct before and after the gameplay. But he unevenly split wholes to create fractions sometimes and justified the uneven splitting in the pre-game interview. After the gameplay, Nick consistently evenly split wholes and claimed that cutting wholes evenly was necessary for creating fractions.

**Figure 36**

*Ada’s splitting in the post-game interview/assessment*

Note. The hexagon was modified from a circle and the item was about marking one-sixth of a whole (a). Item 7 asked for the fraction of the smaller piece out of the whole and Ada’s answer was one-eighth (b). In Item 9, the whole was not provided and the fraction of the given piece out
of a whole was to be figured out (c). Ada considered each slice to be one-tenth and her final answer to be six-tenths.

6.2.4.4 Understanding Improper Fractions. Both interviews and problem-solving of the iterative fraction scheme (IFS) items were considered when evaluating the participants’ understanding of improper fractions since verbal expression was limited and one’s understanding might be implicit. Five SF1 players (all but Maya) understood that a fraction was “part of a whole” but could not make sense of fractions beyond one. The IFS items (Item 17-20) of the pre-game assessment concerned figuring out a larger quantity out of a smaller quantity or drawing a larger quantity given its multiplicative relationship to a smaller quantity (see Figure 21 on p. 49). None of the five SF1 players could solve any item. Nash correctly solved $5 - 7/4$ procedurally during the pre-game interview, but it remained questionable whether he could make sense of the improper fractions involved in the problem-solving process due to his poor performances on the IFS items. Merik understood mixed fractions as a combination of a whole number and a fraction. He knew that in improper fractions the numerator was bigger than the denominator. On the other hand, he solved $2/3 + 2$ by changing 2 into $2/1$ and adding the numerators and the denominators respectively. The incapability to properly solve $2/3 + 2$ indicated his lack of understanding of fractions beyond one. Consistently, Merik did not receive any score on the IFS items in the pre-game assessment.

One SF1 player (Maya), one SF2 player (Alana), and Kate had an unstable understanding of improper fractions as iterations of a unit fraction. They all made the same mistake on Items 17-18 by writing down the fraction of the smaller quantity out of the bigger quantity but correctly solved Items 19-20 in the pre-game assessment. In addition, during the interview, when asked to convert $1 1/2$ into an improper fraction Maya carried out the correct procedure of
multiplying the integer by the denominator and adding the result to the numerator but was not sure how to proceed. Alana considered that fractions could not be larger than one in the interview.

The remaining six SF2 players all conceptually understood improper fractions, for they all scored 3.5 (out of 4) on the IFS in the pre-game assessment. Additional evidence was found in two SF2 players’ interview responses. As seen in his clarification of six-thirds, Neo could explicitly make sense of an improper fraction as a number of the same unit fraction. So could Tim. When generating an example of a fraction larger than one, Tim added a small square to the right of a larger square he previously drew and labeled the total quantity five-fourths (Figure 37). The larger square represented one. Tim previously cut it into four square-shaped quarters and further cut each quarter into halves, showing that one could be split into fourths or eighths. The newly added small square stood alone and was marked "1/4" together with the other square-shaped quarters. The drawing and labeling showed that five-fourths comprised five one-fourths or four one-fourths (one) and an additional one-fourth, which suggested that Tim could think of a fraction as a certain number of a unit fraction.

**Figure 37**

*Tim’s pre-game illustration of five-fourths*
After playing SF1, among the five SF1 players who could not solve any IFS item or explain the meanings of improper fractions before the gameplay, two participants improved tremendously in their IFS score and two participants improved a little in their problem-solving. Zoe left all of the four items measuring the IFS blank in the pre-game assessment. After playing SF1, she properly solved all four items in the post-game assessment. For instance, to solve Item 18, Zoe split both the sector and the circle into one-eighth (Figure 38a). She counted the number of one-eighth in the sector (three) and the circle (eight) and added them up to get her final answer eleven-eighths. Before the gameplay, Nash skipped Items 19-20. But he properly solved them on his own after the gameplay, even though he voiced some uncertainty in his answer. Both Harry and Merik showed some emerging flimsy understanding of improper fractions after playing SF1. Merik struggled to solve Items 18-19, but the drawings he made (Figure 38b-c) showed that he was on the right track. Independently, Harry managed to solve Item 19 (Figure 38d) correctly.
Note. Merik equally split the circle and knew that the left piece was one-third of the right piece (b). When the researcher adapted the question and asked for the total amount, he was 50% certain that the total amount was “four over three”. If he were to answer the question as it was worded, he thought the answer was one-third. On Item 19, Merik knew that “his stick” was longer than the given one by three more pieces, each piece being one-fifth (c). When asked to mark “his stick”, Merik highlighted the given stick only and was not sure whether to include the three additional pieces (c). Harry first solved Item 19 by partitioning the given bar into six parts
and then he added three vertical lines outside (d). When clarifying his drawing, the last piece of
the given bar was purposefully excluded since he meant to cut the given bar into five equal parts
and added three additional parts of the same size (d).

Three participants, one SF1 player (Maya) and two SF2 players (Kate and Alana), did not
have a solid understanding of improper fractions and incorrectly solved two of the four IFS items
in the pre-game assessment. After the gameplay, their performance on the IFS items remained
the same. However, in the post-game interview, Kate reported a better understanding of mixed
fractions. According to Kate, she used to struggle with mixed numbers, thinking of them as "a
bunch". In her illustration of "1 1/4," she cut a bar into five parts and then highlighted one part
(Figure 39 top). In her verbal explanation, she "add(ed) one and four" and "put the one
(presumably the whole number part) into five (the total quantity)"; after the splitting, she needed
"one out of the one-fourth". It appeared that she vaguely knew the whole number part needed to
be cut into the same unit fraction as the fractional part or about the additive relationship between
the whole number and the fraction parts, but she was unclear about what a mixed number as a
group signified. In the post-game interview, her understanding became "one, and then one out of
four". Her drawing became a shaded bar that represented one and a shaded one-fourth from a
second bar (Figure 39 bottom). She remained unconfident in converting a mixed number to an
improper fraction.
Among the six SF2 players who scored 3.5 on the IFS in the pre-game assessment, four participants’ post-game IFS scores increased by .5; one participant’s score remained the same, and one participant’s decreased by .5. In the interview, Nora explicitly demonstrated her understanding of improper fractions as iterations of a unit fraction. During Nora's post-game interview, she created a drawing of five-fourths that comprised five squares, each representing one-fourth (Figure 40). Different from Tim's drawing in the pre-game interview (Figure 35, p. 90), Nora attached all squares of one-fourth and presented five-fourths as one group. But they both showed a multiplicative relationship between one-fourth and five-fourths. Nora's explanation was consistent with her drawing, "You have five blocks. Now we have five-fourths."

6.2.4.5 Fractions in Everyday Life. About the use of fractions in everyday life, at first, the majority of the participants, including four SF1 players, four SF2 players, and Kate, were
baffled, answering with "in math classes", "I don't know", "I don't think I would necessarily need fractions other than for getting a degree." or "I probably do (use fractions) but I am not aware of it." Nora went further and commented, "I think about this all the time when I think about math. How much do we actually apply it in the world?" It appears that during their math learning the connections between basic math topics like fractions and everyday life were seldom explicitly discussed. With some more time, most participants could state some use of fractions in their lives. Maya commented, "I just never really think about it until now that I use fractions outside of math."

Four SF1 players, three SF2 players, and Kate gave at least one example where fractions were used in measuring activities. Specifically, Ada and Zoe noticed that fractions were involved in crafting when planning or using fabric (e.g., one-half-inch seam allowance) or when converting units (e.g., among yards, feet, and inches). Other answers included architecture/construction (Maya, Harry, and Neo), measuring cups and cooking recipes (Ada, Maya, and Tim), shoe sizes (Neo), and quarter dollars/pennies being fractions of one dollar (Neo and Austin). When reflecting on a cooking-themed mobile game she often played, Kate realized that filling up cups to one-third or two-thirds levels involved fractions and could be an example of fractions in her life. In these participants’ answers, Tim and Neo, both being SF2 players, explicitly noted that fractions supplemented whole numbers and allowed more accurate measurements; two SF1 players (Nash and Ada) and Kate implied the same idea.

Three SF1 players and two SF2 players gave examples of fractions being involved in partitioning one object and marking the parts. Nash, Zoe, and Austin mentioned splitting and sharing a pizza. Harry and Maya answered that fractions were involved in reading pie charts. Nora thought that analog clocks could be read by quarters. Two SF2 players, Nora and Nick,
discussed percentages. To Nora, knowledge of fractions was needed when someone thought about how much of a day had passed. Nick's examples included deal labels in stores (“70% off”) and free throw percentages in basketball games. When talking about video games, Nick remembered that a fighting game gave feedback on the percentages of damage caused by each attack and considered this another example of fractions in real life. As was presented earlier, Alana and Kate thought that fractions were often used to characterize multiplicative relationships in everyday life.

The only example that Hannah could think of had more to do with division or ratio. According to her, when seeing the total price for some items in a shop and calculating the price for each item, fractions were used. Merik was the sole participant who could not come up with any example of fractions outside math classrooms. To Merik, "I don't think we should learn fractions...maybe saying ‘shouldn't’ is a bit strong...but I wouldn't say we needed (fractions) as much (as whole numbers)."

During the post-game interviews, all participants could easily generate some examples of fractions in everyday life. Three SF1 players’ and three SF2 players’ answers about fractions in everyday life remained the same as their answers in the pre-game interview. The remaining eight participants (three SF1 players, four SF2 players, and Kate) expanded their answers and provided more examples. These eight participants’ answers are presented. Two SF1 players (Nash and Harry) and one SF2 player (Neo) discussed fractions as indicators of multiplicative relationships. It was noteworthy that the two SF1 players’ examples became less restrictive. Harry previously talked about fractions in measuring activities, and Nash previously only gave an example of splitting pizzas. After playing SF1, both of them noticed that fractions were in our day-to-day lives and could be anywhere. Nash commented, "Fractions are all around us. We just don't
realize it." Three SF2 players and Kate had their answers about the use of fractions in measurement enriched in the post-game interview. Specifically, before the gameplay, two SF2 players (Nora and Hannah) and Kate struggled to come up with examples, and Nick could only think of percentages. After the gameplay, the three female participants thought that fractions were involved in baking/cooking, adjusting recipes, and architecture and construction; Nick mentioned measuring cups. Additionally, Nora recognized that fractions enabled more precise measurements, compared to integers. Hannah’s example was an authentic math problem that she ran into and solved. One time she needed to figure out how to measure one-half cup when she only had one-fourth measurement cups. Lastly, Merik, who could not think of any example about the use of fractions in real life, thought that fractions were involved when sharing things with others.

6.3 Practices and Knowledge of Fraction Problem-solving

Before the gameplay, all participants tended to resort to algorithms or rules to deal with fraction problems, treating operations of fractions as computational or procedural tasks. All but three SF2 players could not explain why the algorithms or rules worked or made sense of the problem-solving at a conceptual level. In other words, when presented with abstract math problems, both SF1 and SF2 players usually activated the resources that suggested math knowledge being passively transmitted. Furthermore, for all but three SF2 players, these were the only resources that the participants possessed and could easily access. While most SF2 players could appropriately and correctly solve fraction problems, SF1 players and Kate frequently omitted or failed to recall certain steps, applied procedures to inappropriate situations, or mixed up the procedures for different situations.
After the one-hour gameplay intervention, the participants’ default problem-solving practices largely remained the same as they still heavily relied on formal algorithms or procedures. However, when prompted to explain and illustrate their problem-solving, SF1 players and Kate improved in solving and explaining various operations of fractions, and SF2 players who could already consistently correctly solve the problems performed better in explaining their problem-solving processes. Four SF1 players, five SF2 players, and Kate made improvements in fraction additions/subtractions. Three SF1 players, one SF2 player, and Kate improved in fraction multiplications. One SF1 player and Kate better solved and explained simple fraction divisions. In the subsequent sections (6.3.1 – 6.3.6), the participants’ pre- and post-game problem-solving of fraction multiplications, divisions, and additions/subtractions in the interviews are presented. Three participants made significant improvements in multiple operations with fractions. Qualitative examinations of five gameplay episodes by these participants showed that their improved problem-solving in the post-game interviews was largely consistent with their knowledge of fractions displayed in the gameplay.

6.3.1 Pre-game Fraction Multiplications

When solving multiplications involving fractions, with certainty and consistency, two SF1 players and six SF2 players drew on the rule of "multiplying straight across", i.e., multiplying the denominators and the numerators respectively. If the problem was an integer multiplying a fraction, they all divided the integer by one first, changing the integer into the fractional format and then applying the procedure. The problem-solving of four SF1 players, one SF2 player, and Kate was inconsistent or unreliable. Among them, two SF1 players and one SF2 player could correctly solve some fractional multiplication problems but lacked confidence in their procedures or answers. The remaining two SF1 players and Kate applied incorrect
procedures. Even when they were aware of the incorrectness, they were unable to correct their problem-solving.

Three SF2 players could explain why "multiplying straight across" worked in simple cases without any assistance. Tim, Austin, and Neo made sense of an integer multiplying a fraction by alternating the problem to adding up the fraction for the integer number of times (e.g., $2 \times \frac{1}{3} = \frac{1}{3} + \frac{1}{3}$). All of them could interpret the multiplication of two fractions by viewing one quantity as the base and then taking a fraction of the base quantity. Specifically, for $\frac{1}{3} \times \frac{1}{2}$ (or vice versa due to the commutative property of multiplication), Tim cut a whole into two halves, with each half being further cut into three equal parts (Figure 41a); Austin cut a whole into three thirds and then split each third into two halves (Figure 41b). Both of their drawings resulted in one whole being equally split into six parts. Slightly different from Tim and Austin, Neo first marked a part of a whole to be one-third and discarded the rest area. Then he split the third into two equal parts (Figure 41c).

Figure 41

Three SF2 players’ productive illustrations of $\frac{1}{2} \times \frac{1}{3}$

Five participants, including two SF1 players and three SF2 players, solved fraction multiplications smoothly and correctly but could not elucidate why the procedure worked. Among them, Nora, an SF2 player, was proficient enough that she could solve such problems automatically in her head. Nonetheless, when asked to make sense of the procedure, Nora said, "Not clear on
that (why it works). I just remember being taught that." When asked to illustrate \( \frac{1}{3} \times 2 \), both Maya (SF1 player) and Alana (SF2 player) drew “1/3” and “2” separately, and “2” was split into thirds (Figure 42). This drawing directly showed the two quantities involved but did not help with the operation. Based on the drawing, both had no clue how to proceed. With assistance, eventually, two SF2 players, Alana and Hannah, were able to interpret the expression \( \frac{1}{3} \times 2 \) as adding one-third two times or having two groups of one-third. Concerning illustration, after drawing one-third, Alana initially thought that she had to “make the fraction bigger because one-half was larger than one-third”. After being reminded that “times 2” was “two groups of something”, Alana was able to interpret \( \frac{1}{3} \times \frac{1}{2} \) as “half of 1/3” and illustrate it by cutting each one-third into two halves. To sum up, two SF1 players and three SF2 players could solve fractional multiplication correctly by following the procedure “multiplying straight across”, but they all struggled with making sense of it or illustrating the problem-solving. Among these participants, with assistance, two SF2 players ended up being able to make sense of simple fractional multiplications during the pre-game interviews.

**Figure 42**

*Representative unhelpful illustration of \( \frac{1}{3} \times 2 \) (Maya’s work)*

Two SF1 players (Ada and Merik), and one SF2 player (Nick) sometimes could solve fraction multiplications correctly. Their problem-solving seemed unreliable, even though sometimes their procedures and solutions were correct. Ada solved \( 2 \times \frac{1}{3} \) correctly, "you keep the bottom and multiply at the top". But she was not sure about it because she also thought the
correct procedure could be "multiplying both the denominators and the numerators (respectively)". On the same question, unlike the rest, Merik did not convert 2 into 2/1. He directly applied the procedure of multiplying the whole number by the numerator and keeping the denominator. For 2/3 * 1/3, Merik got 2/3 as the answer, since he multiplied the numerators but kept the denominator. And he felt more certain about his answer to this question than his answer to the earlier question. It appeared that Merik was partially drawing on the rule for addition/subtraction, where the shared denominator remained the same and the numerators were operated on. Nick initially applied cross multiplication to 2/1 * 1/3 and got 6 as the answer. After being prompted to think about 2*3, Nick realized his mistake, did straight multiplication, and got 2/3. However, when it came to 1/2 * 1/3, Nick still applied cross multiplication and got 2/3 as the answer, about which he felt certain. In Nick’s explanation, “You are supposed to cross multiply when both the numerators are the same…multiply straightforward when both denominators are the same.” Among these three participants, it is observed that they remembered and applied multiple rules or procedures to different fraction multiplications. Sometimes the recalled and applied rules were appropriate.

Lastly, two SF1 players and Kate incorrectly solved fractional multiplications or struggled to solve such questions. For 2 * 1/3, Harry converted 1/3 to 2 and got 4 as the final answer. He thought of the denominator 3 as “1 * 1 * 1”, and 1 / 3 (“one over three”) as taking or deducting one out of “three of those ones”, thus leaving 2 behind. Feeling that he missed at least one step in the problem-solving, Harry was very uncertain about the answer. Harry did cross multiplication to 1/2 * 1/3 and got 3/2 (“three over two”). He felt the method was weird and the answer did not seem right. But the judgment did not stem from his thinking of sensemaking of fraction multiplication, but from his conception of fractions. In his words, “There is a greater number on the top, and the number on the bottom is lesser than the number on the top, which makes it an uneven
amount. You cannot take three out of two, because it is impossible.” Zoe first converted the fractions so that they had the same denominator. Then she multiplied the numerators and carried over the shared denominator. By applying these procedures, she got 2 as the answer to $2 * \frac{1}{3}$ with the calculation process being “$2 * \frac{1}{3} = \frac{2}{1} * \frac{1}{3} = \frac{6}{3} * \frac{1}{3} = \frac{(6*1)}{3} = \frac{6}{3} = 2$”, and $\frac{1}{2}$ as the answer to $\frac{1}{2} * \frac{1}{4}$ (i.e., $\frac{1}{2} * \frac{1}{4} = \frac{2}{4} * \frac{1}{4} = \frac{(2*1)}{4} = \frac{2}{4} = \frac{1}{2}$). Zoe was somewhat certain about the processes, but she felt uncertain about the answers. Like Merik’s multiplication of two fractions, it seems that Zoe mixed up the rule for addition/subtraction and that for multiplication. Different from Merik, Zoe applied the same procedure consistently when solving “integer * fraction” and “fraction * fraction”. Kate wanted to apply “keep, change, flip” to fraction multiplications. Like Zoe, Kate also thought that a common denominator was necessary for fraction multiplications. Unable to convert the fractions to have the same denominator, she did not solve either $2 * \frac{1}{3}$ or $\frac{1}{2} * \frac{1}{3}$. After assisting her to make sense of $2 * 3$, Kate still struggled to figure out the answer to $2 * \frac{1}{3}$.

6.3.2 Post-game Fraction Multiplications

Considering fraction multiplications, after the one-hour gameplay intervention, five participants (three SF1 players, one SF2 player, and Kate) provided better explanations and two participants (one SF1 player and Kate) improved in solving some problems. Four SF2 players (Tim, Austin, Neo, and Alana) were not asked to solve such fraction multiplications in the post-game interview, since they already did an excellent job in problem-solving and explaining the meanings before the gameplay, without or with limited external assistance.

Among those who could consistently apply the appropriate procedures but could not explain their problem-solving in the pre-game interview, during the post-game interview, one SF1 player (Nash) and one SF2 player (Hannah) had no change in their practices or (lack of)
explanation, while another SF1 player (Maya) and another SF2 player (Nora) improved in explaining their problem-solving. Previously, Maya did not know how to visually represent $\frac{1}{3} \times 2$ in one picture. After playing SF1, Maya was able to interpret the expression as adding one-third to an existing one-third (Figure 43a). She could represent $\frac{1}{3} \times \frac{1}{2}$ as cutting one-third into two equal parts and taking one-sixth or one part as the result (Figure 43b). Despite the improvements, when asked to show $2 \times \frac{1}{3}$ on a given block that represented 2, she struggled and was not sure whether she was supposed to mark one-third outside or inside the given block (Figure 43c). Therefore, the improvement in Maya’s explanation of fraction multiplications seemed unstable. After playing SF2, Nora was able to interpret “fraction * integer” as adding the fraction for the integer number of times, but she had no idea how to make sense of multiplications of two fractions. If she were to teach it to someone else, she could only do it by stating and showing the “multiplying straight across” procedure.

Figure 43

*Maya’s post-game illustrations of fraction multiplications*

Two SF1 players (Ada and Merik) and one SF2 player (Nick) could correctly solve some fraction multiplications before the gameplay. While Merik and Nick had no improvement after playing SF1 and SF2 respectively, Ada could better explain the expressions where a fraction multiplies an integer after playing SF1. In the pre-game interview, about $\frac{1}{3} \times 2$, Ada felt the procedure insensible and uncertain about her answer, even though her procedure and answer were correct. After the gameplay, Ada could make sense of the expression by partitioning two
ones into thirds respectively and taking one-third from each whole (Figure 44). Consequently, it appears that Ada understood simple multiplication of a fraction and an integer as pulling the fraction from different wholes the integer number of times. However, Ada created one-third by cutting a circle into one-half and two one-fourths, so her improvement was limited to a better understanding of the operation “multiplication”.

**Figure 44**

*Ada’s post-game illustration of 1/3 *2*

Two (one SF1 player, Zoe, and Kate) of the three participants (two SF1 players and Kate) who could not correctly solve fraction multiplications did better at solving and explaining them after the gameplay. Before the gameplay, Zoe was somewhat certain about her self-taught procedure that integrated formal rules for fraction additions. After playing SF1, Zoe solved the same problem by “multiplying straight across”. She explained $\frac{1}{3} \times 2$ as adding one-third “two more times” to one-third, which gave her $\frac{3}{3}$ or 1 as the result. With the researcher’s help, Zoe corrected the mistake and learned that adding one other one-third to one-third was sufficient. About $\frac{1}{3} \times 2$, Kate first thought that she needed to “divide it” and got $\frac{1}{6}$. When prompted to use the game mechanics in SF2 to explain, Kate said that a ghost would spit out one-third twice and she needed to add them together. Concerning $\frac{1}{3} + \frac{1}{3}$, Kate drew two one-thirds from two different wholes and thought that they added up to two-sixths. With assistance and being suggested to think of the process in the game, Kate recognized that the addition result was two-thirds. Although Zoe’s and Kate’s illustrations and clarifications were still inadequate after the
gameplay, they started trying to make sense of fraction multiplication and their explanations were easily improvable.

About $\frac{1}{2} \times \frac{1}{3}$, Zoe first drew one-half and one-third from separate wholes (Figure 45a). After being pushed to show the expression in one drawing, Zoe first pondered on cutting one-half into two halves and creating one-fourths. Being reminded of the problem’s irrelevance to one-fourth, Zoe decided to cut one-half into three equal parts (Figure 45a). She felt uncertain but continued this process to illustrate the expression. Zoe marked one resulting piece one-third (of a half), but she knew that the piece was one-sixth of the whole square. Similarly, during the post-game interview, Kate visually showed $\frac{1}{3} \times \frac{1}{2}$ by cutting each one-third of one whole into two halves (Figure 45b). Both Zoe and Kate knew that each smallest piece was one-sixth but needed assistance to connect the visual representation to the problem-solving. Despite being imperfect, their explanations of multiplications of two fractions improved a lot compared to their pre-game performance. Harry made some changes in how he solved fraction multiplication, but his problem-solving remained incorrect and insensible to himself. During the post-game interview, Harry converted $2 \times \frac{1}{3}$ to $\frac{2}{2} \times \frac{1}{3}$ and then did “multiplying straight across”, getting $\frac{2}{6}$ as the immediate answer. After simplification, he got $\frac{2}{3}$ as the final answer. Harry had no clue how to visually represent the problem-solving.

**Figure 45**

*Zoe’s and Kate’s post-game illustrations of $\frac{1}{2} \times \frac{1}{3}$*
Note. On Zoe’s first attempt (a), she only drew and marked one-half of the left square. After considering splitting one-half into two equal parts, she decided and proceeded to cut the right half of the left square into three equal parts. The top piece was marked “1/3” (of a half). She demonstrated her understanding of the piece being one-sixth of the whole square verbally.

6.3.3 Pre-game Fraction Divisions

To reduce the difficulty in explaining the meanings, the fraction division problems were largely limited to a fraction being divided by an integer, e.g., \( \frac{3}{4} / 3 \). Of the five SF1 players and seven SF2 players who were asked to solve such problems, four SF2 players could consistently correctly solve them by drawing on the procedure “keep, change, flip” (keep the dividend, change the division sign to multiplication, and use the inverse of the divisor). One SF2 player, Alana, could apply “keep, change, flip” to fraction divisions but had uncertainties in the process. Two SF1 players and two SF2 players could solve some division problems consistently and correctly by using a limited strategy, “dividing straight across”. Among these four, one SF2 player (Nick) could not correctly solve fraction divisions when this method no longer worked. It was unclear whether the other three participants knew other viable procedures or not. Three participants, all SF1 players, could not correctly solve fraction divisions.

Tim, Hannah, Austin, and Neo, all being SF2 players, were very familiar and comfortable with applying “keep, change, flip” to fraction divisions. After calculating the results procedurally, they were asked to draw illustrations. Hannah had no idea how to visually represent \( \frac{3}{4} / 3 \). The other three attempted and struggled to draw illustrations. For \( \frac{4}{3} / 2 \), Tim drew two circles and split each into three-thirds (Figure 46). He marked four pieces and considered the illustration complete. In Tim’s explanation, the two circles represented the divisor 2; each circle was split using the unit fraction of the dividend; the numerator of the dividend
indicated the number of pieces wanted. For there were a total of six pieces and four were marked, the answer was $4/6$. Tim’s strategy worked for division problems where a fraction was divided by an integer. However, the illustration was not straightforward, and the explanation was unclear. It appeared that Tim purposefully matched his illustration and explanation with his abstract problem-solving. Austin hesitated with creating a visual representation of $3/4 / 3$, due to a lack of relevant exposure, “I never got taught the visual way. I just knew how to do it algebraically (arithmetically), but I don’t know how I would do this visually.” In his first attempt, he drew three-fourths of one and three ones separately. The division operation was left out. In Neo’s initial try, after drawing three-fourths of one square, he split each one-fourth into four quarters and felt too confused to move forward. Neo knew that division was about “making smaller groups of it.” However, like Tim and Austin, he was unfamiliar with explaining or visually showing fraction division.

**Figure 46**

*Tim’s pre-game independent illustration of $4/3 / 2*

With guidance and assistance, Tim and Austin were able to poorly explain the division problem-solving with illustration, and Neo did slightly better than them. After splitting a circle into four quarters, Tim split each quarter into three equal pieces and circled three pieces out of the same quarter as the answer (Figure 47a). With this illustration, Tim was able to point out all
individual elements (dividend, divisor, division, and result), but the process or the connections among the elements were not explained. Considering that he circled three pieces out of the same quarter rather than picking one piece from each quarter, Tim was probably matching the illustration with the numbers he got from the procedural calculation. Austin’s drawing and explanation were similar to Tim’s. Austin created two drawings. In one version (Figure 47b), he split each quarter of three quarters inside a circle into three (unequal) parts. In the second version (Figure 47c), he split one-quarter of a circle into three (unequal) parts. In both cases, he selected a quarter to be the result. Austin said, “It is gonna be 3/12. Because you divide it by 3, you have to multiply by 3 (one-third). So we split it (each quarter) up into three parts.” The poor clarification suggested that he did not fully understand how to visually represent and explain the division process.

**Figure 47**

*Tim’s and Austin’s pre-game illustrations of 3/4 / 3*

Among all participants, Neo did the best in explaining fraction division. With some assistance, Neo solved 3/4 / 3 without trying to reach 3/12. He still drew a square and split it into four quarters. Then he directly viewed each quarter as a unit and counted three units. About “being divided by 3”, Neo was more comfortable with “keep, change, flip” so he viewed it as “multiplying by 1/3”. And he knew that taking one out of the three parts would visually represent it. Although Neo successfully solved 3/4 / 3 visually, he struggled to explain 3/4 / 2 with a visual
illustration. He admitted that he was relying on the arithmetic solution to figure out the drawing, as he was much more confident in the former.

When dividing fractions, Alana (one SF2 player) was unsure about whether or when she needed to convert fractions to have the same denominator besides doing “keep, change, flip”. Given $\frac{3}{4} / 3$, Alana thought that a common denominator was needed so she first changed the divisor 3 to $\frac{12}{4}$. Then she applied “keep, change, flip”, and got $\frac{12}{48}$ or $\frac{1}{4}$. Alana was not sure whether she should have continued matching the denominators after transforming the problem to $\frac{3}{4} \times \frac{4}{12}$. The extra step of changing fractions to have a common denominator did not impact the correctness of the result, but it made the calculation more complicated. Heavily relying on the procedures that she mechanically but unstably memorized, Alana did not realize this and was uncertain about her problem-solving of fraction divisions. She did not know how to create visual representations to show $\frac{3}{4} / 3$ or $\frac{3}{4} / 2$, even with assistance. From her perspective, the dividend and the divisor were two separate quantities. She could easily view them in the same unit (i.e., by considering the divisors 3 or 2 as repeating $\frac{4}{4}$ three or two times respectively), but she did not know how to integrate the division operation into a drawing.

Two SF1 players (Ada and Merik) and two SF2 players (Nora and Nick) solved fraction divisions by “dividing straight across”, that is, dividing the numerators and the denominators separately. When asked to solve $\frac{2}{3} / 2$ or $\frac{3}{4} / 3$, they all converted the divisor into “divisor over one” and proceeded to “dividing straight across”. Both SF1 players were confident about their division problem-solving, although earlier they were uncertain about their multiplication of an integer with a fraction. When solving fraction multiplications, they both did not bother to convert the integer into a fraction. Furthermore, Merik drew on the rule for fraction addition to solve the multiplication of two fractions with the same denominator. Therefore, it seems that
they did not recognize that “operating straight across” worked in both fraction multiplications and divisions. Unfortunately, Ada and Merik were not further interviewed to solve other fraction divisions where “dividing straight across” did not directly lead to an answer. It was unclear whether they knew the more powerful strategy “keep, change, flip” or could correctly solve other fraction divisions.

Nora was very confident about her fraction divisions and her answers but could not clarify why the procedure worked. Again, Nora admitted that she could easily and quickly solve the problems in her head, but the mental process was not explicable. According to her, “It is not a visual. It just pops up.” It appeared that Nora was so proficient that she had internalized the problem-solving processes and remembered simple calculations as facts. In the post-game interview, Nora was asked to solve \( \frac{2}{3} / (\frac{1}{2}) \) and she converted the fractions to have the same denominators and then applied “dividing straight across”. Considering that such problems were not involved in SF2, it was likely that Nora was capable of using this strategy to deal with all sorts of fraction divisions even before the gameplay. Apart from simple division problems, Nick was asked to solve \( \frac{1}{3} / (\frac{1}{2}) \), where “dividing straight across” would render a fraction (in this case, \( \frac{3}{2} \)) as the denominator of the resulting fraction. This time Nick did “dividing cross”, dividing the dividend’s denominator by the divisor’s numerator to get a numerator and dividing the divisor’s denominator by the dividend’s numerator to get a denominator. Regarding his final answer \( \frac{3}{2} \) or 1 \( \frac{1}{2} \), Nick had some uncertainty because he thought that division should render a number smaller than the dividend. When asked whether the result of dividing a proper fraction would follow this pattern, Nick could not recall and did not know.

Two SF1 players solved fraction divisions incorrectly and one SF1 player left the problems blank. Nash and Maya converted the divisor into “divisor over one” as the first step.
Then, to solve $3/4 \div 2$, Nash multiplied the numerators and the denominators respectively, getting $6/4$ as the answer. He was uncertain about the calculating process but did not know how to solve it otherwise. Not remembering the specific procedures, Maya tentatively solved $2/3 \div (2/1)$ by cross-multiplying the numerators and the denominators and got $6/2$ or 3 as the result. She knew that the result was wrong since dividing a number by an integer should generate a smaller number, but she did not know how to correct it. Zoe did not know or remember how to divide a fraction by an integer and did not make any attempt at problem-solving.

### 6.3.4 Post-game Fraction Divisions

Little improvement was observed in the participants’ problem-solving of fraction divisions. Five participants, all SF2 players, who previously consistently and correctly divided fractions using “keep, change, flip” still resorted to the same strategy after the gameplay. And they still tried to match their explanations and visual illustrations to the calculations, instead of relying on sense-making of the problem-solving. Similarly, another SF2 player, Nora, still drew on “dividing straight across” after the gameplay. There was no improvement in Nash’s (SF1 player) problem-solving as he did not know how to divide a fraction by an integer either before or after the gameplay. Because solving other operations in detail consumed too much time and energy and because the games did not directly target fraction divisions, Ada, Harry, Merik, and Maya (all SF1 players) were not asked to solve fraction divisions in the post-game interviews. It was unclear whether they had changes in their capability to solve fraction divisions. Kate was not prompted to solve fraction divisions during the pre-game interview, because much time was spent in solving other fraction problems. During the post-game interview, she successfully divided a fraction by an integer and explained it properly.
Two participants’ post-game problem-solving exhibited some progress since they sometimes successfully solved and explained fraction divisions. Zoe (SF1 player), who previously had no idea how to divide fractions, managed to solve and explain $\frac{3}{4} / 3$ correctly after playing SF1. She split a square into fourths, marked three pieces “$\frac{1}{4}$” and cut the last fourth into three equal parts (Figure 48). When asked to explain her problem-solving, Zoe said, “dividing it by three, wouldn’t this just be one?” She concluded the result was one-fourth. It appears that Zoe calculated the division by thinking in terms of one-fourth while ignoring the unit. Even though Zoe’s knowledge of fractions suggested by the pre-game interview was much weaker than many other participants, her way of solving and explaining $\frac{3}{4} / 3$ in the post-game interview was the simplest, for she did not draw on the detoured “keep, change, flip” procedure in this situation.

**Figure 48**

*Zoe’s post-game illustration of $\frac{3}{4} / 3$*

![Zoe's post-game illustration of 3/4 / 3](image)

*Note.* Zoe drew three one-fourths and split the excluded upright one-fourth into three equal parts. Her problem-solving was dividing the circled-out three one-fourths into three equal parts, not using the excluded one-fourth or the pieces within.

Nick (SF2 player) attempted to explain fraction divisions, but no clear conclusion could be made about his problem-solving. For $\frac{1}{2} / 2$, Nick first thought that $\frac{1}{2}$ was equivalent to 2 and thus the final answer was 1. With assistance and after being suggested to think of the
problem visually, he recognized that the expression represented breaking down one-half into two equal parts and that one-fourth was the result. When asked to visually show \( \frac{1}{3} / 2 \), Nick shaded a second one-third and thought that the result was two-thirds or one-sixth. Nick appeared unclear about the difference between multiplication and division. Although Nick’s problem-solving of fraction divisions was unreliable, he attempted to make sense of them rather than merely relying on procedures, which could be considered as some improvement from his pre-game performance.

### 6.3.5 Pre-game Fraction Additions/Subtractions

For fraction additions/subtractions, one SF1 player and five SF2 players could consistently correctly solve them by converting the fractions to have a common denominator, adding/subtracting the numerators, and carrying over the common denominator. Three SF2 players among these six participants could explicate why common denominators were needed for fraction additions/subtractions. Among the eight participants (five SF1 players, two SF2 players, and Kate) who could not consistently correctly add/subtract fractions, two SF2 players and Kate sometimes could correctly figure out the answers but were uncertain about their problem-solving. Five SF1 players could not figure out the correct answers.

It was still Tim, Austin, and Neo (all SF2 players) who could solve fraction additions/subtractions and explain the necessity of finding common denominators. Tim thought that “different denominators mean different portions of objects” and common denominators were needed to find out precise results. However, when illustrating \( \frac{1}{2} + \frac{1}{3} \), Tim first drew \( \frac{3}{6} \) of one and \( \frac{1}{3} \) of a half and got \( \frac{4}{6} \) as the answer (Figure 49). With assistance, Tim corrected his illustration and solution (Figure 49). Austin explained that adding fractions with different denominators would not make sense and that the fractions needed to be split up into “equal
parts” so that the two quantities could be measured by a “fair amount of piece”. At first, Neo forgot about how to add fractions, but he knew that adding the numerators and denominators respectively would be wrong. When asked for more clarification of $\frac{1}{2} + \frac{1}{3}$, Neo drew one-half and one-third separately and pointed out that the total amount was certainly more than two-fifths. Then Neo remembered the procedure and solved the problem. According to him, a common denominator was necessary because the initial illustrations of one-half and one-third could not be “layered on top of each other” and the fractions needed to be “all confined to one chart” to make the resulting amount “easier to see”.

**Figure 49**

*Tim’s pre-game illustrations of $\frac{1}{2} + \frac{1}{3}$*

Familiar with the procedures used for adding/subtracting fractions, one SF1 player (Nash) and two SF2 players (Nora and Hannah) could consistently and correctly solve these problems. But they could not explain why common denominators were needed. In their justification, “That is just the way I learned. That is the rule I was taught.” In contrast to these three participants, two SF2 players (Nick and Alana) and Kate were not as familiar with the procedures or rules. They might correctly solve fraction additions/subtractions sometimes, but there was a lack of evidence suggesting reliable performance. Kate knew that common denominators were needed for fraction additions/subtractions. Given her statement “I do not know how to get the same denominator in fractions,” Kate was not asked to solve specific addition/subtraction problems. Earlier when
working on $2 \times \frac{1}{3}$, she had difficulty in figuring out $\frac{1}{3} + \frac{1}{3}$ being equal to two-thirds on her own. So, although Kate knew the use of common denominators in adding/subtracting dissimilar fractions, it was unclear whether she could consistently and practically work through fraction additions/subtractions. Initially, Nick got $\frac{3}{3}$ for $2 + \frac{1}{3}$, for he added the integer and the numerator and carried over the denominator. He explained that the integer was not converted to the fractional form because cross operation only took place in multiplication. It appeared that he misused the rule for multiplying an integer by a fraction in the adding situation. Nick felt uncertain about this procedure or his answer. After being pointed out that he got $\frac{3}{3}$, which was equivalent to 1 and bigger than 2, Nick realized his mistake, corrected his problem-solving, and got $\frac{7}{3}$. When prompted to solve $\frac{1}{2} + \frac{1}{3}$, Nick knew that the common denominator was 6 but he was not sure whether he was supposed to use the common denominator.

Alana properly added and subtracted dissimilar fractions. She was firm about converting the fractions to have the same denominator, “You cannot just add across (dissimilar fractions) … because they are two different fractions. You can only multiply across (fractions).” When subtracting fractions, Alana knew that she could focus on the numbers of parts once the two fractions were measured in the same way. Based on Alana’s explanations, she seemed to understand the rationale of finding a common denominator when adding/subtracting fractions, but her uncertainty about when to apply “keep, change, flip” interfered with her problem-solving of addition and left her uncertain about it. After converting $\frac{1}{2} + \frac{1}{3}$ to $\frac{2}{6} + \frac{3}{6}$, Alana voiced her concern, “I don’t know if I have to turn it into multiplication, like ‘keep, change, flip’ again.” Surprisingly, Alana solved fraction subtraction without similar concerns.

Five SF1 players incorrectly solved or were unable to solve fraction additions or subtractions. When adding an integer and a fraction, Ada added the integer to both the
denominator and the numerator, thinking that the answer to $2 + 1/3$ was $3/5$. Harry and Merik knew that the sum of an integer and a fraction was equal to a mixed number, but neither knew how to convert a mixed number to an improper fraction. Zoe also could not solve such additions, “Do you just add across? I don’t remember.” Concerning additions of two fractions, Ada applied “keep, change, flip” by keeping the first number, changing the sign from addition to multiplication, and flipping the second number. Following this rule, to solve $1/2 + 1/3$, Ada changed the problem into $1/2 * 3/1$ and got $3/2$ as the answer. It appeared that Ada mistook the procedure for fraction divisions as the procedure for fraction additions. Meanwhile, considering Ada changed addition to multiplication, she might lack the understanding of “addition and subtraction” and “multiplication and division” being two pairs of inverse operations. Feeling unsure, Zoe did not try to solve fraction additions or subtractions. Harry and Merik remembered the procedure being adding/subtracting the numerators and the denominators respectively. Therefore, Harry got $3/6$ or $1/2$ when calculating $1/2 + 2/4$, and Merik got $1/1$ or $1$ when solving $2/3 – 1/2$. Maya applied the same procedure when adding two fractions, but she did not know how to do subtractions. Apart from this method, Merik also tried “keep, change, flip”, which happened to result in $1$ as well (his specific problem-solving process was $2/3 – 1/2 = 2/3 + 2/1 = (2+2) / (3 + 1) = 4/4 = 1$) and strengthened Merik’s confidence in his answer. Even though Ada and Maya solved fraction additions/subtractions incorrectly, they remembered the term "common denominator". In Ada’s words, "If they (the denominators) are the same, you could carry it over and just do the top". Unfortunately, it was not clear to them when fractions needed to have the same denominator.
6.3.6 Post-game Fraction Additions/Subtractions

Two SF1 players and five SF2 players improved in explicating their problem-solving process of fraction additions/subtractions, and four SF1 players did better in solving some fraction additions. Among the three SF2 players (Tim, Neo, and Austin) who could consistently correctly solve and explain fraction additions/subtractions before the gameplay, one person demonstrated an improved explanation of problem-solving after the gameplay. Tim and Neo were not asked to solve such problems in the post-game interview since their explanations in the pre-game interview already suggested a solid understanding of fraction additions/subtractions. Before the gameplay, Austin verbally and procedurally explained how he added fractions with different denominators, but his illustration only included two separate drawings of one-half and one-third (Figure 50a). After the gameplay, merely using illustrations, Austin showed and solved $1/4 + 5/6$ (Figure 50b). He drew two rectangles as two ones and marked one-fourth and five-sixths in each rectangle. Then he made vertical cuts to split each fourth into three equal parts and a horizontal cut to split each sixth into two equal parts. Having converted both fractions into twelfths, Austin answered that adding three-twelfths and ten-twelfths together would be thirteen-twelfths.

Figure 50

Austin’s pre- and post-game illustrations of fraction additions
Note. In Austin’s post-game drawing (b), the left rectangle was initially split into fourths with a vertical cut and a horizontal cut. To create twelfths, Austin made two vertical cuts (one cut was missed and added later) in the left/right half. The right rectangle was initially split into sixths with five vertical cuts. Four parts were shaded by mistake since verbally he expressed the intention of shading five parts to indicate five-sixths.

Before the intervention, one SF1 player (Nash) and two SF2 players (Nora and Hannah) could proficiently correctly solve fraction additions/subtractions but could not decipher the procedures. After playing the games, they all exhibited a better comprehension of the procedures, especially about finding common denominators. Before the gameplay, Nash could correctly solve additions or subtractions of fractions, but he could not explain why common denominators were needed. In contrast, during the post-game discussion, Nash began to view fractions with different denominators as fractions in different groups. In his terms, fractions were about “grouping” and common denominators were the “connections” between different fractions.

One-half is one group. Three-fourths, that's another group...Certain things don't fit...If you do (add) a fraction, you have to fit it into a group...At the moment (about $1/3 + 1/2$) they don't have the same group, so they don't go with each other...We make them into the same group (by changing both into sixths).

Hannah played SF2, but her improvement in understanding was similar to Nash’s. In the post-game interview, Hannah knew that converting the fractions to have the same denominator was necessary for finding an accurate answer to fraction additions/subtractions. It was insensible to directly add/subtract fractions with different denominators, for “the sizes of the boxes” (the basic units of the two fractions) were unequal. Nora was not directly asked to solve or explain fraction additions/subtractions during the post-game interview. However, her general discussion
about fractions suggested an understanding of common denominators. Based on a square that was first partitioned into two halves and then into four-fourths, Nora explained that taking one block away from the four blocks resulted in three-fourths and that adding one extra block of the same size to the four blocks resulted in five-fourths. Although the original square (Figure 40, p.99) could be considered in ones, halves, or fourths, Nora intentionally talked about the addition and the subtraction all based on one-fourth and her discussion moved from mechanically describing the steps to explaining the meaning based on a visual representation. Hence, Nora’s explanation of fraction additions/subtractions was deemed to have improved.

Two SF2 players and Kate sometimes could correctly solve fraction additions or subtractions before the gameplay. The two SF2 players better explained additions/subtractions of different fractions to some extent after the gameplay, while Kate had no change. Nick’s post-game interview took place several days after he played SF2, so he vaguely recalled the game at first. He solved fraction addition by adding the numerators and the denominators separately. Then he was prompted to play a few selected SF2 levels to help him recollect the gameplay. Later, he expressed that finding common denominators was necessary for precisely comparing two fractional quantities and solving fraction additions/subtractions. Alana used to feel confused when working with fractions with different denominators. After the gameplay, she felt that she could better deal with them. When asked to compare three-fourths and two-thirds, Alana thought that the latter was bigger since it took up more space. The conclusion was incorrect, but Alana attempted to visualize the two quantities and compare their sizes accordingly, which was an improvement. When pushed to figure out their difference, Alana knew that in the game she would cut them up and make their denominators bigger. But, outside the game context, she was not clear about the precise steps to take. Although her problem-solving was incomplete, Alana’s
understanding of the situation and the process seemed to have improved. Kate knew that common denominators were needed for adding/subtracting fractions before the intervention. She remained unclear about why this was the case after the intervention.

Five SF1 players could not figure out the correct answers to fraction additions or subtractions in the pre-game interviews. After playing SF1, three of them did somewhat better in their problem-solving; Ada learned some addition facts from the game; Merik’s problem-solving did not change but he attempted to make a diagram. Maya did better in both problem-solving and explaining it. At first, she still solved $1/2 + 1/3$ by adding the numerators and the denominators respectively (Figure 51), in the same way that she solved it before the gameplay. After reflecting on her gameplay, she realized that they could not “go together” and needed to be broken up (Figure 51). Maya split one-third into two one-sixths. About one-half, she first wanted to break it up into two one-fourths but recalled that in the game she broke it up into three one-sixths. By adding up two-sixths and three-sixths, she got five-sixths as the result. Considering the two different solutions, Maya felt that $5/6$ was correct because the visual representation made sense.

Figure 51

*Maya’s post-game solution without and with the game in mind*

Previously Harry’s and Zoe’s problem-solving of adding/subtracting fractions had nothing to do with finding common denominators. After playing SF1, they both knew that
fractions with different denominators did not “go into each other”. To Harry, “the math game really just screwd up my adding.” Although he vaguely remembered that he needed to do something before trying to add things together in the game, he could not recall or describe the process. Zoe remembered from the game that to add fractions “the denominators have to be the same; (the fractions) had to be out of the same amount”. Even though they remained incapable of solving the problems independently, their conceptual understanding of how to solve the problems improved.

Ada remembered some simple addition facts from the gameplay, but the recollection might be incorrect. For example, Ada recalled that \( \frac{1}{4} + \frac{1}{4} = \frac{1}{2} \) and that three sets of one-sixths added up to three-sixths or three-eighteenths. She also thought that \( \frac{1}{3} + \frac{1}{3} = \frac{1}{6} \). About the addition of an integer and a fraction, Ada’s problem-solving method remained the same and was incorrect. Merik continued adding the numerators and the denominators separately when solving \( \frac{1}{2} + \frac{1}{4} \). In his attempted visual representation, he cut a whole into two halves and then cut the right half into four equal slices creating one-fourth of a half rather than that of one whole (Figure 52). Merik felt weird about drawing it out and struggled to connect the drawing to his problem-solving. Although the drawing was incorrect in representing the problem, it could provide the basis for a discussion and contained opportunities for developing a better understanding of fractions.

**Figure 52**

*Merik’s post-game illustration of \( \frac{1}{2} + \frac{1}{4} \)*
6.3.7 Summary of Findings and the Gameplay

Across different operations with fractions, six SF2 players could consistently apply appropriate algorithms regardless of the gameplay. Specifically, Alana’s procedures for multiple operations included additional conversions that did not impact the overall correctness (e.g., converting two fractions to have the same denominator before applying “keep, change, flip” when solving fraction divisions) so her final results remained correct. Nora and Hannah could effectively utilize various algorithms but not make sense of them. Tim, Austin, and Neo could illustrate and sensibly explain different operations with fractions except for complicated fraction divisions. Among these six participants, Hannah, Austin, and Alana improved in explaining fraction additions/subtractions after playing SF2; Nora did better in visually representing and clarifying fraction multiplications and adding/subtracting dissimilar fractions; no change was observed in Tim’s or Neo’s responses. The other participants’ fraction knowledge was more fragmented, as they recalled and applied inappropriate or wrong procedures in various operations with fractions. After the gameplay, for four participants, the degree to which their knowledge was fragmented and inconsistent reduced as they improved in making sense of certain operations with fractions and thus were able to solve certain problems properly through the sense-making approach. Specifically, Zoe (SF1 player) and Kate (who played both games) improved in solving and explaining at least two types of operations with fractions. Ada and Maya (both SF1 players) performed better in solving fraction additions and interpreting fraction multiplications. Nash (SF1 player) and Nick (SF2 player) better explained why fractions needed the same denominator during additions/subtractions. There were some changes in Harry’s and Merik’s problem-solving, but the changes were minor, and their improvement was limited.
The gameplay data could not reveal the improvements in explanations as the participants seldom verbalized their thinking during the gameplay. Nonetheless, the gameplay data could elucidate the improvements in problem-solving if we evaluate the extent to which fraction problem-solving in the gameplay is consistent with that in the post-game interviews. This section presents five gameplay episodes by Zoe, Maya, and Kate, who had significant improvements in solving various operations with fractions.

During the pre-game interview, Zoe had incorrect ideas about how various fraction operations were carried out and mixed up the procedures for addition and multiplication. When playing SF1, Zoe repeatedly demonstrated her capability to solve these problems. For instance, she solved SF1.IV.14 (Figure 53a) on her first try. To achieve this, she needed to know that two one-thirds were equivalent to four one-sixths \((1/3 \times 2 = 1/6 \times 4)\) and that one-third was equal to two-sixths. It was unclear whether she was thinking about “four one-sixths” in an additive \((1/6 + 1/6 + 1/6 + 1/6)\) or multiplicative \((1/6 \times 4)\) way. Nonetheless, it appeared that she could solve simple fraction addition or multiplication when she reached SF1.IV.14.

As another example, both Zoe and Maya independently solved SF1.VI.18 (Figure 53b) with their knowledge of fractions. Due to the game setting at this level, one must slice two one-sixths from the one-half on the left, since the two-sixths need to merge with the fallen chunks to form four-sixths first before being pulled to resolve the hidden lava. During the gameplay, before slicing one-sixth off one-half, Zoe hesitated a little bit but still decided to do so. Maya did this without any hesitation. The action suggested their knowledge that one-third of one-half was one-sixth \((1/3 \times 1/2 = 1/6)\) or that one-half divided by three led to one-sixth \((1/2 / 3 = 1/6)\). After slicing off one-sixth, they both knew that the remaining one-third needed to be sliced to release another one-sixth \((1/3 / 2 \text{ or } 1/2 \times 1/3 = 1/6)\). Meantime, neither sliced one-half on the right or
two-thirds, which suggested their knowledge that the resulting quantities from slicing them, one-fourth and one-third respectively, could not be directly added to two-sixths. The fraction knowledge Zoe’s and Maya’s gameplay suggested was consistent with their performance and responses during the post-game interview.

Figure 53

*SF1.IV.14 and SF1.VI.18 setups*

Kate was another participant who could not solve or explain various fraction operations during the pre-game interview. Although Kate still struggled to solve and explain fraction additions during the post-game interview, when playing SF1.VI (“Addition”) and SF2.III (“Common Denominator”), she actively drew on her understanding and solved 64% and 80% of puzzles completely on her own. When solving SF1.VI.23 (Figure 54a), Kate initially tried to slice off the entire one-half, for she knew that one-half was equivalent to three-sixths. Once she noticed that only two one-sixths (1/6 * 2 or 1/6 + 1/6) could be sliced off the one-half ice, she sliced off one-third from the two-thirds ice on the left. Then she quickly sliced the one-third into two equal parts and tapped the bubble so that only one-sixth could join the other two pieces to form the needed three-sixths, in striking contrast to her pre-game interview performance where she struggled to find out \( \frac{1}{3} + \frac{1}{3} = \frac{2}{3} \). Also, Kate was the sole participant solving this puzzle independently with her existing understanding among the five participants who played this level.
This episode showed at least her knowledge of basic fraction equivalence ($1/2 = 3/6$), fraction divisions/subtractions ($2/3 / 2$ or $2/3 - 1/3 = 1/3$; $1/3 / 2$ or $1/3 - 1/6 = 1/6$) and additions ($1/6 + 1/6 + 1/6$ or $1/6 * 3 = 3/6$).

**Figure 54**

*SF1.VI.23 and SF2.III.11 setups*

To solve SF2.III.11 (Figure 54b), Kate first tapped the narwhal on the right side of the ice and saw three one-fourths being cut into six one-eighths. Then she withdrew this narwhal’s horn and used the left narwhal to cut the ice into nine one-twelfths. Without using either narwhal around the lava that was in thirds, Kate sliced off three one-twelfths ice, resolved part of the lava, and left five-twelfths lava behind. After tapping the left narwhal again and changing the ice from three two-twelfths to six one-twelfths, she made two cuts and dropped four-twelfths and one-twelfth ice respectively to clear the path. The fact that Kate only used one narwhal to convert ice into twelfths and not eighths suggested her knowledge about the relatively more straightforward connection between twelfths and thirds, as compared to the connection between eighths and thirds. Both episodes indicated Kate’s capability to convert fractions to have a common denominator to get ready for the subsequent comparison/addition/subtraction.

SF2.III.15 (Figure 55a) was one of the more challenging levels. On Kate’s first try, she sliced one-fourth off one with the help of the leftist narwhal (Figure 55b) and quickly realized
that it would not work. On her second try, she tapped the center narwhal to create thirds, knowing that the thirds could help create ninths. After slicing off two one-thirds or two-thirds (Figure 55c), the down-facing narwhal was used to convert two-thirds into three two-ninths (Figure 55d). Then she sliced off two nine-fourths to resolve four-ninths lava on the ground. In this episode, Kate demonstrated her understanding of the multiplicative relationship between thirds and ninths and the lack of this relationship between fourths and ninths. In addition, her actions suggested her knowledge that two one-thirds \((1/3 \times 2)\) were sufficient to create four-ninths \((1/3 \times 2 > 4/9)\) and that two two-ninths were equal to four-ninths \((2/9 \times 2 \text{ or } 2/9 + 2/9 = 4/9)\). The fraction multiplication and division knowledge suggested in her gameplay and explicated in her post-game interview was consistent.

**Figure 55**

*SF2.III.15 setups and Kate’s solutions*

*Note.* Before making slices, the two narwhals around “1” converted the one whole into four one-fourths and three one-thirds respectively (similar to d). When slicing, the narwhal’s horns withdrew, and the resulting quantities were shown. A player could either slice two-thirds off by
using the center narwhal once and making one cut (b) or by using the center narwhal twice and making two cuts (not shown; then two one-thirds would slide off).

The gameplay and the interview responses were not always congruous. During the post-game interview, Kate admitted that she did not understand mixed numbers before and that the game helped her understand mixed numbers as two quantities being added together. However, during the gameplay, Kate was the only participant who solved all the puzzles concerning improper fractions and mixed numbers (SF2.III.20-28) without any aid from the game. Her gameplay did not disclose any clue that suggested a lack of understanding of mixed numbers.

6.4 Views of Math Learning

Earlier in 6.2.1, the participants’ recollections about learning fractions, a particular math topic, were presented. This section focuses on one’s subject-specific perceptions. Regardless of the specific criterion used, there was no change in the participants’ self-identification as a “math person” (or not) before and after the gameplay, because the gameplay was limited in terms of the math topic covered and the length/intensity. Nonetheless, the participants would like to see more SVGs like SF1 and SF2 in formal math education.

One interview question asked whether a participant considered oneself to be a “math person”. Two different ideas about what qualifies someone as a math person emerged from the participants’ responses. Based on the responses of eleven participants (five SF1 players, five SF2 players, and Kate who played both games), one evaluation criterion was whether one could quickly grasp every math topic and easily be good at doing math. Using this criterion, three participants considered themselves as math persons. Nash and Tim thought so because their math performance was relatively higher than their performances in other subjects at school. Another three participants (two SF1 players and one SF2 player) firmly self-identified as non-math
persons. Surprisingly, Neo, one of the highest performers among the participants, was in this group. Although his knowledge of fractions was solid, he admitted that he lacked confidence in dealing with math and was not comfortable with it. Five participants (two SF1 players, two SF2 players, and Kate) felt that they were math persons to some extent, depending on the specific math topics. For instance, Kate claimed to have a “love-hate relationship” with math. While pre-calculus was loved, fractions were part of the hated. Ada’s current answer was “Maybe I am.” According to her, this was influenced by her EOP instructor who actively advocated the idea that “To be a math person, do math and be a person.” Also, her confidence in math recently increased because of the validation and encouragement from a teaching assistant.

According to six participants (one SF1 player and five SF2 players), another criterion of being a math person was having a high level of enjoyment and motivation in learning math. With this assumption, none of the six participants whose expressions suggested this criterion identified as a math person. Nora and Tim recognized that they were good at math, but they did not enjoy it to the degree of actively seeking it in their free time. In contrast, Tim actively participated in the online communities of some games and tried to attend in-person events. Hannah considered math less useful or understandable as it got more advanced. Alana hated math before and was not motivated to learn more math, but she enjoyed math in that academic year because of the math teacher she had. In Alana’s opinion, her teachers in the past used to move on without trying to support students to develop an understanding. On the contrary, her current teacher offered plenty of office hours and opportunities to correct mistakes, which made her feel that the teacher cared and that she “got the chance to understand” (in her own words).

After the gameplay, when asked again whether they considered themselves math persons, the participants reported no change in their answers. Nonetheless, three SF1 players (Ada, Zoe,
and Harry) and Kate felt slightly more comfortable and confident with fractions. Maya (SF1 player) and Hannah (SF2 player) still disliked math, but the game made it more enjoyable. To Harry (SF1 player), Neo (SF2 player), and Kate, fractions were only one of many math topics and the gameplay experience was insufficient to change their confidence in learning the entire subject. However, Neo’s confidence in explaining math increased. In the opinion of Tim (SF2 player), math was not involved as heavily as strategy in SF2. Merik (SF1 player) attributed the lack of change to the limited gameplay that did not significantly impact his understanding of fractions and lasted only one hour. Incidentally, Nora (SF2 player) gained more interest in games because playing SF2 elicited her competitiveness and motivation to make improvements.

All of the participants would like to see more SVGs like SF1 and SF2 in formal math education. They brought up various ways for SVGs to be incorporated into classroom teaching and learning. For instance, if the games were introduced and played before students learned the formal math concept, gameplay could serve as a warm-up activity to intrigue students’ interest and to “have something to relate to when formally learning the subject matter”. Alternatively, from the participants’ perspectives, students could play SVGs after learning the basics of a math topic, as the games could supplement lectures and bring “a different type of learning”. The participants thought that playing such games might be particularly helpful for visual learners who fell behind during conventional lectures. In the participants’ discussions, the third way of incorporation was to intertwine gameplay and traditional instructions, each time playing the parts of the games closely related to the specific math topic to be learned or just learned. Furthermore, the participants suggested that teachers informally evaluate students’ understanding using the games. Students could discuss or write out what they learned in the games, based on which
teachers could devise subsequent instructions. Finally, if the games were used after fractions were thoroughly taught, they could be an assessment tool or a refresher.
7 Discussion

Slice Fractions 1 & 2 are well-designed mobile serious video games (SVG) with many design features that elevate their potential in supporting basic mathematics learning. This study investigates the Educational Opportunities Program (EOP) college freshmen’s experience with SF1 and SF2 and the impact of the gameplay on their knowledge of fractions, fraction problem-solving, and views of math learning. Pre-game and post-game interviews, assessments, and gameplay of fourteen participants were collected. Based on their pre-game interview responses and assessment performances, six participants played SF1, seven played SF2, and one participant (Kate) played parts of both games. The next four sections featured summaries and interpretations of the findings on each research question. Then, this study’s limitations, implications, and directions for future research are discussed.

7.1 Discussion of Findings on Experiences and Views of the Games

Most participants (all six SF1 players and four SF2 players) were or are avid video game players, and three SF2 players and Kate merely played casual mobile games infrequently. No participant recalled prior experience with well-designed SVG where the subject matter and the game mechanics were intertwined. Within SF1 and SF2, the average number of tries it took to solve a puzzle rose and fell somewhat regularly across the fractional puzzles. This pattern was likely to be the result of the interaction between two game design features. First, periodically the game presents a new game element/mechanic or piece of fractional knowledge. Due to a lack of instruction, it might take trial and error to learn how the new element/mechanic works or the fractional knowledge, hence the increases in the average tries. Second, immediately following a puzzle involving a new game or fractional element are one or a few puzzles to reinforce the newly presented knowledge and skills, which contribute to the decreases in the average tries. The
alternation of these two game design features is consistent with the video game design principles for flow experience (Kiili et al., 2012). By alternating the presentations of new things with the opportunities to practice, a dynamic balance is maintained between a player’s skill level and the game’s difficulty, maximizing the possibility of providing a flow experience.

Comparing Nash, Harry, and Zoe’s regular SF1 gameplay and additional SF2 gameplay, their average try/puzzle in SF2 was smaller than that in SF1 while their speed remained roughly the same. Also, these three participants more frequently relied on knowledge of fractions to independently solve fraction puzzles in SF2 than in SF1, where they more often used trial-and-error and visual feedback within the game. Furthermore, these three participants, lower math performers in the pre-game interview and assessment, exhibited lower or similar average try/puzzle, faster gameplay speeds, and similar self-reliant puzzle-solving rates in SF2 than the seven SF2 players who were considered higher math performers pre-game. These findings suggest that some EOP freshmen might not experience SF2 as being more difficult than SF1, unlike the conclusion derived from the theoretical analysis of the games. On the other hand, since Nash, Harry, and Zoe played SF2 after playing SF1, SF1 gameplay might have primed their fraction knowledge and skills and thus eased their SF2 gameplay.

Considering the participants’ prior experience with video games, regardless of game, the participants who were more familiar with video games were faster in their puzzle-solving. This is sensible because experienced gamers might be used to assessing different situations in a game and acting as soon as possible. Among SF2 players, the experienced gamers used fewer tries to solve fraction puzzles than the inexperienced gamers. Again, the players’ prior game experience might support them in making sense of novel game elements and mechanics faster, so it took them fewer tries to fully comprehend and solve the puzzles. No such difference was observed
among SF1 players, as the gamers’ mean try/puzzle was slightly larger than the non-gamer’s. Considering there was only one non-gamer who played SF1, the result might be peculiar and unrepresentative. It appears that the self-reliant puzzle-solving rate was independent of a player’s (lack of) prior video game experience, as gamers and non-gamers of each game drew on one’s knowledge of fractions in puzzle-solving at similar frequencies.

Similar to prior findings based on SF1 (Gresalfi et al., 2017), in this study, SF1 and SF2 were enjoyed by the players as these were considered playful and engaging. On the other hand, a few EOP freshmen who were experienced gamers considered these games not challenging enough because of limited game elements, plenty of support, and slow-paced animation, in contrast to previous findings where third graders played SF1 and gave no such feedback (Gresalfi et al., 2017). This is reasonable since the games were designed with K-12 learners, gamers or not, in mind. Also, as SVGs, SF1 and SF2 need to balance playfulness with the subject matter. Therefore, older players experienced in playing complicated video games might consider SF1 and SF2 not captivating enough.

Regarding SF1 and SF2 as math learning tools, the EOP freshmen found the visualizations and interactions helpful in better understanding fractions and fractional processes. This echoes Cyr et al.’s (2019) finding that playing SF1 enhanced students’ capability to visualize fractions and supported students to perform better on the test items where diagrams were not given. It is also consistent with Zhang et al.’s (2020) finding that playing SF1 or Motion Math enabled students to analyze and solve more complicated test items. The EOP freshmen heavily appraised the interactivity in these games as it made fraction learning active and pleasant, in line with Gresalfi et al.’s (2017) finding that students who played SF1 or Motion Math reported higher levels of enjoyment and their observation that all students opted to play
SF1 in their free time. Distinct from previous research that focused less on the players’ experiences, this study shows that playing SF1 or SF2 was powerful to some EOP freshmen as the games provided an eye-opening and positive math learning experience. The participants also voiced some criticisms or concerns about using these games as math learning tools, but the negative comments were isolated.

Perhaps because SF1 was easier and more basic, four SF1 players mentioned that the game could be used to introduce fractions, while only two SF2 players considered so. Nine participants felt that at least some formal teaching of fractions was required before playing the games. This opinion might have to do with their past math learning experiences where exploration was limited, and formal notations and rules were directly taught. In the interview, after some pondering, one participant (Kate) thought that players who had not learned fractions probably might go through many levels by observing and following the patterns.

During a personal trip, my cousin, a first grader in China, played SF1.II. She was familiar with counting, addition, and subtraction, but had not learned multiplication or division yet. Nonetheless, she managed to finish SF1.II, where the basic fraction setup (a numerator over a denominator) was introduced. Not knowing fractions, she read fractions as “one (over) two” or “one (over) four”. This showed that formal knowledge of fractions was unnecessary for solving the basic puzzles in SF1. It might be interesting if future studies could investigate young learners’ gameplay and informal learning of fractions through playing SF1 and SF2. Arena and Schwartz (2013) found that community college students benefited from playing a video game when instructions were purposefully designed to build on the statistical intuitions gained from the gameplay. Neither earlier empirical studies involving SF1 (Cyr et al., 2019; Gresalfi et al., 2017; Zhang et al., 2020) nor the current study included tailored discussions or instructions. It
might be fruitful for future research to examine instructional designs that could help learners develop intuitive ideas about fractions acquired from SF1 and SF2 into formal knowledge of fractions.

7.2 Discussion of Findings on Recollections, Attitudes, and Conceptions of Fractions

Based on the participant’s recollections, diagrams or pictures were involved when fractions were introduced. However, operations with fractions were exclusively taught and learned abstractly in the form of memorizing and following standard rules or procedures. Only two participants remembered using manipulatives and drawing diagrams. No one recalled that their teachers clarified the meanings or the rationales of the formal algorithms. Two female participants voiced their strong dislike of following rules and not understanding mathematics. Two male participants explicitly preferred standard algorithms and considered them more efficient than solving fraction problems with drawings.

The participants’ accounts of past experiences in learning fractions aligned with their problem-solving practices, since recalling and applying memorized procedures were dominant. The emphasis on steps, rules, or procedures implied an absolutist, static, and objective view of mathematics knowledge (Depaepe et al., 2016), and implied math teaching and learning to be a knowledge transmission process, with teachers being authorities and imparting knowledge to students. On the other hand, a few participants lamented their past math learning experiences. They attributed their lack of understanding of certain math topics mostly to teachers’ pedagogy. This suggested that the participants were at least implicitly aware of the possibility of learning math in a sensible way that was more in line with a constructive perspective of mathematics knowledge and math learning (Depaepe et al., 2016).
Before the gameplay, about half of the participants, primarily low math performers who played SF1, were uncomfortable and unconfident in solving fraction problems. After the gameplay, the participants reported either a slight increase or no change in their comfort and confidence level toward fractions. One participant reported increased confidence in explaining and illustrating fractional problem-solving. The limited change is reasonable for at least two reasons. First, the gameplay intervention only lasted for one hour, which was negligible compared to the lengthy hours the participants spent learning and practicing fractions in schools. Second, there was no systematic facilitated reflection or discussion about the math in the games or the connections between formal algorithms and the gameplay. Solving visual fractional problems in an interactive game environment with plenty of feedback differs from solving abstract fractional expressions using standard algorithms. Therefore, the positive experience with fractions in the gameplay might not be enough to immediately lead to more confidence and comfort in tackling conventional fractional problems.

In this study, playing SF1 or SF2 was determined by the researcher based on a quick informal evaluation of a participant’s pre-game interview responses. SF2 players were supposed to be higher math performers than SF1 players. This was confirmed in the formal evaluation of the participants’ assessment scores since those who played SF2 had constructed more fraction schemes/operations than those who played SF1 in either pre- or post-game assessments. Both SF1 and SF2 players improved in the assessment of fraction schemes/operations, but the improvement was statistically insignificant (SF1 players) or marginally significant (SF2 players). This might have happened because of the small sample sizes and the large variances, especially among SF1 players. Half of all participants (five SF1 players, one SF2 player, and Kate who played both games) had a sizeable increase, at least 6%, in their assessment scores. Interestingly,
one SF1 player’s (Harry) assessment score decreased by 21%. The decrease in his performance might be caused by the relatively higher increase in the difficulty of the assessment items compared to the limited growth in his knowledge of fractions. For example, considering his weak grasp of equal partitioning, one item in the pre-game assessment was adapted and he was asked to mark “1/4” (rather than “1/5”) of a circle. The corresponding item in the post-game assessment required him to mark “1/7 or 1/3” of a circle. The participant properly solved the easier item in the pre-game assessment but not the harder one in the post-game assessment. If the pre-game item had not been adjusted, he probably would not have scored on the item either before or after the gameplay. Because of the adaptation, his performance on this item appeared to have decreased after the gameplay.

A closer examination of the participants’ scores on each fraction scheme/operation showed that, after the gameplay, the previously statistically significant differences between SF1 and SF2 players’ performances in the part-whole scheme (PWS) and the splitting operation (SO) were no longer significant. Also, SF1 players significantly increased their scores on the iterative fraction scheme (IFS). Based on these findings, playing SF1 seemed particularly helpful for the constructions of the PWS, the SO, and the IFS. Furthermore, increases in mean scores were observed among SF1 players in the partitive fraction scheme (PFS) and among SF2 players in the SO, the IFS, and the reversible partitive fraction scheme (RPFS). However, these increases were not significant, probably influenced by the small sample size and the large variance of each group. Since SF2 players’ pre-game scores on all fraction schemes/operations except the RPFS were high (3.5 or higher out of 4), the room for improvement was limited and thus the impact of playing SF2 on fraction schemes/operations might not be fully revealed in this study. In future research, learners with weak knowledge of fractions could be invited to play SF2 to further test
its influence on players’ construction of various fraction schemes/operations. Only one participant played parts of both games. So, there was not enough data for explorations concerning the differences between playing only one game and both games. Lastly, at first glance, it was surprising that SF1 players scored lower in the RPFS after the gameplay. An examination showed that this decrease occurred because two SF1 players solved an RPFS item before but not after the gameplay. The pre-game RPFS item was adjusted at the time of the assessment (“The piece of pie shown below is 1/2 [or 1/3] as big as your pie. Draw your pie.”) to be much easier than the corresponding post-game RPFS item (where the fraction used was “3/4”).

The interviews and the assessment performances showed that the participants progressed in developing a more general and productive conception of fractions. In the pre-game interview or assessment, half of the participants (mostly SF1 players) could not easily come up with a verbal explanation about the meaning of “fraction” and most of the remaining participants provided a valid but limited answer, “part of a whole”. After the gameplay, all participants could easily give some explanation of “fraction” and the explanations were often more general and more complex. While three SF2 players’ responses strongly suggested an understanding of fractions as indicators of multiplicative relationships in the pre-game interviews, six additional participants’ (three SF1 players and three SF2 players) post-game interview responses indicated this. Before the gameplay five participants, four of whom were SF1 players, considered equal partitioning to be optional when creating fractions. Later, after the gameplay, three of them improved in this area and more frequently did equal partitioning. Concerning improper fractions, eight participants (mostly SF1 players) showed either an unstable or a superficial understanding (i.e., the numerators are bigger than the denominators). In the post-game session, four
participants (all SF1 players) exhibited progress in understanding improper fractions as iterations
of a unit fraction beyond one, and one participant (Kate) explicitly commented on her improved
understanding of mixed fractions.

This study investigated in depth the influence of playing SF1 and SF2 on the participants’
fraction schemes/operations and explicit conceptions of fractions. Utilizing both the assessment
of fraction schemes/operations and the interview comprehensively revealed the participants’
implicit and explicit knowledge of fractions, which might be inconsistent sometimes. While the
assessment items were specifically designed to elicit and reveal the participants’ understanding
or constructed knowledge, the open-ended interview questions permitted the participants to draw
on different epistemic resources. For example, when participants explained the meanings of
fractions by describing the formal written formats, they utilized the epistemic resource that
suggested knowledge as propagated stuff. When participants discussed “part of a whole” or what
numerators/denominators represented, they activated the epistemic resource that implied
knowledge as constructed stuff. The interview findings suggested that not all participants
possessed both types of epistemic resources on certain fractional topics (e.g., before the
gameplay Merik could describe the setups of improper fractions but not make sense of them
verbally or in the assessment). Some participants appeared to have developed certain fractional
knowledge as constructed epistemic resources to different extents after the gameplay (e.g., four
SF1 players including Merik improved in the IFS items). These findings imply that epistemic
resources are topic-specific or concept-specific, if not more fine-grained. Another finding was
that the participants drew on the knowledge as constructed epistemic resources more often and
more comfortably in the post-game interviews. Two factors might have contributed to this. First,
playing the games frequently required activations of such epistemic resources, which primed the
activations of the same resources in the post-game interviews. Second, the participants might have learned about the interviewer’s expectations and thus focused more on meanings and understandings afterward. Despite the benefits of using both assessments and interviews, cautions are needed when considering the changes in the participants’ improved performances or responses. For instance, six additional participants’ post-game interview responses suggested an understanding of fractions as indicators of multiplicative relationships. This change might not be a result of playing SF1 or SF2. Instead, maybe the participants’ responses were affected by the assessment items that described multiplicative and fractional relationships of two quantities (e.g., Item 1 was “Make 1/8 of the chocolate bar shown below”).

Based on the findings, the EOP freshmen experienced an artificial gap between school mathematics and street/everyday mathematics, consistent with previous findings (Nunes et al., 1993), and that playing SF1 and SF2 helped reduce the gap to some extent. In everyday activities, fractions and closely related concepts (percentages, proportions, and ratios) are directly involved in cooking (e.g., measuring cups, proportions/ratios of various ingredients), arts and crafts (e.g., measuring or designing something, selecting needles, paint brushes, screws or bolts of different sizes), driving (e.g., reading an oil tank gauge, price signs at gas stations, road signs), music (e.g., notes and rhythms), reading an analog clock (quarters) and converting money (among pennies, nickels, dimes, and dollars), to name a few. At first, most participants struggled to come up with examples of fractions in their everyday lives. After the initial struggles, many participants realized or recognized the utility of knowledge of fractions for the first time. In other words, those participants’ knowledge of fractions was likely not built on their intuitive understanding. Gained abstractly without connections to personal experiences, their knowledge pieces were flimsy and easily forgotten once relevant tests were over. As seen in many pre-game
interviews, fraction problem-solving was primarily about recalling and applying isolated rules with extremely limited understanding-based underpinning. After the gameplay, most participants could easily generate more examples of fractions in real life. Furthermore, two SF1 participants quickly came up with examples based on their observations of nearby objects (tables). As artificial as SF1 and SF2 were, playing them led to a better understanding of fractions and their presence in the learners’ lives. The improved understanding might contribute to an enhanced sense of the numbers in between integers and tighten the connection between one’s formal math knowledge gained from lectures and one’s intuition or informal knowledge developed from personal experiences. This would consolidate one’s knowledge base to support more advanced math learning. More importantly, it might support the development of a sense of agency and confidence in math learning, which might stay with a learner for years and enable the learner to seek math learning without fear whenever necessary.

7.3 Discussion of Findings on Practices and Knowledge of Fraction Problem-solving

Regardless of the game or the gameplay, all participants were inclined to draw on formal algorithms or procedures to solve fraction arithmetic problems. However, the participants’ verbal and visual sensemaking and explanation of certain procedures improved after playing SF1 and SF2. The most improvement was observed among SF1 players or the participants whose prior knowledge of fractions was relatively weak. While they often recalled wrong procedures or inappropriately applied algorithms before the gameplay, they could better rely on their sensemaking to properly solve some operations with fractions after the gameplay. Among the participants who could proficiently use fraction algorithms, before the gameplay, only three SF2 players could explain why various procedures worked. Later, many of these participants could better explain and illustrate the algorithm used in fraction additions/subtractions. Three
participants exhibited the most progress. Their improved problem-solving and explanations in the interviews were in line with their knowledge of fractions as reflected in the gameplay. More detailed summaries of the findings will be presented, followed by discussions.

Before the gameplay, eight participants, mostly SF2 players, could consistently and correctly solve fraction multiplications. Among them, only three participants (all SF2 players) could properly explain and illustrate their problem-solving. With assistance, two more SF2 players could make sense of the multiplication of an integer and a fraction. After the gameplay, two more participants (one SF1 player and one SF2 player) could make sense of such fraction multiplications. Among the six participants who could not reliably solve or explain fraction multiplications before the gameplay, three of them (two SF1 players and Kate) showed improvement in problem-solving or explanation. Two participants’ improvements and post-game responses were noteworthy. Zoe mechanically recalled and applied a self-taught incorrect procedure before the gameplay but drew on her understanding to correctly solve fraction multiplications after playing SF1. After the gameplay, Kate correctly interpreted the fraction multiplication using the elements and the mechanics in SF2. The participants’ limited improvement in fraction multiplications might be attributed to at least two reasons. First, most SF2 players had little room for improvement as they could interpret fraction multiplications properly before the gameplay. Second, SF1 does not directly tackle multiplication, unlike SF2, and thus SF1 players’ progress was modest. Although numerous SF1 puzzles involve or show the outcome of combining a number of a unit fraction (e.g., two 1/4s are equivalent to 2/4), a player might not recognize it as multiplication (1/4 * 2) but as addition (1/4 + 1/4). Future studies could purposefully engage learners with weak knowledge of fraction multiplications in
playing SF2.II or investigate whether a discussion on the meaning of “multiplication” (using integers) positively influences learners’ interpretation of fraction multiplications.

Among the twelve participants who were asked to solve fraction divisions in the pre-game interviews, five (all SF2 players) could consistently draw on the standard procedure “keep, change, flip”; one-third (two SF1 players and two SF2 players) drew on a relatively limited procedure “dividing straight across”; the remaining three participants (all SF1 players) were unable to figure out the correct answers properly. All participants struggled to illustrate and explain the process of fraction divisions. After the gameplay, little, if any, improvement was observed in the participants’ fraction divisions. Most of them still recalled the formal procedures and were somewhat uncomfortable with drawing or explaining the process. Only two participants (one SF1 player and one SF2 player) improved in their sensemaking of fraction divisions. Considering that more participants could properly interpret fraction multiplications but not fraction divisions, many participants appeared to lack the knowledge that multiplication and division are inverse operations. Even though neither game was explicit about division, equal splitting was common in puzzle-solving, which probably explained the participants’ improvement in the Splitting Operation in the assessments. In the meantime, there was little change in their capability to interpret and solve fraction divisions in the interviews. Maybe there was a disconnection between the term “division” and the concept of “splitting” from the participants’ perspectives. Alternatively, the epistemic frame of applying “keep, change, flip” or “divide straight across” was so predominant in solving fraction division problems that activating knowledge as constructed stuff epistemic resource required more thoughtful task design. For example, it might help if a researcher clarifies that a precise numeric answer is unnecessary or if solving fraction divisions is primed with solving integer divisions with a focus on the meaning.
Concerning fraction additions/subtractions, before the gameplay, three participants (all SF2 players) could successfully solve the problems and explain the processes; another three participants (one SF1 player and two SF2 players) could consistently reach correct answers but not explain the processes; three participants (two SF2 players and Kate) could properly add/subtract fractions sometimes but had uncertainties in their procedures; five participants (all SF1 players) did not know how to solve fraction additions/subtractions properly. After the gameplay, half of the participants (primarily SF2 players) improved in making sense of fraction additions/subtractions and in explaining the necessity of finding common denominators; four participants (all SF1 players) solved certain simple fraction additions better for they learned or remembered the number facts from their gameplay. Maya, an SF1 player, made prominent progress. When recalling and applying algorithms, her solution to fraction additions remained the same and incorrect. When prompted to reflect on the gameplay and situate the problem-solving in the game, she successfully and appropriately solved the same fraction addition problem. From the findings, playing SF1 or SF2 mostly helped those who were already familiar with the procedures, as such players better understood common denominators after the gameplay. Their justification for converting fractions to have the same denominator shifted from citing authorities (teachers) to evaluating the step’s sensibility. Correspondingly, the epistemic resource activated changed from knowledge as propagated stuff to knowledge as sensible or constructed stuff.

Taking the participants’ performances at various operations with fractions into account, they exhibited different degrees to which their knowledge of fraction problem-solving was connected (Ma, 1999). Even before the gameplay, Tim, Austin, and Neo demonstrated connected fraction knowledge, as they flexibly switched simple multiplications to additions (e.g., \( \frac{1}{3} \times 2 = \frac{1}{3} + \frac{1}{3} \)) and switched simple divisions to multiplications (e.g., \( \frac{1}{3} / 2 = \frac{1}{3} \times \frac{1}{2} \)). In
comparison, several other participants (e.g., Harry, Zoe, Kate, and so on) showed fragmented and inconsistent fraction problem-solving (Erlwanger, 1973), particularly in the pre-game interviews. For example, Merik solved $2 \times \frac{1}{3}$ and $\frac{2}{3} \times \frac{1}{3}$ using different and conflicting algorithms, which showed that he did not master a general procedure applicable to all fraction multiplications. As another example, Maya correctly solved $2 \times \frac{1}{3}$ but wrongly solved $\frac{2}{3} / 2$, using recalled procedures in both cases. This implied her weak knowledge of the connection between simple fraction multiplications and divisions. After the intervention, four participants (three SF1 players and Kate who played parts of both games) more properly solved some operations with fractions, which laid the foundation for connecting different fraction knowledge pieces.

The second area of change among the participants was their capability to justify operations with fractions. All but three participants primarily drew on testimony from authorities like teachers before the gameplay. Afterward, most participants interpreted abstract math expressions and their problem-solving processes using one’s prior knowledge. Nora, Hannah, and Alana, who already could solve various fraction problems using rules before playing SF2, could clearly explain why common denominators were needed in fraction additions/subtractions after playing SF2. Sometimes changes in justifying and making sense of problem-solving seemed related to developing a connected understanding of different operations with fractions. For example, after the gameplay, five participants (three SF1 players, one SF2 player, and Kate who played parts of both games) could interpret the multiplication of a fraction and an integer as adding a number of the fractions together, which they could not do previously. In this case, the sense-making of simple fraction multiplications is intertwined with relating multiplication to addition.
Three participants made noteworthy progress in multiple operations with fractions after playing SF1 or SF2. Five gameplay episodes were purposefully selected and qualitatively examined. Their knowledge of fractions exhibited in the gameplay was absent in their pre-game interviews but present in the post-game interviews, which suggests that playing SF1 or SF2 did contribute to the improvement in their fraction problem-solving. This finding was not surprising. As discussed earlier, independently solving the puzzles in the games and understanding-focused fraction problem-solving in the paper-and-pencil context shared the commonality in activating knowledge as constructed stuff epistemic resources, given the availability of such epistemic resources. On the other hand, when the gameplay was supported by trials and errors, visual feedback, and additional hints, gameplay became similar to a conventional learning activity facilitated by a more knowledgeable figure. It was possible that before the gameplay, these participants did not have easy access to their fractional knowledge as constructed stuff epistemic resources, because such activation was rare, if the resources were not absent, in their usual fraction problem-solving activities. From SF1 and SF2, the participants learned, possibly implicitly and unstably, that they had and could activate alternative epistemic resources when solving operations with fractions.

Unlike previous empirical studies, arithmetic operations of fractions were included in the interviews in this study. In those empirical studies (Cyr et al., 2019; Gresalfi et al., 2017; Zhang et al., 2019), third graders were the participants and gameplay was contrasted with traditional instruction. Probably because arithmetic with fractions was not formally taught in the math classrooms yet, such questions were excluded from the assessments. Based on the findings in this study, the participants still preferred to activate knowledge as propagated stuff epistemic resources after the gameplay due to familiarity and habit. However, more participants were
capable of activating knowledge as constructed stuff epistemic resources to make sense of operations with fractions. Although the one-hour gameplay was unlikely to alter the participants’ practices in fraction problem-solving, it enriched some players’ fraction epistemic resources and enabled them to resort to alternative ways of knowing under certain circumstances. Consider Maya’s post-game problem-solving of fraction additions (p. 124-125). At first, she tried to recall and apply the standard algorithm. It was unsuccessful as the recalled procedure was incorrect and she could not evaluate the problem-solving process or result without receiving feedback from an authoritative figure. Then, prompted by the interviewer, she considered how the problem would be presented and how she would solve it in the game. In the second attempt, she solved the problem visually and sensibly where she independently evaluated the answer to be correct because of the sound process.

Similar to previous studies, this study showed that the activation of epistemic resources was sensitive to context and that the defining features of a unique context might be subtle (Kawasaki et al., 2014; Louca et al., 2014). Both the assessment and the fraction problem-solving took place in a paper-and-pencil format, but they triggered activations of different epistemic resources. In the meantime, the assessment and the games (SF1 and SF2) appeared to be distinct contexts, but the participants activated the same set of epistemic resources. The assessment items in this study were similar to those in other empirical studies (Cyr et al., 2019; Gresalfi et al., 2017; Zhang et al., 2019) since they all focused on measuring one’s understanding of fractions. In contrast, the fraction problems in the interviews were abstract math expressions and looked like practice questions on a worksheet. For example, an assessment item might look like this, “Draw your stick that is 1/3 as long as the stick shown below.” In this problem, the available information is the multiplicative relationship between a given stick and an unknown
stick. The exact length of the given stick or the unknown stick does not matter. Hence, an estimated result along with a justification was sufficient. Solving such a problem requires actions and appropriate actions reflect one’s understanding. This resembles some gameplay episodes in SF1 or SF2, when the puzzles are solved independently of game-bound feedback or hints. In the interviews, a fraction problem might be “Solve 1/3 times 2; explain and illustrate the process.” There was no specific context to situate the abstract expression “1/3 * 2”. Probably because such problems were mostly, if not always, treated as calculation tasks, the sight of such expressions reliably triggered the activation of knowledge as propagated stuff epistemic resource and the pursuit of a numeric answer. Admittedly, in retrospect, even I, the researcher, automatically expected precise numeric answers for such problems in the interviews. This suggests that the EOP freshmen and the interviewer had formed an epistemic frame that guided their expectations and practices when seeing abstract math expressions. On the other hand, solving abstract math expressions could activate knowledge as constructed stuff epistemic resources, if the task prioritizes contextualization and sense-making of the expressions and the problem-solving processes rather than the results acquired by following standard algorithms.

Most participants whose fraction problem-solving and sense-making benefited from playing SF1/SF2 were females. Also, it was female participants who expressed strong frustration about not understanding and feeling left behind in math classrooms. On the contrary, most male participants felt much more comfortable following standard algorithms than illustrating and making sense of problem-solving. These observations corroborate the findings in previous studies, where Boaler (1997a, 1997b) found that abstract and decontextualized math teaching was more alienating for high school girls than boys and that underachieving girls felt being denied access to understanding by the traditional rules-focused math instructional approach. The
findings in this study imply that promoting understanding-focused math teaching and learning, using SVGs or not, might be particularly helpful for female students. Considering that mathematics remains a gatekeeper to STEM majors and careers (Douglas & Attewell, 2017), it is important to continuously revamp K-12 math education to focus on sensemaking, which may contribute to higher retention of females in math-related studies in colleges and address the gender gap in STEM fields.

7.4 Discussion of Findings on Views of Math Learning

Most participants thought that being good at math and learning math quickly made one a “math person”. Several participants considered enjoyment and motivation to learn math to be a math person’s hallmark. Regardless of the criteria, there was no change in the participant’s self-perception, because fractions were a tiny fraction of mathematics and the intervention was short. Despite the lack of change in their self-identifications, the participants would like more SVGs like SF1 and SF2 in formal math education. They thought the games were versatile and could benefit math teaching and learning in multiple ways.

The lack of change in their self-identification was understandable. Muis and Duffy (2013) employed instructions that promoted constructivism in a statistics course and found that students began to report more sophisticated epistemic beliefs from the eighth week. In this study, as earlier sections showed, triggering shifts in fractional practices or epistemic frames with one-hour gameplay was rather difficult. Considering that beliefs are generalizations made based on past experiences (Elby, 2010; Hammer et al., 2005), it was reasonable that the short intervention was insufficient to induce drastic changes in their explicit beliefs about mathematics or about one’s relationship with mathematics. Another reason for the lack of change in self-perception might be because the gameplay and the interviews took place outside of formal math education,
which might have reduced the perceived legitimacy of the math or their gameplay performances. Nonetheless, the participants improved in their fraction schemes/operations after the gameplay. They also more frequently activated *knowledge as constructed stuff* epistemic resources in problem-solving, strengthening their grounds for more sophisticated mathematical epistemic beliefs.

**7.5 Significance of the Study**

In this study, two commercially available mobile games, Slice Fractions and Slice Fractions 2, are examined meticulously concerning their design features and potential to support college freshmen of an Educational Opportunities Program (EOP) in developing a better understanding of fractions and fraction problem-solving. The study greatly adds to our knowledge of these two commercially available mobile games by having them theoretically and empirically analyzed, which might help educators make better-informed decisions about whether and how to use them to support students’ learning of fractions. This study reveals college EOP students’ knowledge of basic mathematics, struggles or baggage from past math education, and views of math learning. Curriculum designers and educators of EOP math courses could draw on the findings to adapt or devise curricula geared towards developing EOP students’ self-efficacy in math learning and knowledge of basic math, within and beyond the scope of mathematics. The findings might also inspire game designers to improve existing games or to create novel serious video games based on the same principles.

Using both assessments of fraction schemes/operations and interviews, the impact of the gameplay on the participants’ implicit and explicit knowledge of fractions was thoroughly investigated. The findings contribute to our understanding of the scope of learners’ knowledge as it exists in different forms and is involved in various activities. Furthermore, they deepen our
understanding of the situatedness of the activations of epistemic resources. It is not format or environment (paper-and-pencil or game) but the nature of a task or problem that determines the types of epistemic resources activated, given the availability and accessibility of various epistemic resources.

7.6 Limitations and Future Research

From the perspective of quantitative research, the small sample size in this study might limit the validity of some of the findings. There were only fourteen participants in this study. Since two serious video games were involved, the size of each group was further reduced. As mentioned earlier, the small sample sizes along with the relatively bigger variances within each group might have contributed to statistical insignificance on some occasions. Also, the small samples might not truthfully reflect the distribution of the population and left nonparametric statistical tests to be the only appropriate options. While most participants’ pre-game assessment performances were consistent with their interview responses, Kate’s data displayed high inconsistency, possibly due to the disconnection between her implicit and explicit knowledge of fractions. Therefore, she was instructed to play parts of both games, making her a peculiar case. This further complicated the quantitative analyses and possibly undermined the findings.

There might be concerns about ecological validity because many participants’ gameplay did not follow the intended sequence. The intervention (gameplay) lasted merely one hour, and most participants could not finish the entire game within the one-hour timeframe. To ensure sufficient gameplay data about later parts of the games, several participants were instructed to skip levels/sections or to play later parts/sections of the games first. This went against the game designers’ intention because level-jumping was prohibited in the original versions of the games. If a learner downloaded and played a brand new SF1 or SF2, the entire gameplay would follow a
specific sequence intended by the game designers, as successfully solving a puzzle would unlock the next puzzle. In this study, the version of SF1 or SF2 the participants played already had all the puzzles unlocked and thus enabled skipping or switching the orders of puzzles/sections. Because of the limited timeframe, gameplay might have been rushed at the end of the gameplay sessions. Even though many participants’ gameplay did not strictly follow the intended sequence, most of the findings about their gameplay remained valid since adjustments were made only if the skipped levels or sections would not heavily negatively impact subsequent gameplay.

To improve the study, the problem-solving of fraction additions and subtractions could be separated, and the math expressions used in the interviews could be more systematic. In the current study, the first few participants’ interview performances showed that a participant knowing how to add dissimilar fractions properly could subtract dissimilar fractions properly as well. In the subsequent interviews, to save time, if a participant’s fraction additions were solved improperly, the participant was not asked to solve fraction subtractions. It was assumed that the participant could not subtract fractions properly or demonstrate an understanding of addition and subtraction as inverse operations. In retrospect, the latter assumption might be incorrect. If a participant consistently adds or subtracts the numerators and the denominators of two fractions respectively (e.g., \( \frac{1}{3} + \frac{1}{2} = \frac{2}{5} \); \( \frac{2}{5} - \frac{1}{3} = \frac{1}{2} \)), the problem-solving would be wrong but still suggests an understanding of addition and subtraction as inverse operations. Instead, the improper fraction additions/subtractions might suggest a disconnection between addition and multiplication or between subtraction and division, if the person could correctly solve fraction multiplications/divisions (e.g., \( \frac{1}{3} \times 2 = \frac{2}{3} \)) but not additions/subtractions (e.g., \( \frac{1}{3} + \frac{1}{3} = \frac{2}{6} \), unequal to the result of \( \frac{1}{3} \times 2 \)). In this study, some participants’ fraction knowledge was rather fragmented. If the knowledge fragmentation/disconnection were to be more systematically
measured, a variety of fraction problem-solving would be needed. For instance, concerning multiplications involving fractions, a set of basic problems might include $2 \times \frac{1}{3}$ (integer * fraction), $\frac{1}{3} \times 2$ (fraction * integer), $\frac{1}{2} \times \frac{1}{3}$ (multiplication of two dissimilar fractions), $\frac{1}{3} \times \frac{2}{3}$ (multiplication of two similar fractions), and $\frac{3}{2} \times \frac{1}{3}$ (improper fraction * fraction).

Future research may consider recruiting younger learners or student teachers. In this study, all the participants were college freshmen from an EOP program. Most of them already had a firm grasp of the part-whole scheme (PWS), the partitive unit fraction scheme (PUFS), and the partitive fraction scheme (PFS) before the gameplay. Therefore, it was unclear whether the games could support the development of these more basic schemes. It would be interesting for future research to recruit young learners who have no exposure to fractions at schools and examine whether and how these games elicit their fractional intuitions and support their construction of basic fraction schemes. Apart from young learners, it might be fruitful to introduce SVGs like SF1 and SF2 to student teachers who intend to become elementary school teachers or middle school math teachers. Student teachers with weak math knowledge might gain a deeper understanding of math from playing such games. Also, the gameplay experiences might prompt student teachers to reflect and devise math learning activities that could better engage learners in problem-solving and elicit learners’ knowledge as constructed stuff epistemic resources.

Besides the formal findings, some informal observations were made about fraction readings, based on which future research may be conducted to carefully investigate the interplay between language/linguistics and fraction learning. First, the participants were more likely to read fractions as “m over n” when describing the calculation procedures and as “m n-th” when explaining the meanings of their problem-solving. The former way of reading stressed the top-
down setup/structure, and the latter seemed more helpful in conveying sensemaking. Second, all
the participants were comfortable with either way of reading if they were dealing with simpler
fractions like one-half or one-third. However, when the fractions involved were more complex,
SF2 players or learners with a better grasp of fractions seemed to be more comfortable with
reading fractions as “m n-th” or using unit fractions like “thirds” or “fourths” in their discussions
compared to SF1 players or learners with weaker knowledge of fractions. If these informal
observations are valid, it might help learners to better understand fractions and operations with
fractions if readings of fractions are restricted to “m n-th” during instructions and practices. Han
& Ginsburg (2001) found that experts considered the Chinese way of reading fractions or
percentages, e.g., “out of n, take m”, to be clearer and less confusing than the English way, m-
nth. The method of reading is likely to impact one’s way of writing a fraction. Both ways of
reading in English support this writing sequence, “numerator, bar, denominator”. In contrast, the
Chinese reading supports an alternative sequence, “bar, denominator, numerator”. In my opinion,
the Chinese reading might better support developing a part-whole understanding of fractions but
the English reading (“m n-th”) appears superior in supporting an understanding of fractions as
iterations of a unit fraction. It might be interesting to conduct cross-cultural research to
investigate whether the different ways to read fractions play different roles in fraction learning
and how to utilize different readings to support fraction learning.
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169
Appendix A. Pre-game and Post-game Interview Protocols

Pre-Game Interview Protocol

Welcome and introductions
Consent (recording)
Purpose of the study
Procedures, voluntary participation, and confidentiality

1. What does the word “fraction” make you feel?
   a. We are about to solve some problems involving fractions, on a scale of 1 to 10, from very uncomfortable/uncertain to very comfortable/confident, where are you at?
2. What do you know or remember about fractions?
   a. What does “fraction” mean? How do you define “fraction”? Can you give me some examples?
   b. What are some rules or algorithms? Can you explain why these rules/algorithms work? (Do they make sense to you?)
      (Multiplying an integer and a fraction; multiplying two fractions; dividing a fraction by an integer, dividing an integer by a fraction, dividing two fractions; adding an integer and a fraction; adding two fractions; subtracting a fraction from an integer, subtracting an integer from a fraction; subtracting two fractions)
3. What do you remember about learning math, especially fractions, at school?
   a. How did the teacher teach fractions? (pics or manipulatives? Lecture and drill?)
   b. What activities or tools were used during learning of fractions? (Worksheets? Watch lecture videos and do practice questions?)
4. In your opinion, why do we need to learn this concept?
   a. Where in real-life do you see and use fractions?
   b. How does it help or hinder your learning of other math topics?
5. How often do you play video games (on games consoles, computers, mobile devices)?
   a. What genres of video games do you play? Can you give me some examples?
   b. Do you play educational video games? What are some examples?
   c. Would you consider yourself a math person and or a gamer? Why?

Thank you for sharing your views and experience. Now we’ll work on some problems. There are a total of 20 items. There is no complex calculation involved. How you come to your answer is the most important thing.
Post-Game Interview Protocol

1. What do you think of this game, as a game? As a math-learning tool?
   a. What did it feel like to play the game (solo, solving a series of puzzles by slicing or tapping objects)?
   b. What were some of the most interesting, challenging, or surprising moments, if any? What happened and how did you overcome the challenge?
   c. What do you think the game does well and not so well?
      Compared to other games that you played?
      Compared to how math is usually taught and learned?

2. About fractions, what do you think you have learned from the game, if anything?
   a. You talked about what fraction means to you earlier. What are your thoughts now, any change? Can you give me some examples?
   b. Looking at the rules or algorithms you mentioned earlier. Now, could you explain why these rules/algorithms work? (Do they make sense to you?)

3. Now, after playing the game, how do you feel when hearing “fractions”?
   a. We are about to solve some more problems involving fractions, on a scale of 1 to 10, from very uncomfortable/uncertain to very comfortable/confident, where are you at now?
   b. Now, why do you think we need to learn this concept? Any change from your earlier answer?

4. Would you like to see more games like SF to be used in math learning?
   a. What might be some potential benefits and drawbacks?
   b. How would you like games like SF to be integrated in math learning?
   c. What do you think teachers’ role would become?

5. Earlier you described yourself as ______ (a math/game person?), after playing SF, how would you identify or perceive yourself? Any change?

Thank you for sharing your thoughts. Lastly, like the pre-game session, there are some problems for you to solve. How you come to your answer is the most important thing.
Appendix B. Pre-game and Post-game Assessments

Pre-Game Assessment

1. Make 1/8 of the chocolate bar shown below.

2. What fraction is the smaller bar out of the larger bar?

3. Make 5/8 of the chocolate bar shown below.

4. What fraction is the smaller bar out of the larger bar?
5. What fraction is the smaller stick out of the longer stick?

6. Your piece of pie is 1/5 as big as the piece shown below. Draw your piece of pie.

7. What fraction is the smaller pie piece out of the whole pie?

8. Your stick is 1/3 as long as the stick shown below. Draw your stick.
9. What fraction is the piece of pie shown below out of a whole pie?

10. Your stick is 3/5 as long as the stick shown below. Draw your stick.

11. What fraction is the smaller stick out of the longer stick?

12. Your piece of pie is 3/5 as big as the piece shown below. Draw your piece of pie.
13. The stick shown below is 3 times as long as another stick. Draw the other stick.

14. The stick shown below is 5 times as long as another stick. Draw the other stick.

15. The amount of pizza shown below is 6 times as big as your slice. Draw your slice.

16. The amount of pizza shown below is 3 times as big as your slice. Draw your slice.
17. What fraction is the longer stick out of the shorter stick?

18. What fraction is the amount of pie shown below out of one whole pie?

19. Your stick is $7/4$ as long as the stick shown below. Draw your stick.

20. Your pie is $4/3$ as big as the piece shown below. Draw your pie.
21. The piece of pie shown below is $\frac{2}{3}$ as big as your piece of pie. Draw your pie.

22. The stick shown below is $\frac{3}{5}$ as long as a whole candy bar. Draw the whole candy bar.

23. The piece of pie shown below is $\frac{4}{5}$ as big as your piece of pie. Draw your pie.

24. The stick shown below is $\frac{4}{7}$ as long as a whole candy bar. Draw the whole candy bar.
Post-Game Assessment

1. Make 1/10 of the chocolate bar shown below.

2. What fraction is the smaller bar out of the larger bar?

3. Make 7/10 of the chocolate bar shown below.

4. What fraction is the smaller bar out of the larger bar?
5. What fraction is the smaller stick out of the longer stick?

6. Your piece of pie is $\frac{1}{7}$ as big as the piece shown below. Draw your piece of pie.

7. What fraction is the smaller pie piece out of the whole pie?

8. Your stick is $\frac{1}{5}$ as long as the stick shown below. Draw your stick.
9. What fraction is the piece of pie shown below out of a whole pie?

![Pie Diagram]

10. Your stick is $\frac{5}{7}$ as long as the stick shown below. Draw your stick.

![Smaller Stick Diagram]

11. What fraction is the smaller stick out of the longer stick?

12. Your piece of pie is $\frac{3}{5}$ as big as the piece shown below. Draw your piece of pie.

![Larger Pie Diagram]
13. The stick shown below is 5 times as long as another stick. Draw the other stick.

14. The stick shown below is 7 times as long as another stick. Draw the other stick.

15. The amount of pizza shown below is 6 times as big as your slice. Draw your slice.

16. The amount of pizza shown below is 5 times as big as your slice. Draw your slice.
17. What fraction is the longer stick out of the shorter stick?

18. What fraction is the amount of pie shown below out of one whole pie?

19. Your stick is 8/5 as long as the stick shown below. Draw your stick.

20. Your pie is 5/2 as big as the piece shown below. Draw your pie.
21. The piece of pie shown below is $\frac{3}{4}$ as big as your piece of pie. Draw your pie.

22. The stick shown below is $\frac{4}{7}$ as long as a whole candy bar. Draw the whole candy bar.

23. The piece of pie shown below is $\frac{4}{7}$ as big as your piece of pie. Draw your pie.

24. The stick shown below is $\frac{5}{9}$ as long as a whole candy bar. Draw the whole candy bar.
## Appendix C. Comparison of Pre- and Post-game Assessment Items

<table>
<thead>
<tr>
<th>Targeted Scheme/Operation</th>
<th>Item</th>
<th>Representation</th>
<th>Pre-game Fractions</th>
<th>Post-game Fractions</th>
<th>Notes*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Part-whole scheme (PWS)</td>
<td>1</td>
<td>Bar</td>
<td>1/8</td>
<td>1/10</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>Bar</td>
<td>1/7</td>
<td>1/9</td>
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<td></td>
<td>3</td>
<td>Bar</td>
<td>5/8</td>
<td>7/10</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>Bar</td>
<td>2/7</td>
<td>3/11</td>
<td></td>
</tr>
<tr>
<td>Partitive unit fraction scheme (PUFS)</td>
<td>5</td>
<td>Bar</td>
<td>1/5</td>
<td>1/6</td>
<td></td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>Pie</td>
<td>1/5</td>
<td>1/7</td>
<td></td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>Pie</td>
<td>1/6</td>
<td>1/8</td>
<td></td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>Bar</td>
<td>1/3</td>
<td>1/5</td>
<td></td>
</tr>
<tr>
<td>Partitive fraction scheme (PFS)</td>
<td>9</td>
<td>Pie</td>
<td>4/7, 3/5, 5/8 etc.*</td>
<td>Multiple correct answers</td>
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<tr>
<td></td>
<td>10</td>
<td>Bar</td>
<td>3/5</td>
<td>5/7</td>
<td></td>
</tr>
<tr>
<td></td>
<td>11</td>
<td>Bar</td>
<td>2/5, 3/7, 4/9 etc.*</td>
<td>Multiple correct answers</td>
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<td>12</td>
<td>Pie</td>
<td>3/5*</td>
<td>Different sectors are used</td>
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<tr>
<td>Splitting operation</td>
<td>13</td>
<td>Bar</td>
<td>1/3</td>
<td>1/5</td>
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<tr>
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<td>14</td>
<td>Bar</td>
<td>1/5</td>
<td>1/7</td>
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<td>15</td>
<td>Pie</td>
<td>1/6*</td>
<td>Different sectors are used</td>
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<tr>
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<td>16</td>
<td>Pie</td>
<td>1/3*</td>
<td>1/5*</td>
<td>Same nonstandard sector</td>
</tr>
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<td>Iterative fraction scheme (IFS)</td>
<td>17</td>
<td>Bar</td>
<td>3/2</td>
<td>5/4</td>
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<tr>
<td></td>
<td>18</td>
<td>Pie</td>
<td>5/4</td>
<td>4/3</td>
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<td>19</td>
<td>Bar</td>
<td>7/4</td>
<td>8/5</td>
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<td>Pie</td>
<td>4/3</td>
<td>5/2</td>
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<td>Pie</td>
<td>2/3</td>
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<td>4/5*</td>
<td>4/7</td>
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<td>4/7</td>
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