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Information-Based Physics: An Intelligent Embedded Agent's Guide to the Universe

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Influence

I know about the universe because it influences me

In fact, **everything** I know about the universe is conveyed via such influences.

Moreover, I cannot come to know about what does not influence me.

Agent-Centric View

Everything I can know
is completely describable
in terms of how it influences me

Information

Information acts to constrain our beliefs

You can believe anything you want…
until you obtain information

Physical Laws are shaped by three factors:

- The nature of influence
- Constraints on the quantification of such influences
- Inferences that can be made from the information obtained via influences

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- The nature of influence
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Information-Based Physics

Progress

Derivation of Probability Theory
as a quantification of the Boolean algebra of statements

Knuth, K.H., Skilling, J. 2012. *Axioms* 1:38-73. arXiv:1008.4831 [math.PR]

Derivation of the Feynman Path Integral Formulation of **Quantum Mechanics**
as a quantification of measurement sequences

Goyal P., Knuth K.H., Skilling J. 2010. *Phys. Rev.* A 81, 022109.
arXiv:0907.0909v3 [quant-ph]
Goyal P., Knuth K.H. 2011. *Symmetry* 3(2):171-206.

Quantification

"Measure what is measurable,
and make measurable that which is not so."

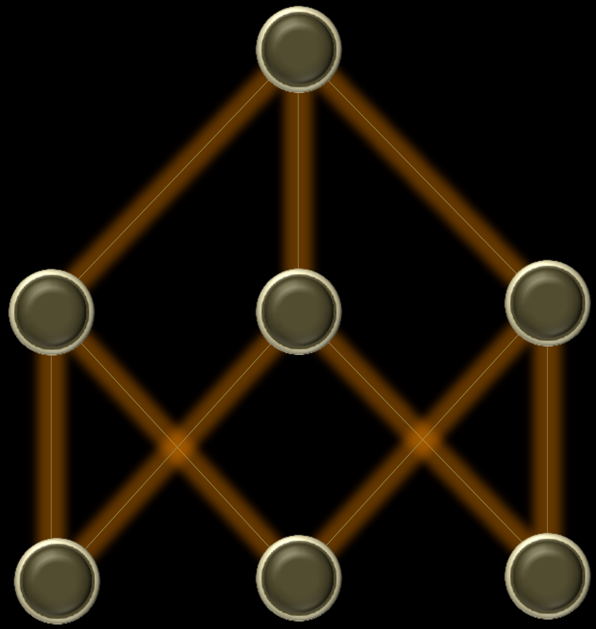
Galileo Galilei

Map order-theoretic or algebraic entities to sets of real numbers, and encode operations on those entities in terms of operations on numbers (*Laws*).

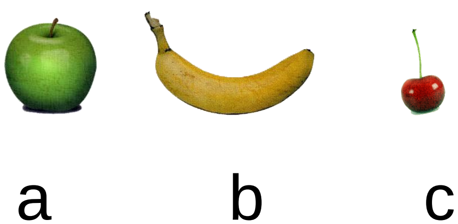
The challenge is to do this in an apt and consistent manner.

The utility lies in the fact that many problems possess similar symmetries, which lead to identical laws.

Inference

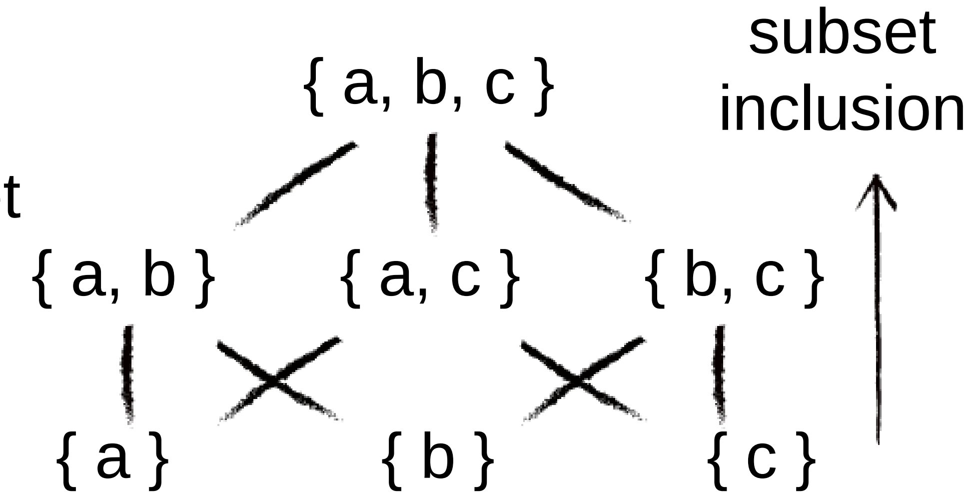


States and Statements



states
of a piece of fruit

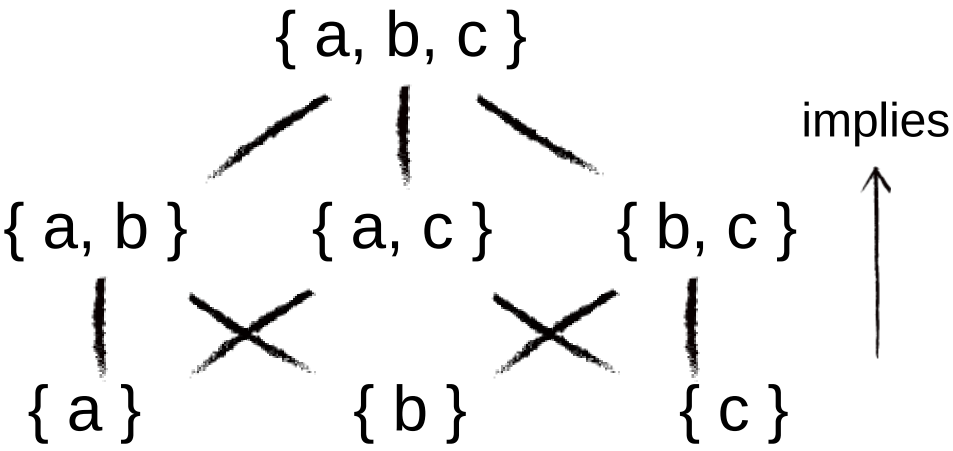
powerset



statements
about a piece of fruit

statements describe potential states

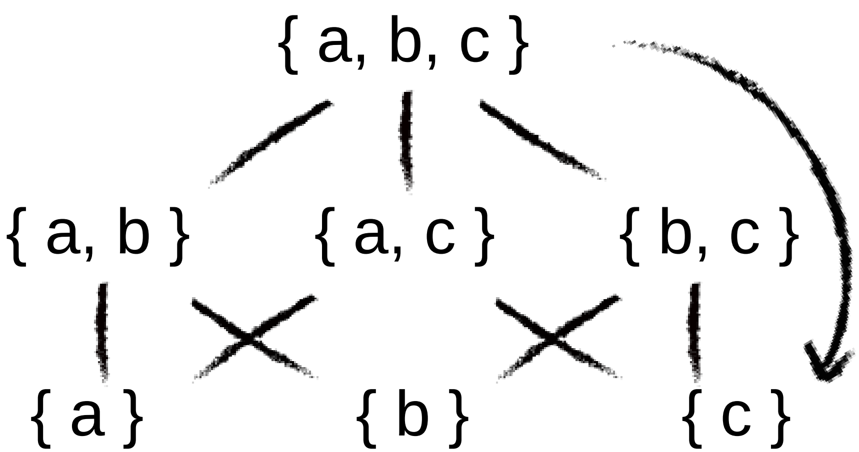
Implication



statements
about a piece of fruit

ordering encodes implication

Inference



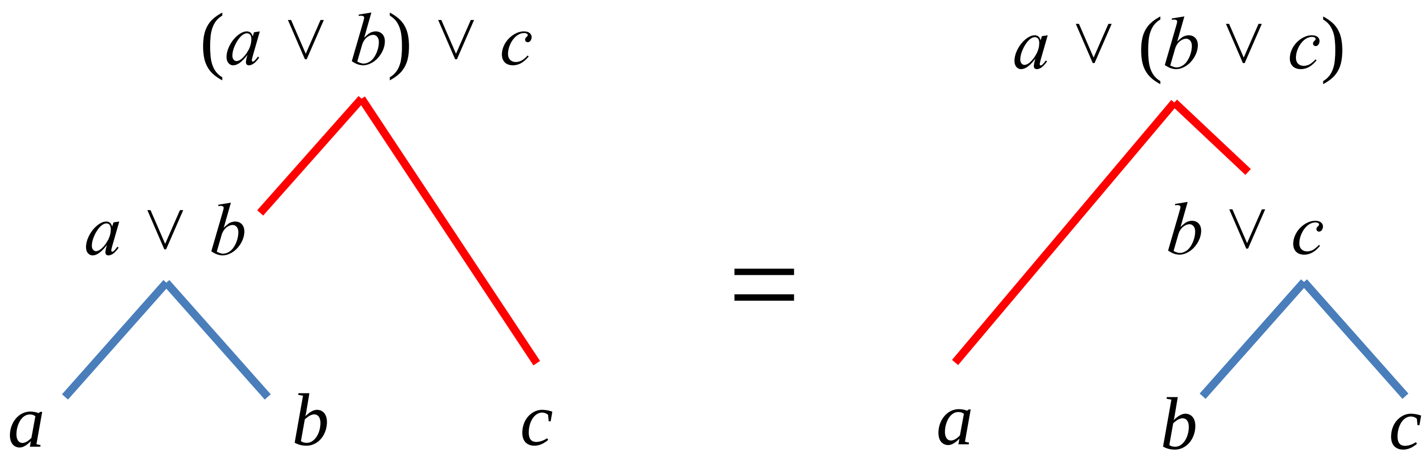
statements
about a piece of fruit

Quantify to what degree knowing that the system is in one of three states {a, b, c} implies knowing that it is in some other set of states

Bi-Valuation: p({c} | {a,b,c})

inference works backwards

Associativity



$$(a \vee b) \vee c \;=\; a \vee (b \vee c)$$

Assume $v(a \vee b) = v(a) \oplus v(b)$

Then

$$\big(v(a) \oplus v(b)\big) \oplus v(\mathrm{c}) \;=\; v(a) \oplus (v(b) \oplus v(\mathrm{c}))$$

The Associativity Equation

The algebraic symmetry of associativity
along with a concept of ordering

$$(a \vee b) \vee c = a \vee (b \vee c)$$

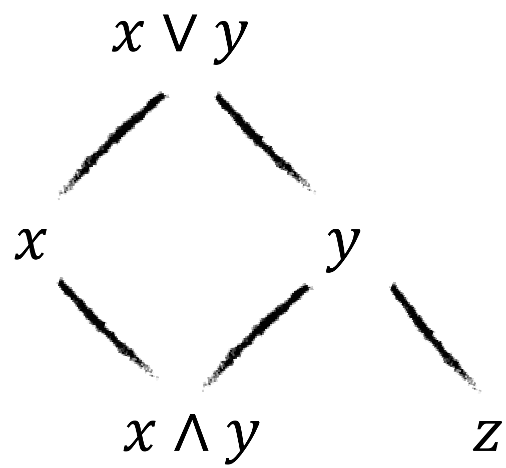
must be preserved by our quantification

$$\big(v(a) \oplus v(b)\big) \oplus v(\mathrm{c}) = v(a) \oplus (v(b) \oplus v(\mathrm{c}))$$

This means that the operation $\oplus$ is a transform of **addition**
(Aczel 1966, Knuth & Skilling 2012):

$$h\big(v(a \vee b)\big) = h\big(v(a) \oplus v(b)\big) = h\big(v(a)\big) + h\big(v(b)\big)$$

General Case



$$v(y) = v(x \wedge y) + v(z) \qquad v(x \vee y) = v(x) + v(z)$$

$$v(x \vee y) = v(x) + v(y) - v(x \wedge y)$$

Probability

Associativity of Join

$$p(a \vee b \mid i) = p(a \mid i) + p(b \mid i) - p(a \wedge b \mid i)$$

Associativity of Direct Product of Hypothesis Spaces

$$p(a, b \mid i, j) = p(a \mid i)\, p(b \mid j)$$

Associativity of Context

$$p(a \mid c) = p(a \mid b)\, p(b \mid c)$$

which can be used to derive Bayes theorem

Knuth, K.H., Skilling, J. 2012. *Axioms* 1:38-73. arXiv:1008.4831 [math.PR]

Why Sums Rule

$$p(x \vee y \mid i) = p(x \mid i) + p(y \mid i) - p(x \wedge y \mid i)$$

$$I(X;Y) = H(X) + H(Y) - H(X,Y)$$

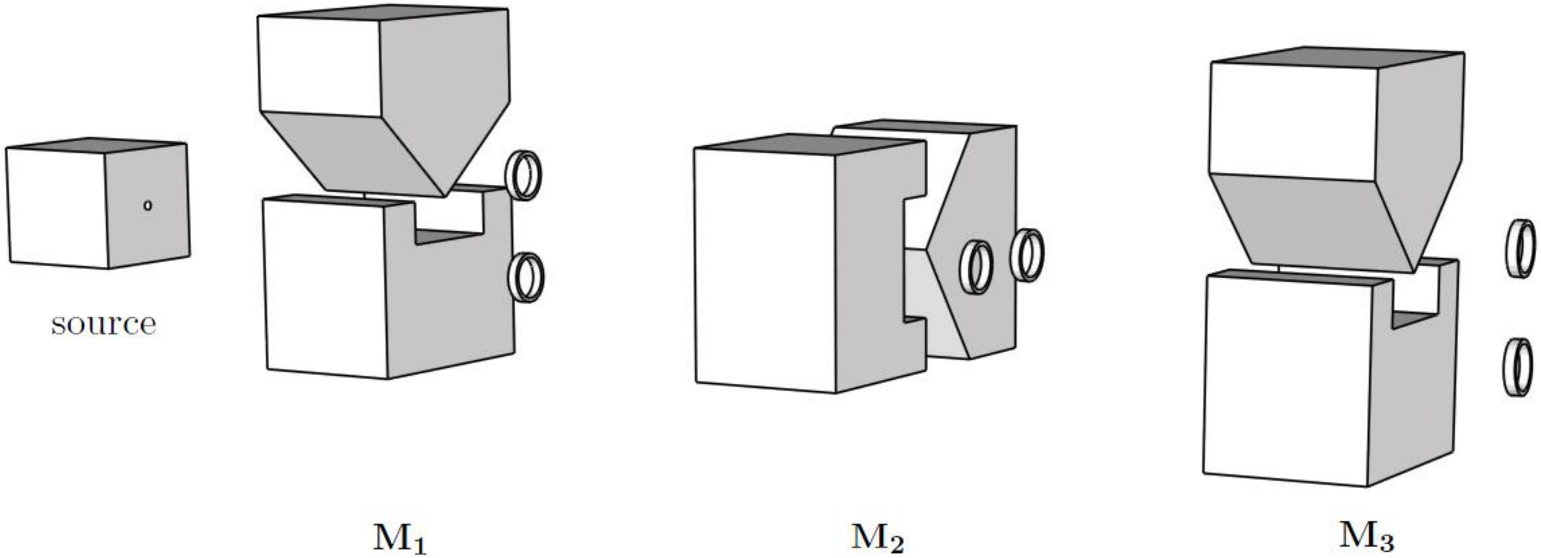
$$\max(x, y) = x + y - \min(x, y)$$

$$\chi = V - E + F$$

$$\log(\gcd(x, y)) = \log(x) + \log(y) - \log(\mathrm{lcm}(x, y))$$

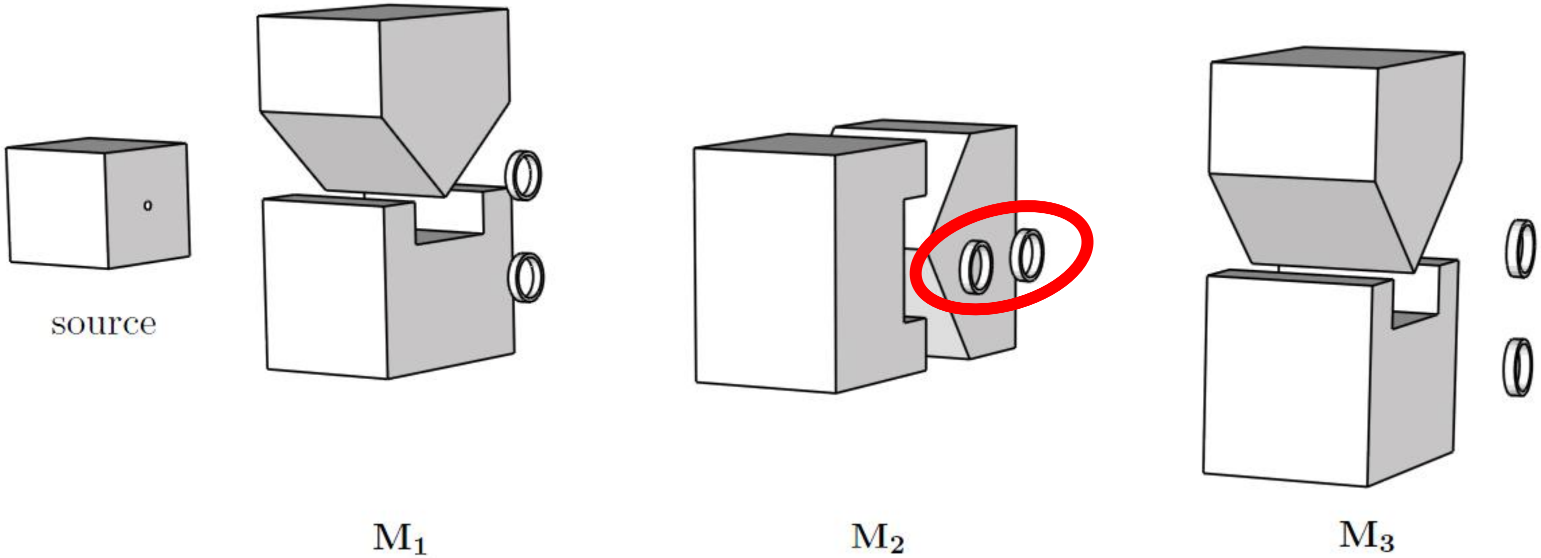
Quantum
Measurement Sequences

Measurement Sequences

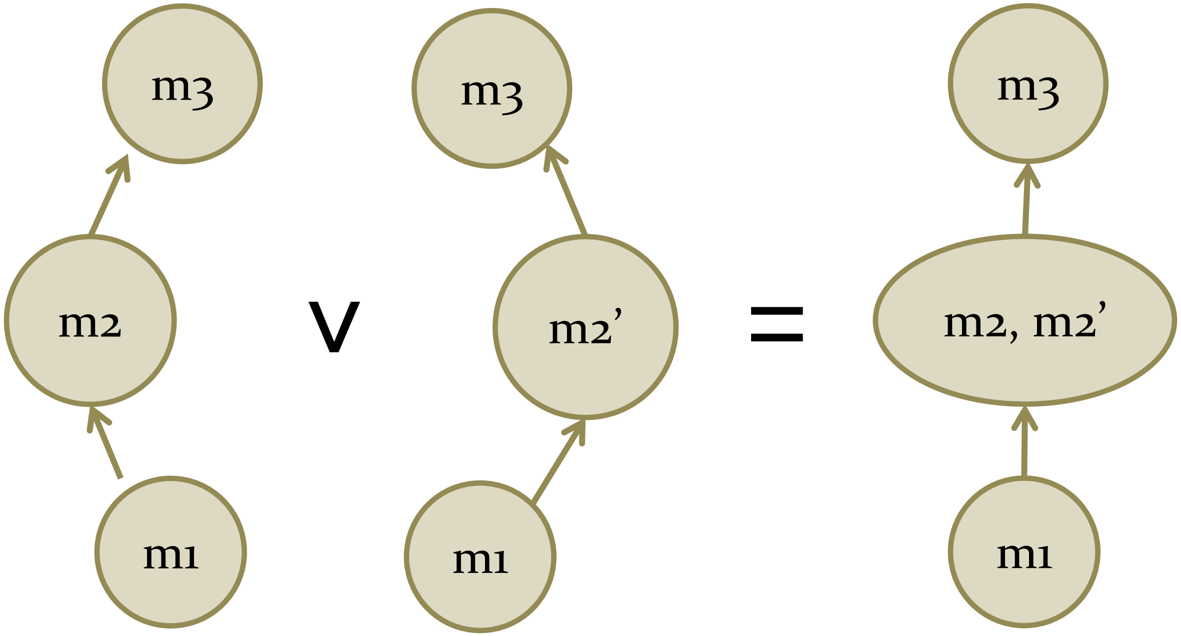


Quantify a quantum mechanical measurement sequence [m1, m2, m3] with a pair of real numbers (a_1, a_2).

Parallel Combination of Measurements



Parallel Combination of Measurements

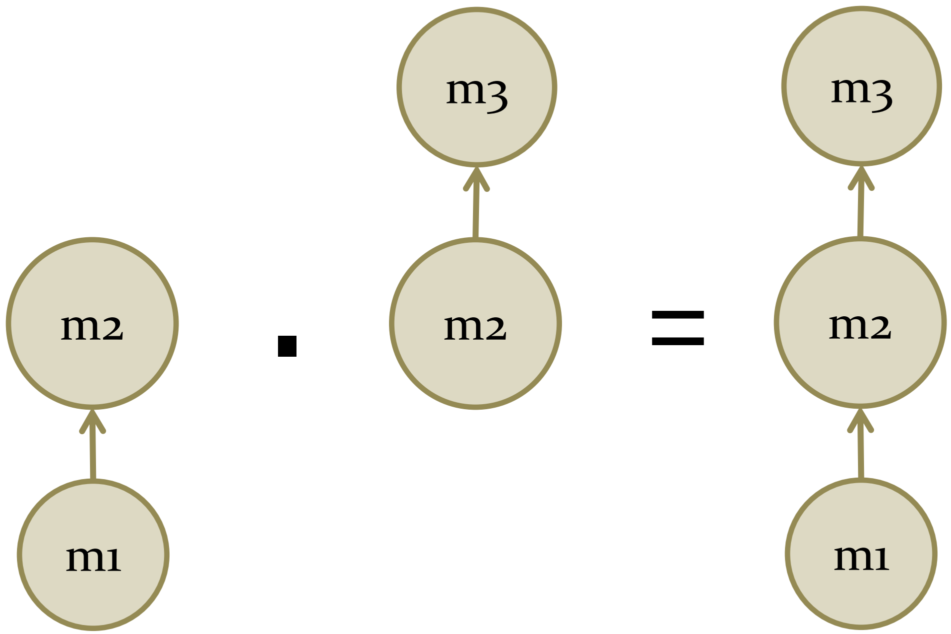


Associativity Again!

$$\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \oplus \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} a_1 + b_1 \\ a_2 + b_2 \end{pmatrix}$$

Goyal, Knuth, Skilling, 2010. PRA 81, 022109, arXiv:0907.0909v3 [quant-ph]
Goyal, Knuth, 2011. Symmetry, 3(2):171-206. http://www.mdpi.com/2073-8994/3/2/171

Serial Combination of Measurements

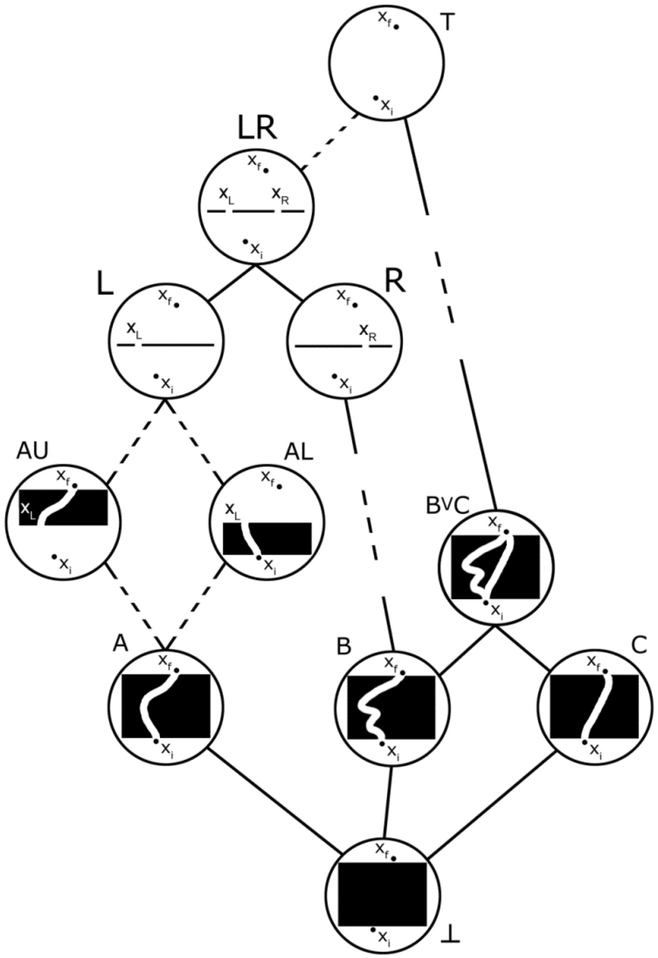


Distributivity,
Reciprocity,
Agreement
with probability

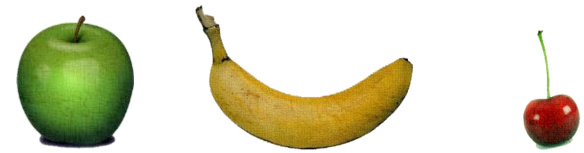
$$\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \odot \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} a_1 b_1 - a_2 b_2 \\ a_1 b_2 + a_2 b_1 \end{pmatrix}$$

Goyal, Knuth, Skilling, 2010. PRA 81, 022109, arXiv:0907.0909v3 [quant-ph]
Goyal, Knuth, 2011. Symmetry, 3(2):171-206. http://www.mdpi.com/2073-8994/3/2/171

Quantum vs. Classical States



QM Slit Experiments



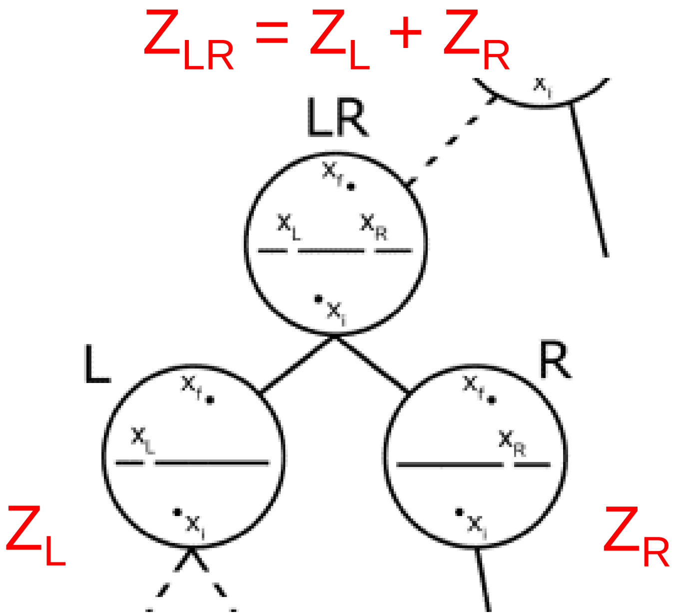
a b c

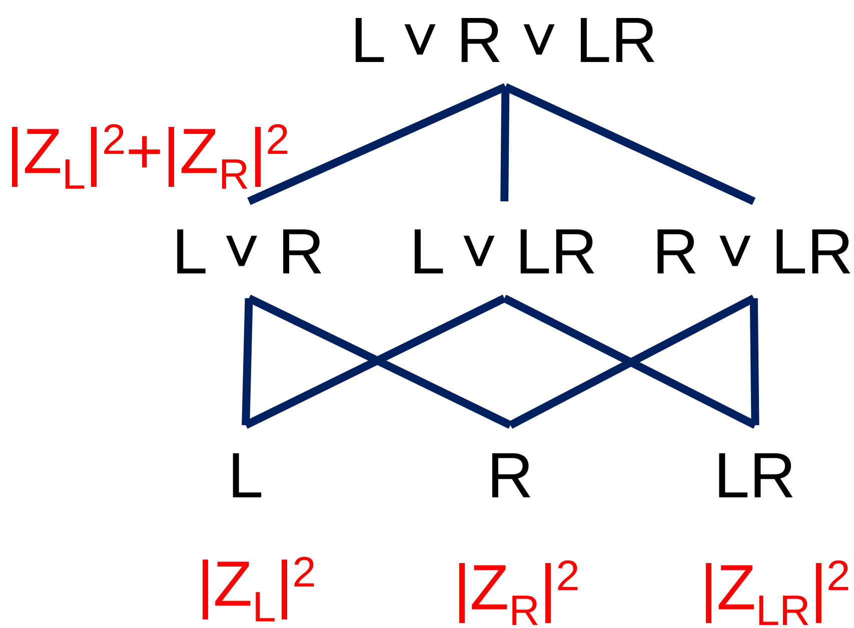
Fruit

The Classical State Space is an Antichain.

Whereas the QM space of measurement sequences is a partially ordered set.

Quantum Amplitudes and Probabilities





Measurement Sequences

Complex amplitudes quantify relationships among sequences

Statements about Sequences

Complex amplitudes are used to compute probabilities

Current Progress

Space-Time Relationships
from a quantification of causal sets of events

Knuth, K.H., Bahreyni. 2012. *in review*. arXiv:1209.0881 [math-ph]

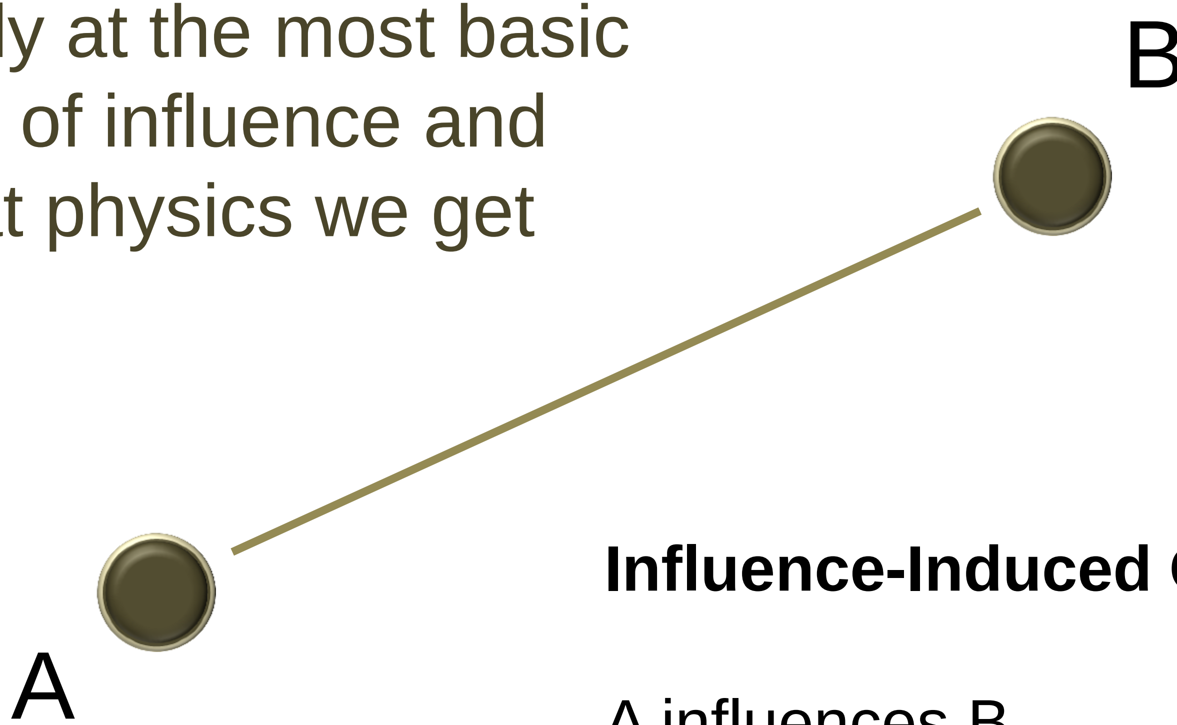
Derivation of the Dirac Equation in 1+1 Dimensions
as a quantification of direct particle-particle influence

Knuth K.H. 2012. MaxEnt 2012 Proceedings. arXiv:1212.2332 [quant-ph]

Influence

Influence

Look only at the most basic property of influence and see what physics we get



Influence-Induced Order

A influences B
B is influenced by A

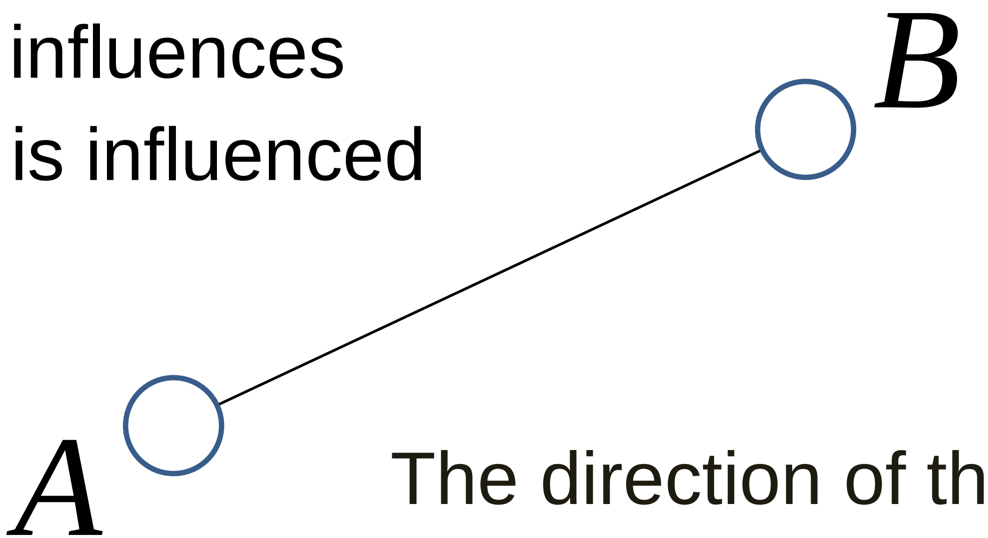
Events

Define a pair of Events as the Boundary of Influence

Event A: A influences

Event B: B is influenced

$$A \leq B$$

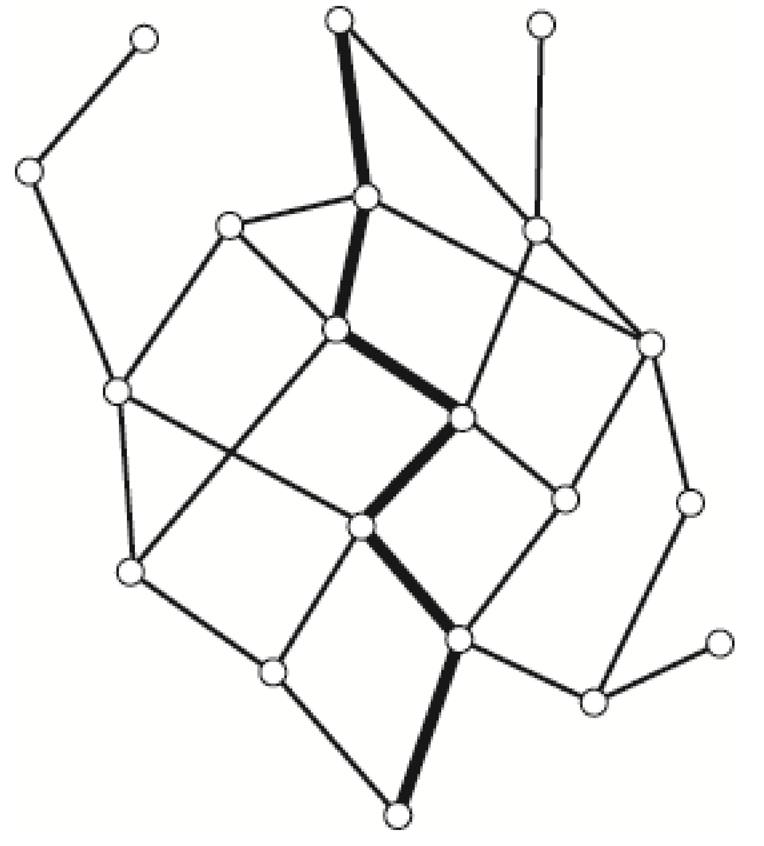


The direction of the ordering relation is arbitrary.
(could write $A \geq B$)
This will be relevant later.

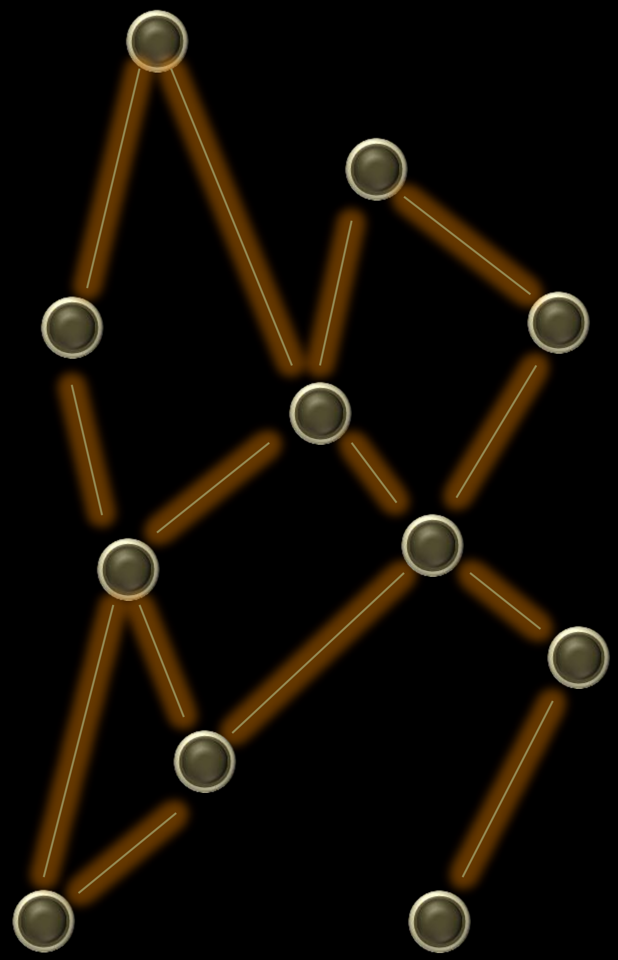
Poset of Events

A set of influences can be described by a **partially-ordered set (poset)** of events.

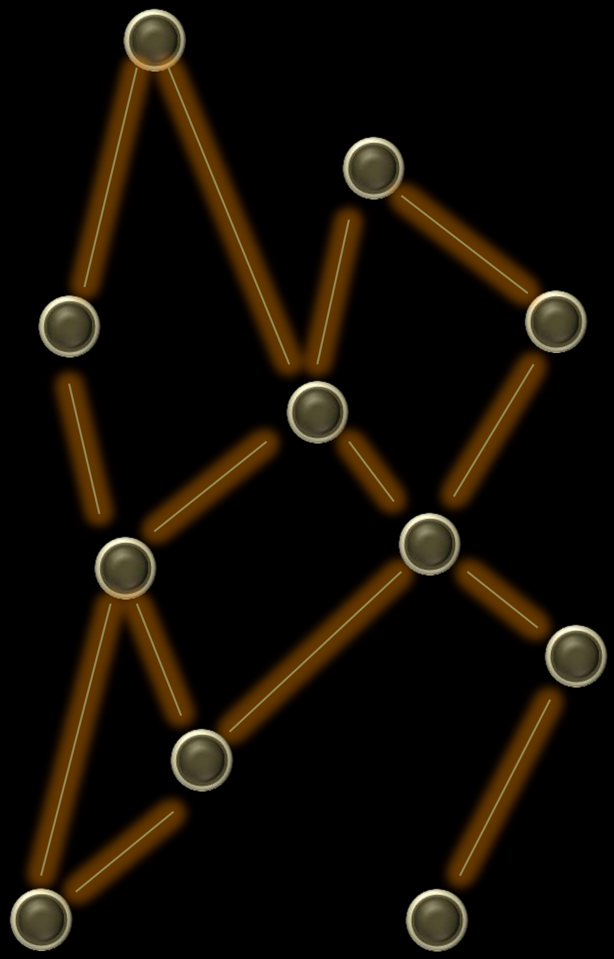
Chains, which are totally ordered, represent a sequence of events.



Quantification

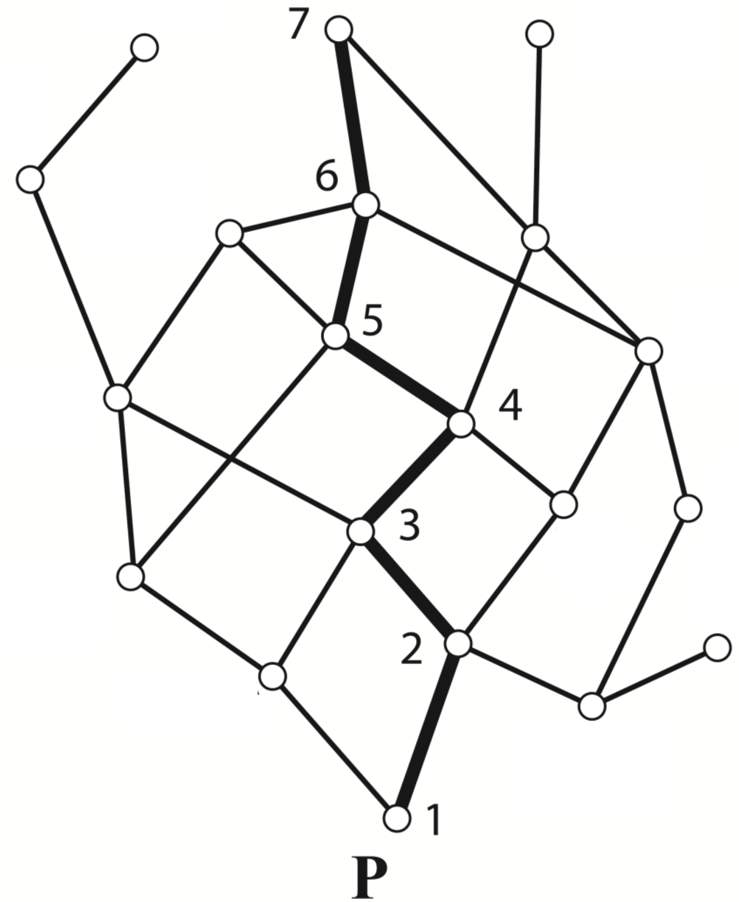


Rather than endowing the poset with additional properties, our goal is simply to identify a consistent means by which events in the poset can be aptly quantified.

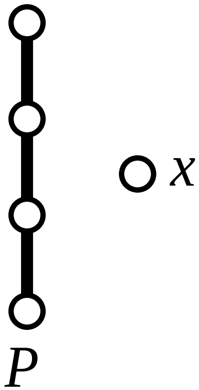


Quantifying a Chain

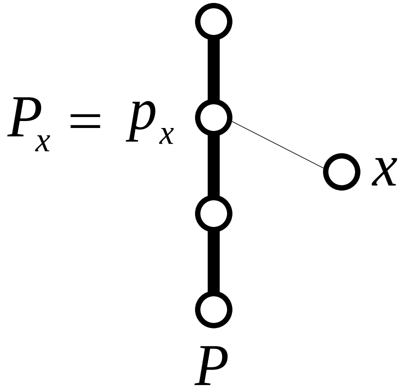
Chains are easily quantified by a **monotonic valuation** assigning to each element a real number



Chain Projection

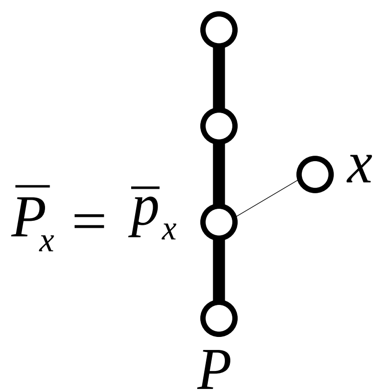


$p_i \,||\, x$ for all p_i



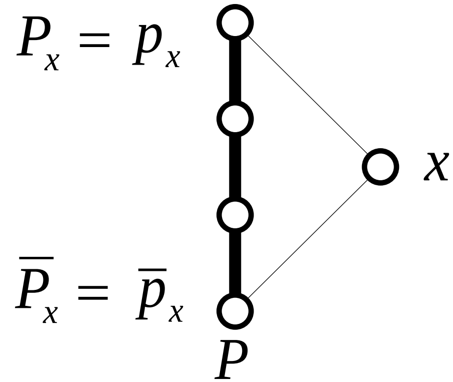
$p_i \geq x$ for all $p_i \geq p_x$

$p_i \,||\, x$ for all $p_i < p_x$



$p_i \leq x$ for all $p_i \leq \overline{p}_x$

$p_i \,||\, x$ for all $p_i > \overline{p}_x$

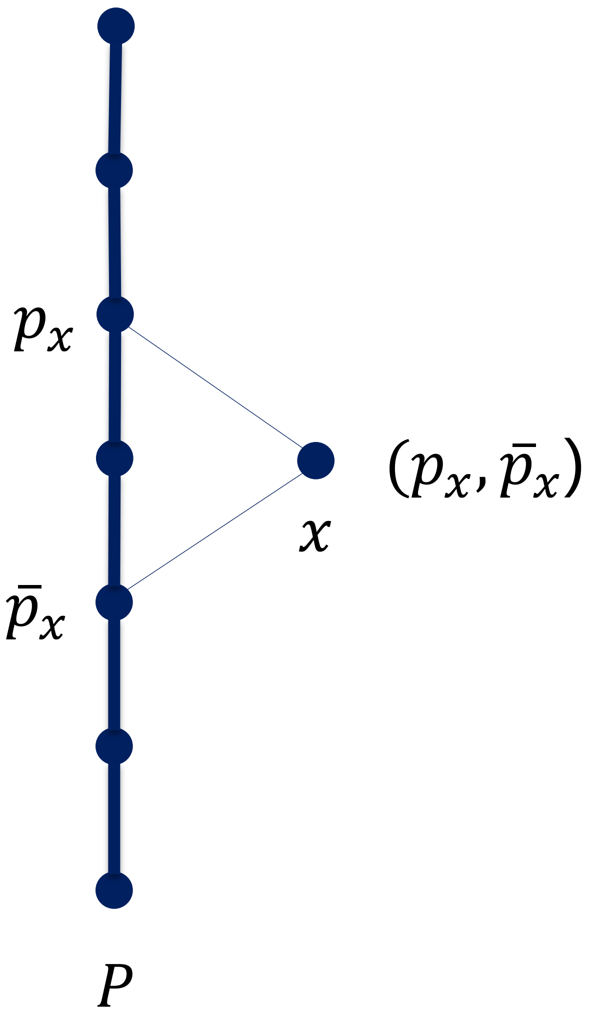


$p_i \leq x$ for all $p_i \leq \overline{p}_x$

$p_i \,||\, x$ for all $\overline{p}_x < p_i < p_x$

$p_i \geq x$ for all $p_i \geq p_x$

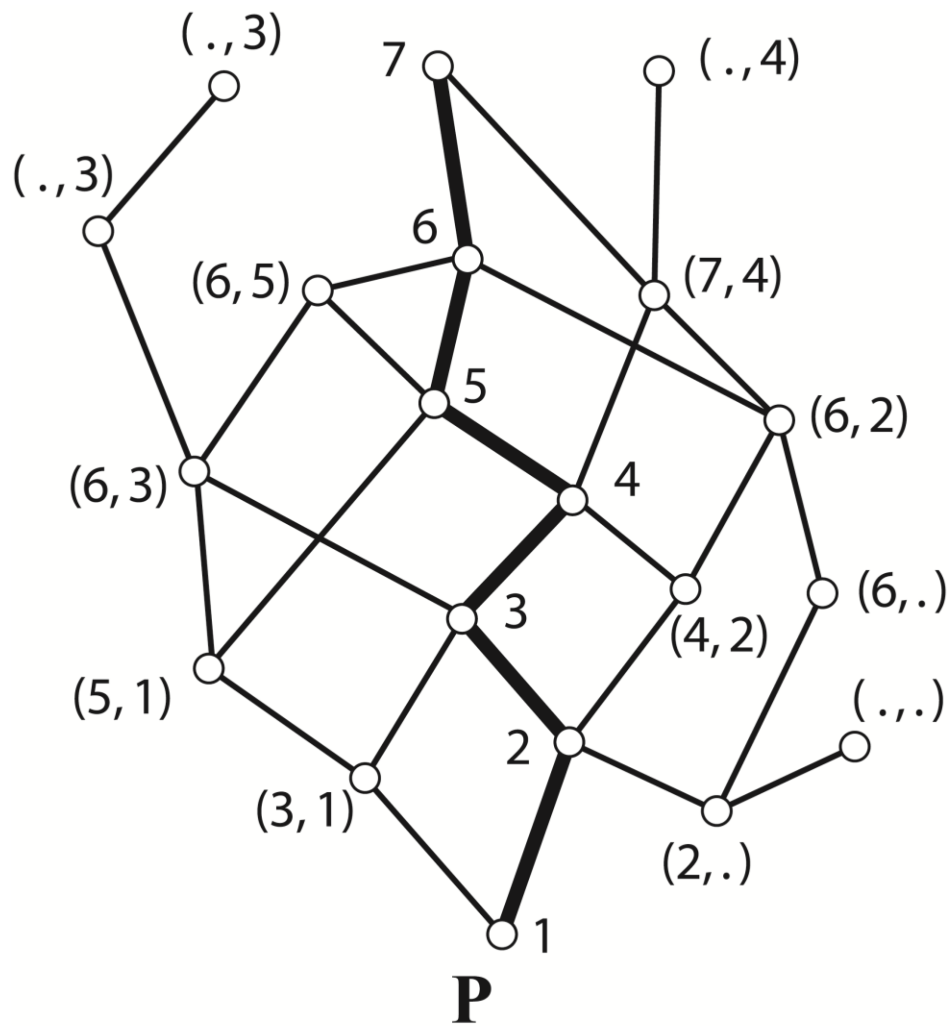
Quantification with Pairs



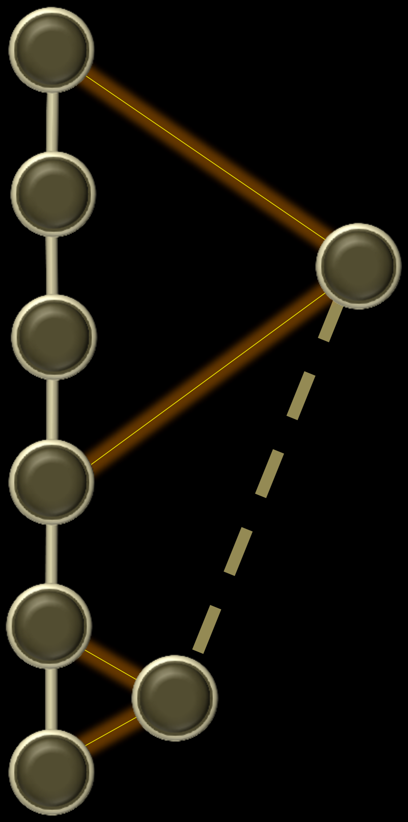
Quantification can be extended by relating poset elements to the embedded chain via **chain projection.**

For an element x, there is the potential to be quantified by a pair of numbers

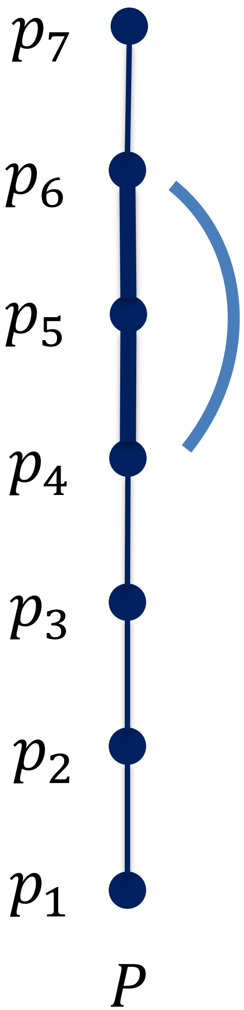
Quantification by Chain Projection



Intervals



Closed Intervals Reside on Chains

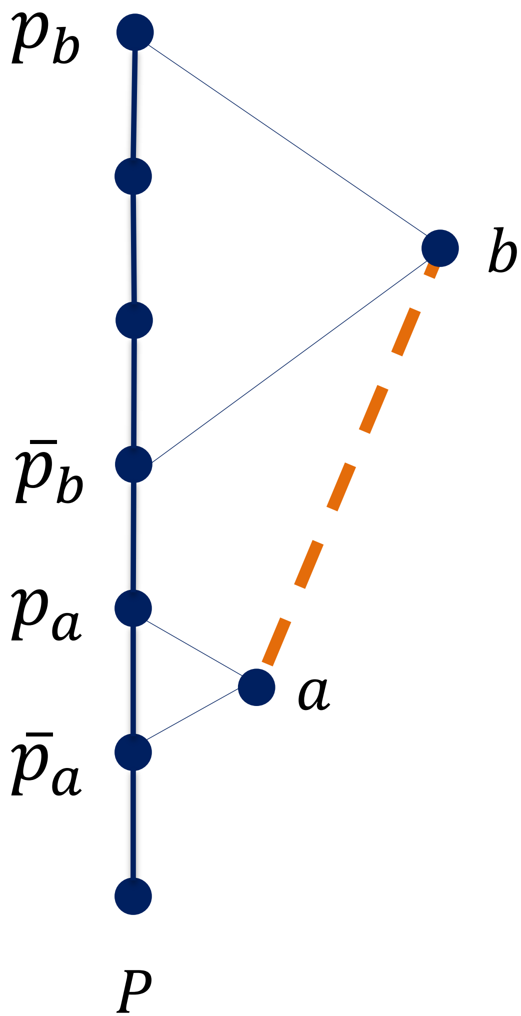


$[p_6, p_4] = \{p_6, p_5, p_4\}$

Closed intervals can be quantified by pairs: (p_6, p_4)

Or by scalars (theorem): $p_6 - p_4$

Generalized Intervals



Generalized intervals are defined by their endpoint elements.

They can be quantified by:

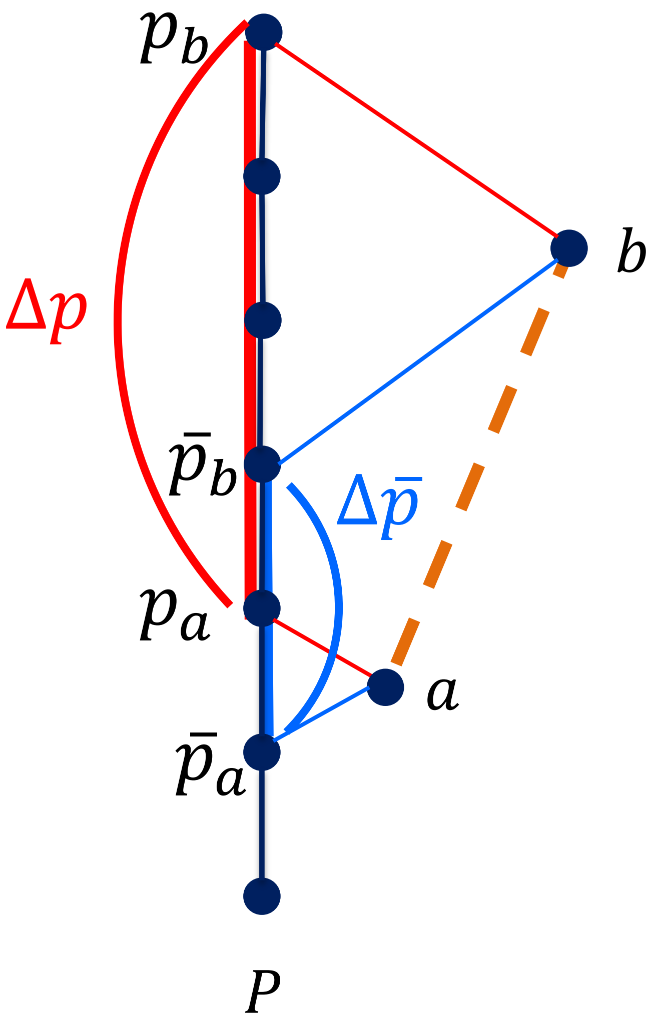
4-tuples: $(p_b,\ \bar{p}_b;\ \ p_a,\ \bar{p}_a)$

Pairs: $(p_b - p_a,\ \bar{p}_b - \bar{p}_a)$

Scalars (theorem):

$$(p_b - p_a)(\bar{p}_b - \bar{p}_a)$$

Generalized Intervals



4-tuple: $(p_b,\ \bar{p}_b;\ \ p_a,\ \bar{p}_a)$

Pair: $(\Delta p,\ \Delta \bar{p})$

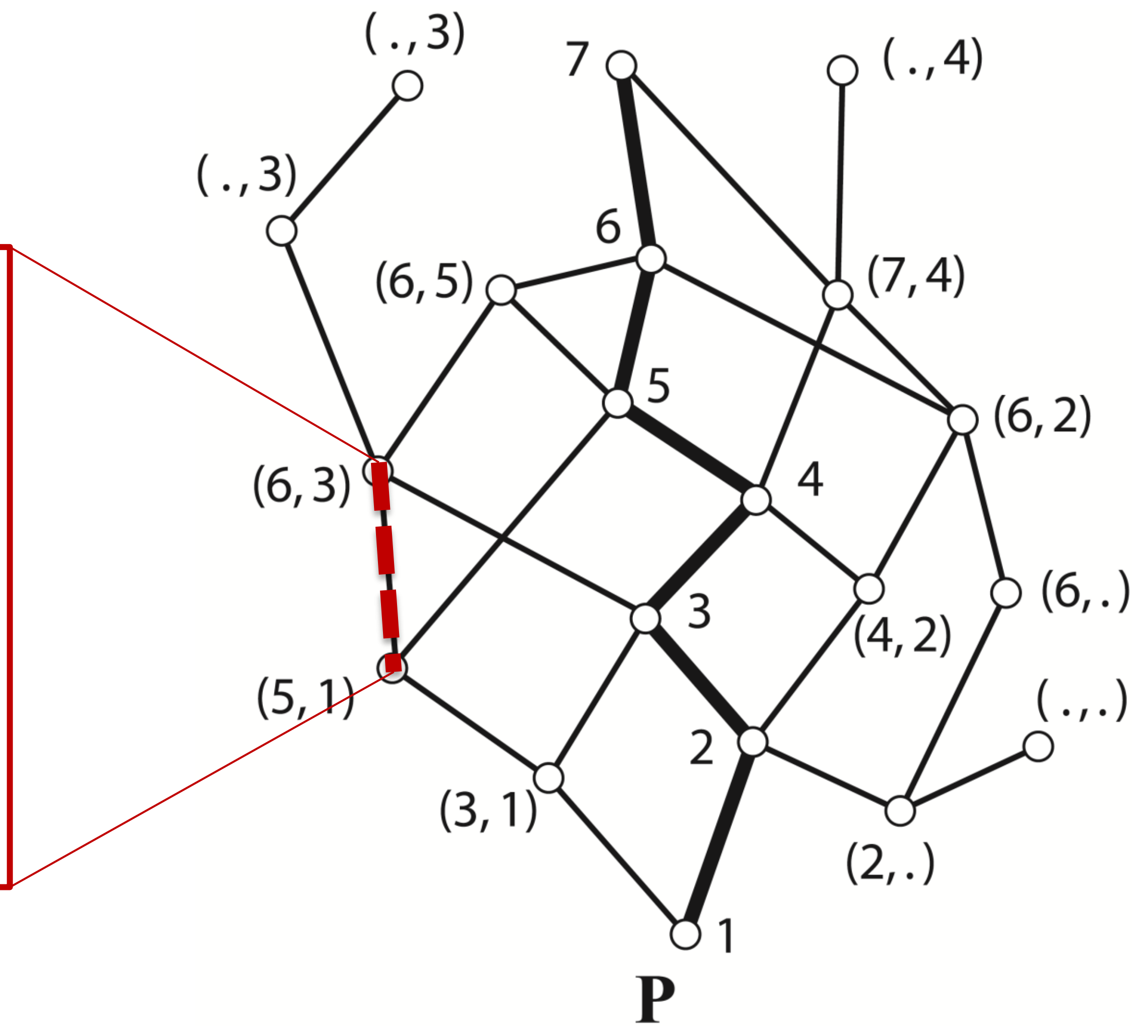
Scalar (theorem): $\Delta p \Delta \bar{p}$

Quantifying Intervals

Quadruple
(6, 3; 5, 1)

Pair
(6-5, 3-1) = (1, 2)

Scalar
(6-5)(3-1) = 2

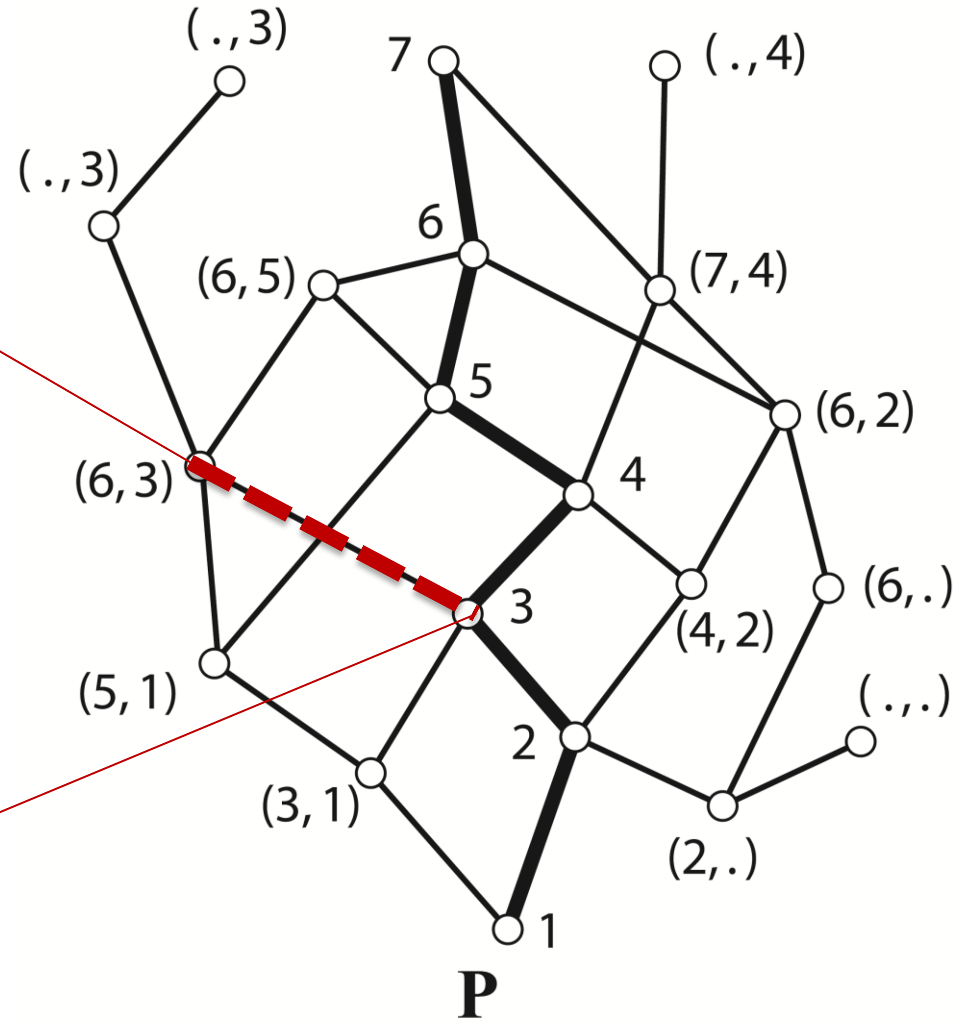


Quantifying Intervals

Quadruple
(6, 3; 3, 3)

Pair
(6-3, 3-3) = (3, 0)

Scalar
(6-3)(3-3) = 0

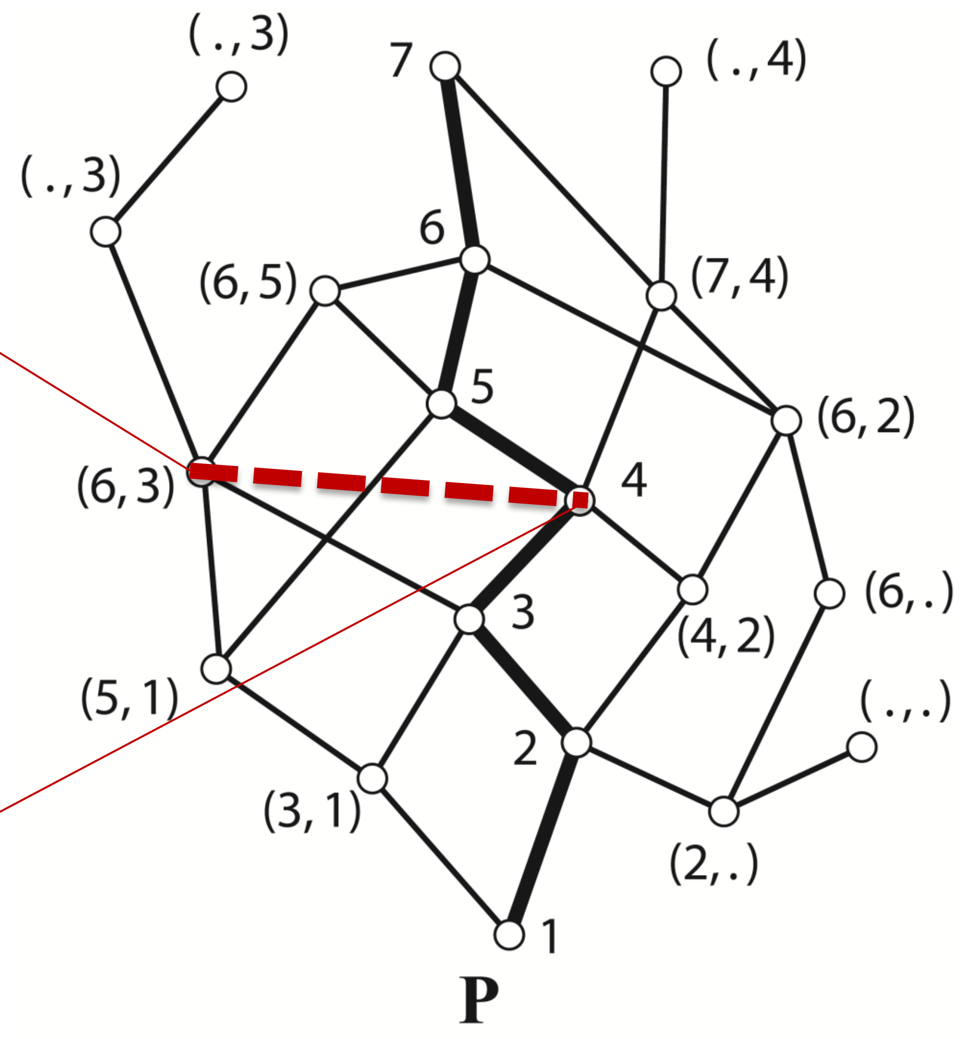


Quantifying Intervals

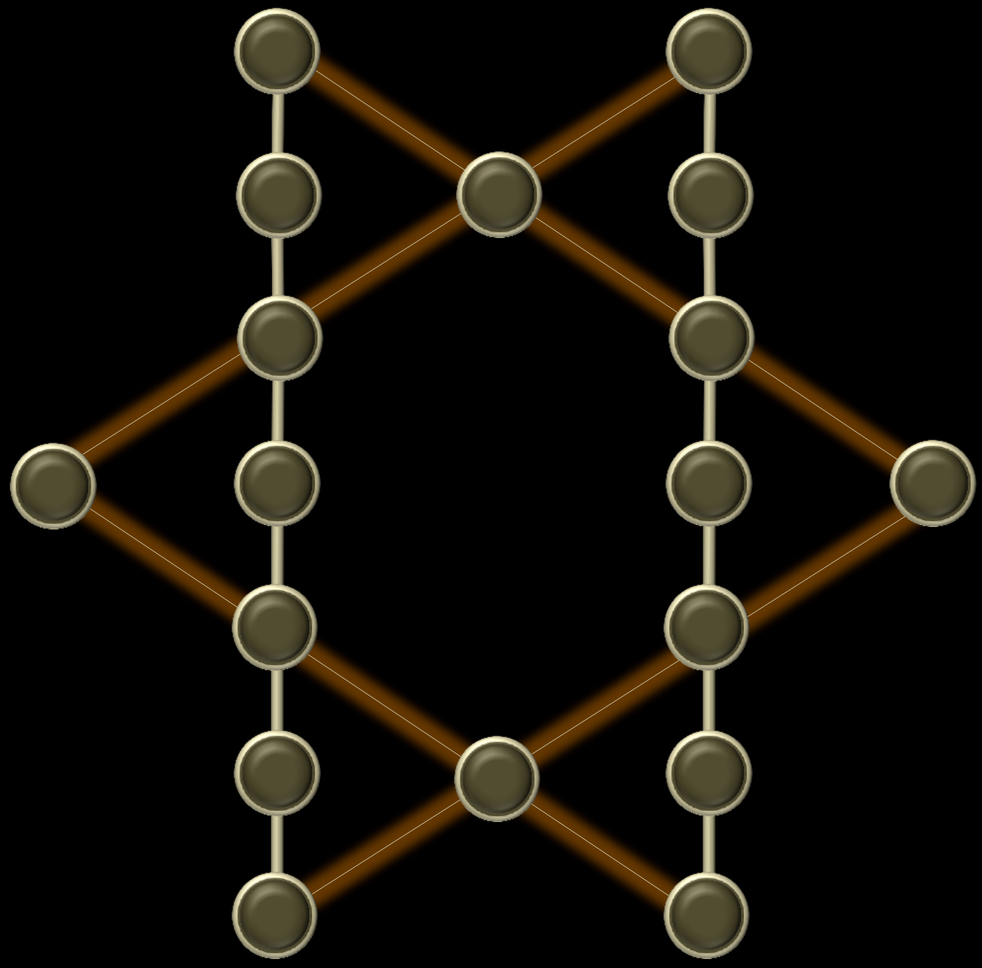
Quadruple
(6, 3; 4, 4)

Pair
(6-4, 3-4) = (2, -1)

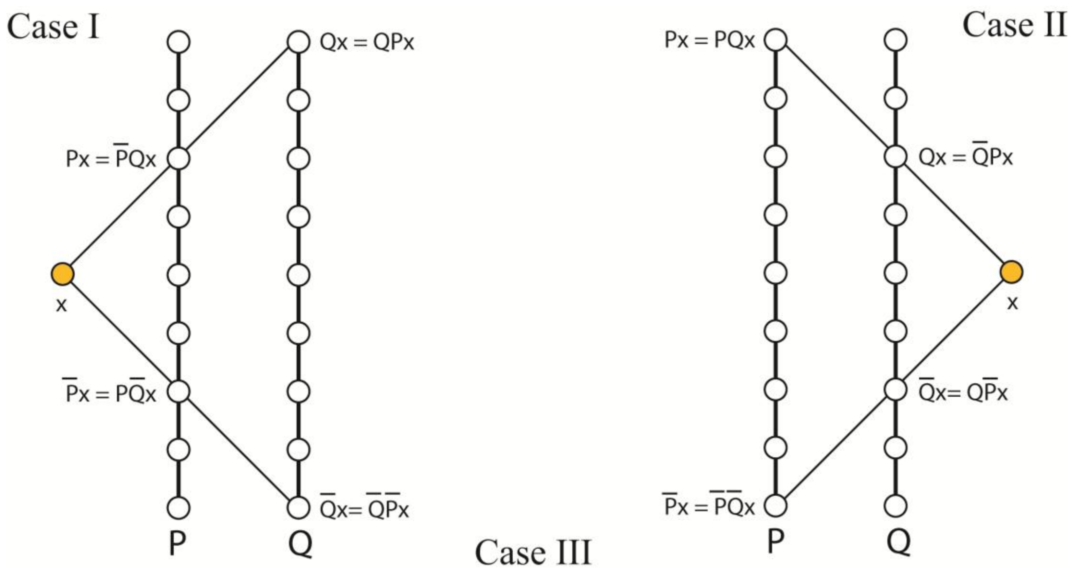
Scalar
(6-4)(3-4) = -2

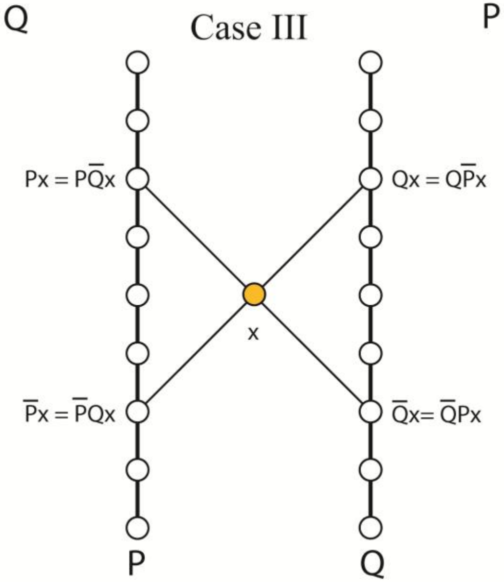


Induced Subspaces



Collinearity

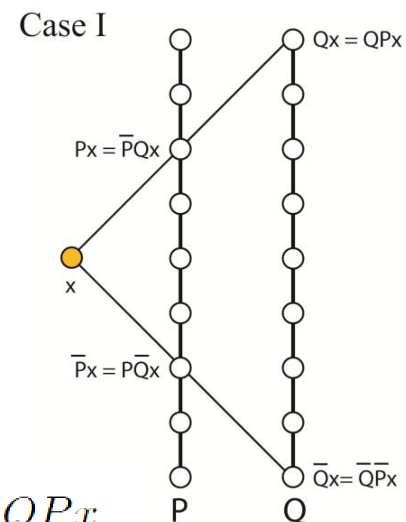


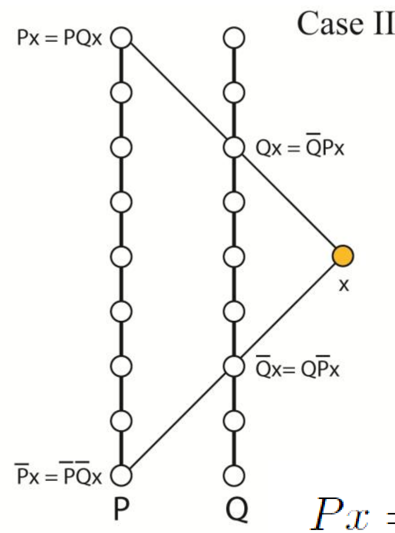


An element *x* is ***collinear*** with finite chains **P** and **Q**, iff the projections P_x and $\bar{P}_x$, can be found by first projecting *x* onto **Q** and then onto **P**, and vice versa.

Betweeness

P-side

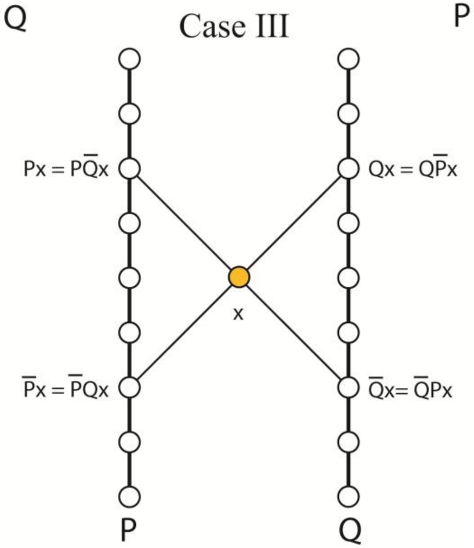




Q-side

$$Px = \overline{P}Qx \quad Qx = QPx$$
$$\overline{P}x = P\overline{Q}x \quad \overline{Q}x = \overline{QP}x$$

$$Px = PQx \quad Qx = \overline{Q}Px$$
$$\overline{P}x = \overline{PQ}x \quad \overline{Q}x = Q\overline{P}x$$

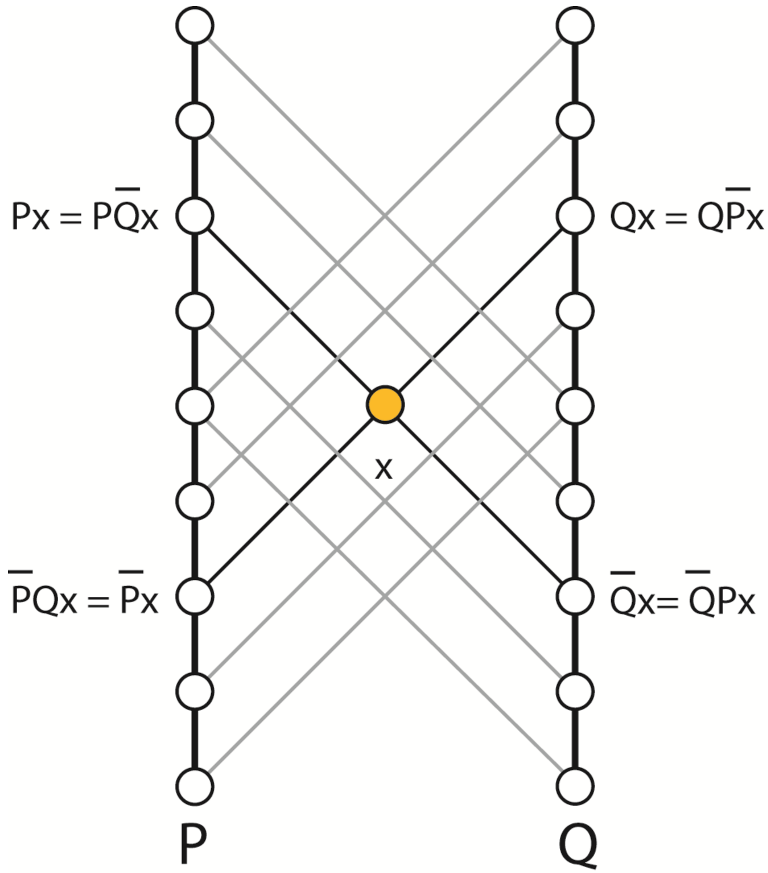


Between

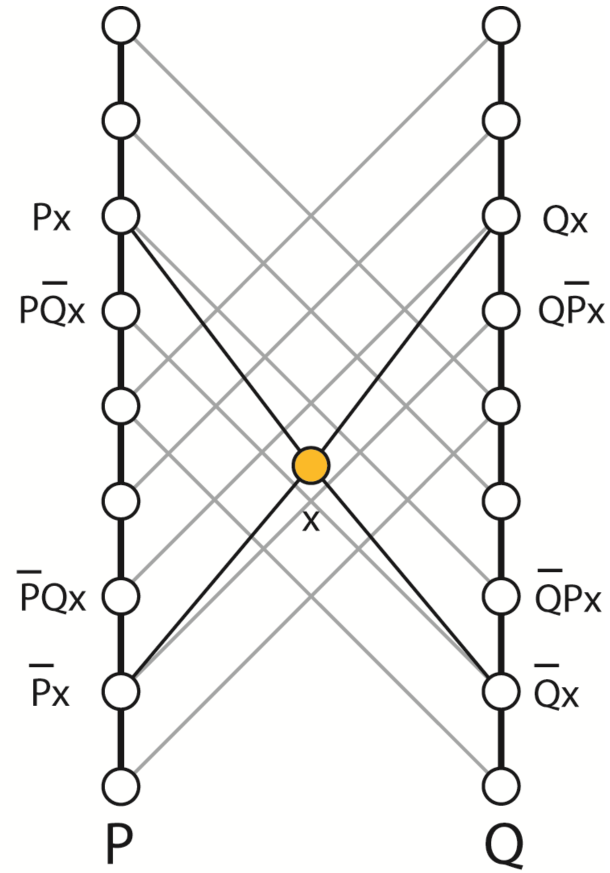
$$Px = P\overline{Q}x \quad Qx = Q\overline{P}x$$
$$\overline{P}x = \overline{P}Qx \quad \overline{Q}x = \overline{Q}Px$$

Induced Subspaces

Every pair of chains induces a subspace in a poset



$$x \in \overline{PQ}$$



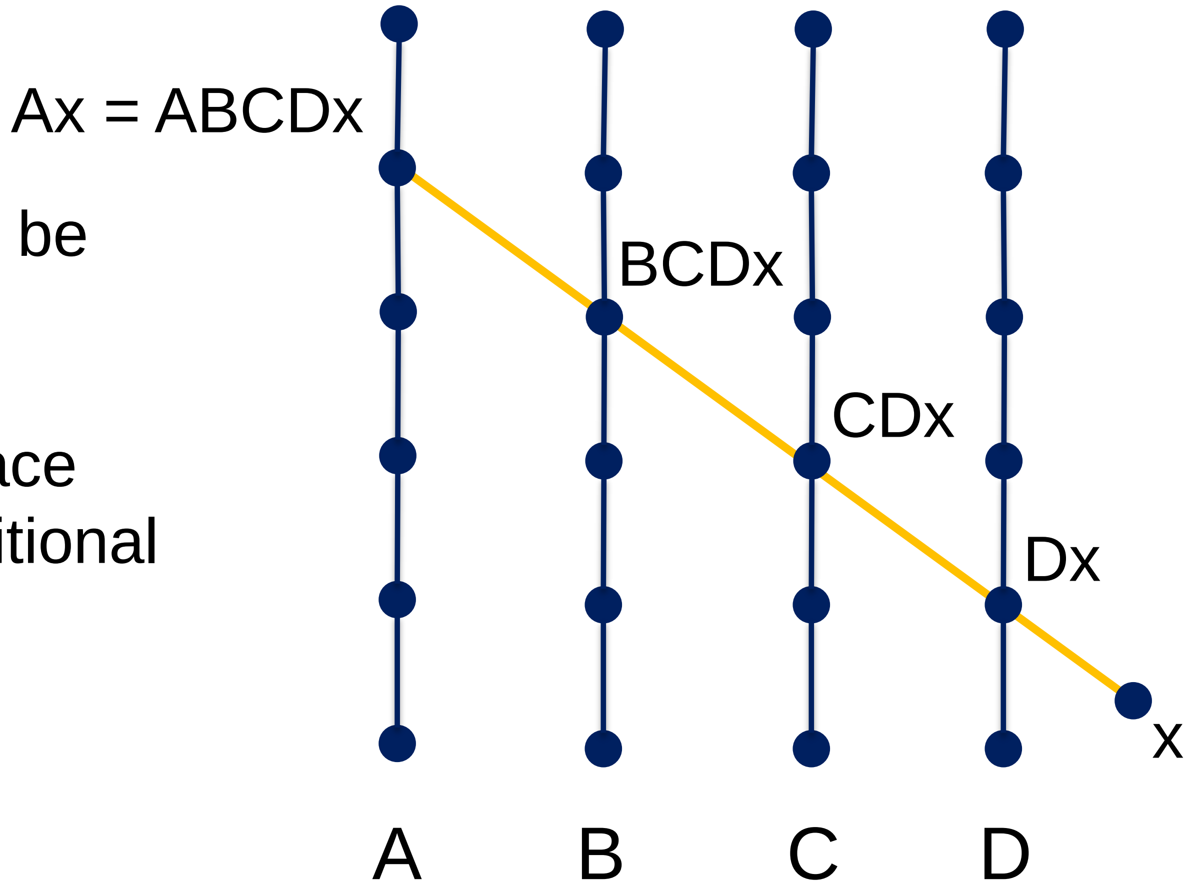
$$x \notin \overline{PQ}$$

1+1 Dimensions

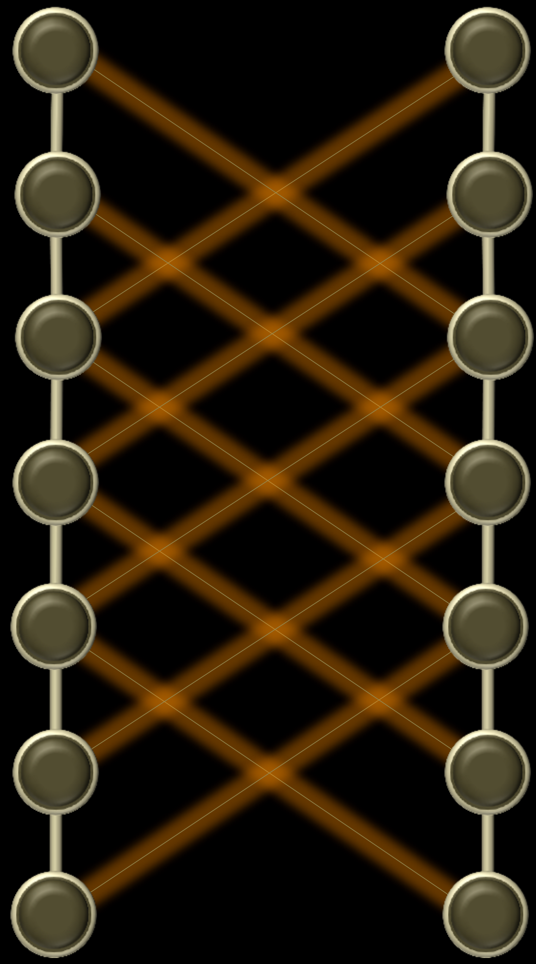
Collinear chains can be ordered.

This induced subspace brings with it an additional dimension.

There can be many induced subspaces.



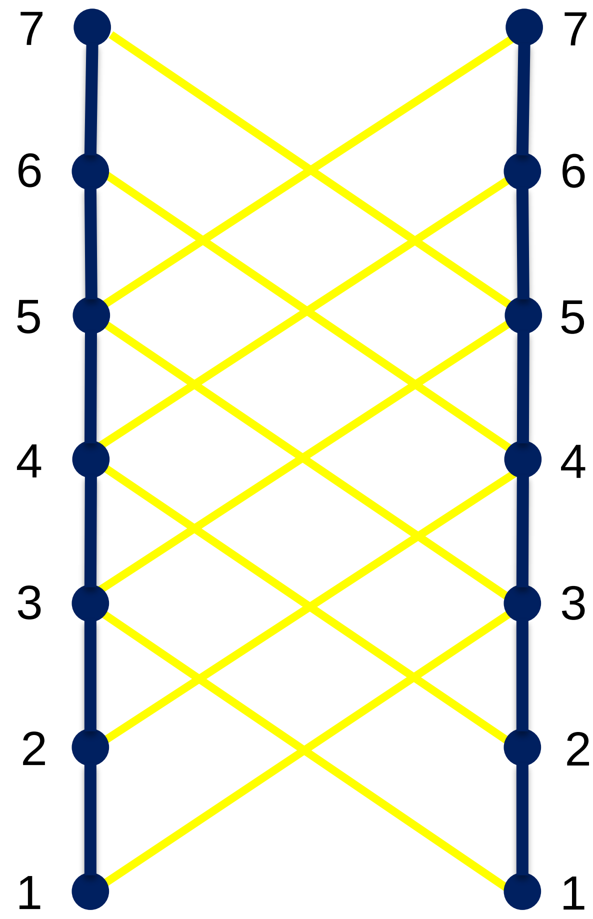
Coordinated Chains



Coordinated Chains

Coordinated Chains
are two chains that agree on lengths of each others intervals

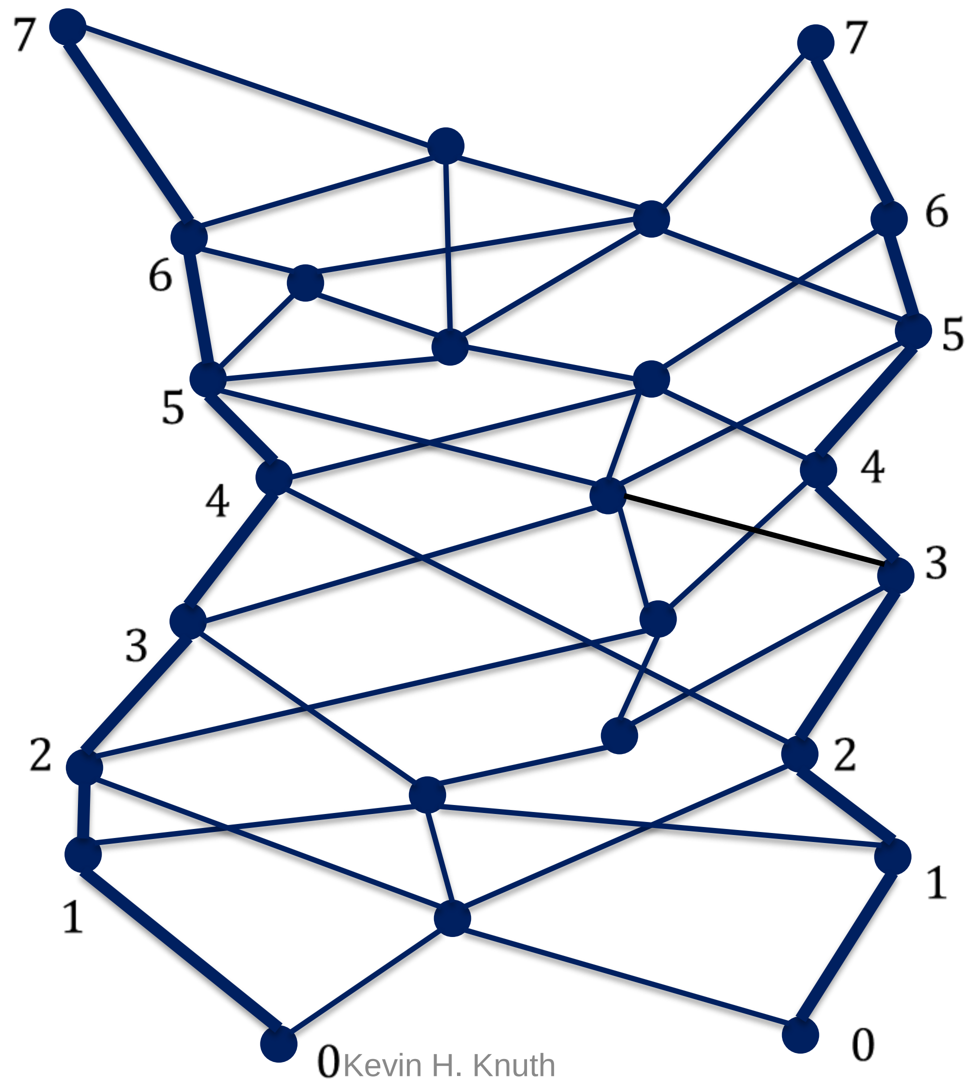
This construct will allow us to explore quantification using only the fact that they are influenced



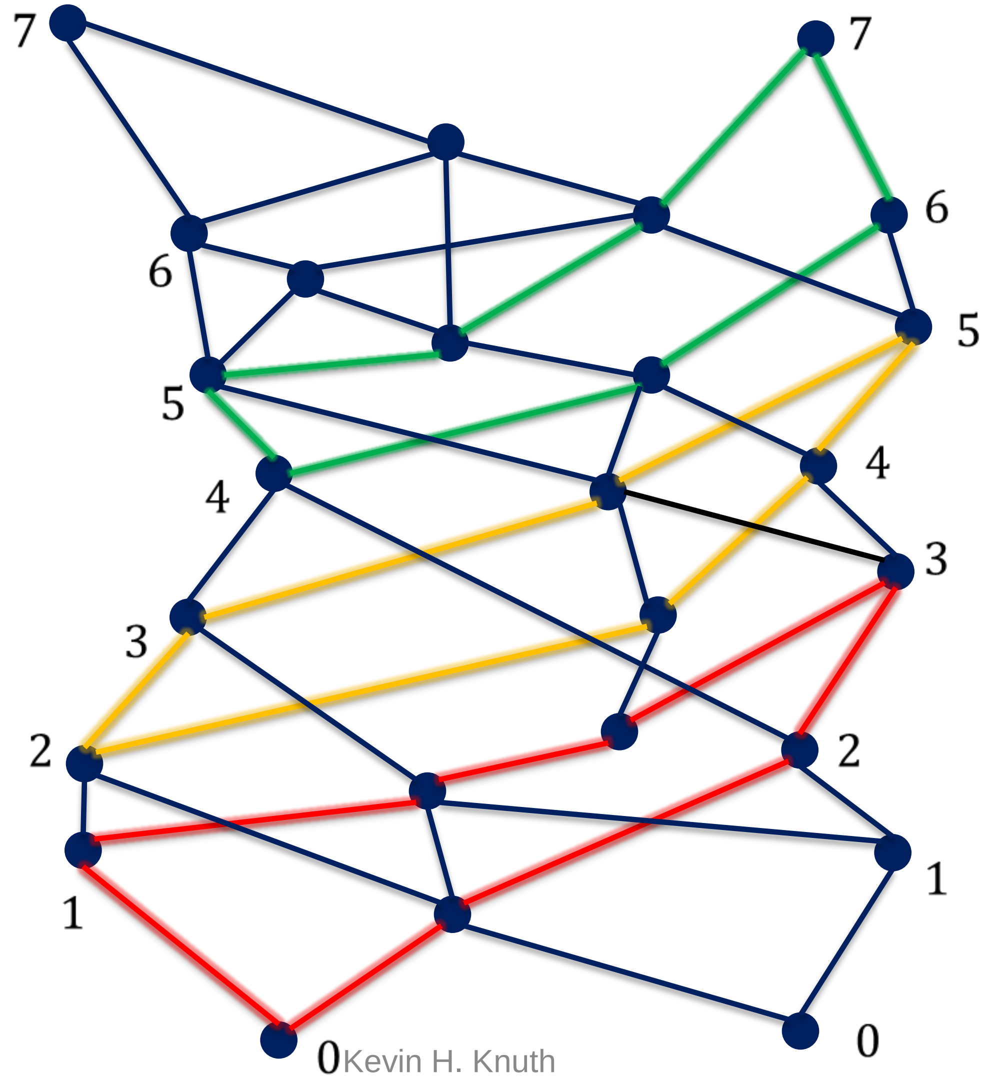
CONSISTENCY PRINCIPLE

If two chains (agents) agree on the quantification of each others' closed intervals, then they must agree on the quantification of every interval they both observe.

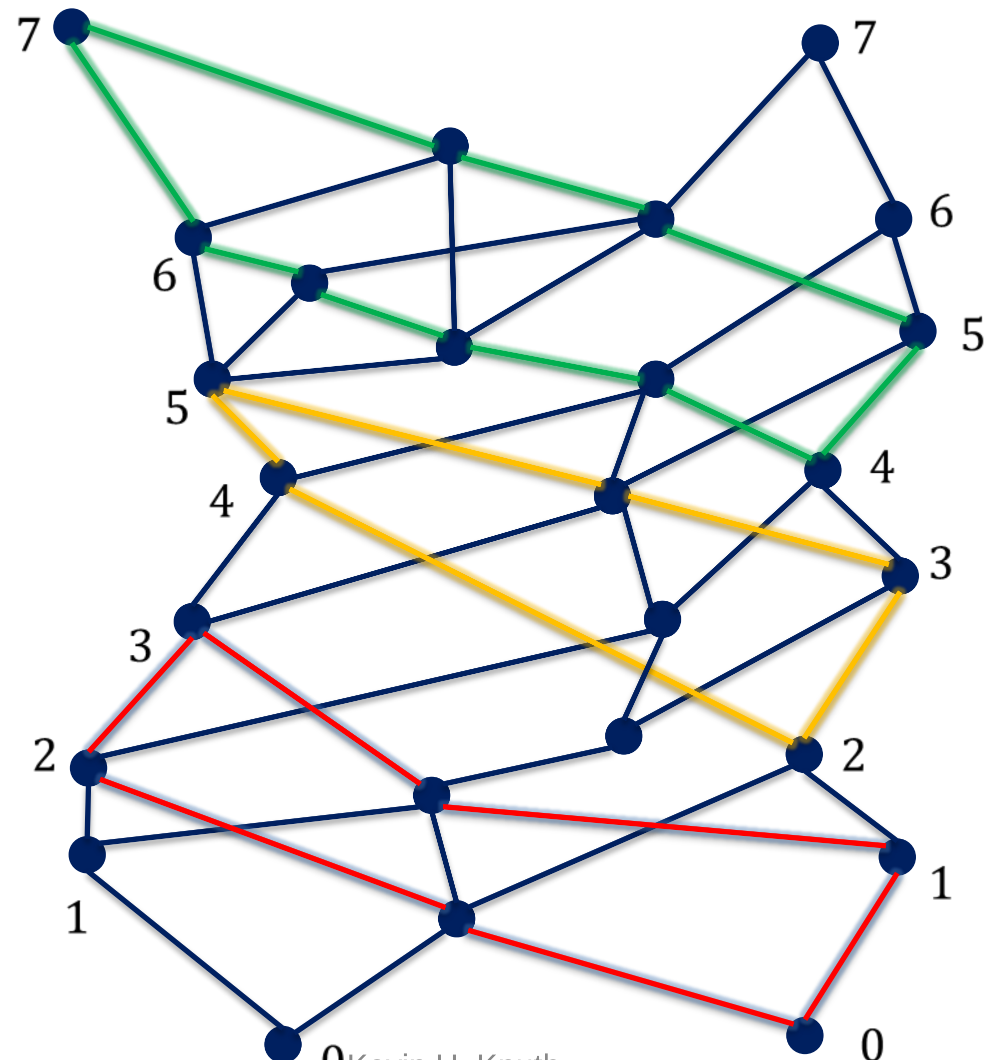
Coordinated Chains



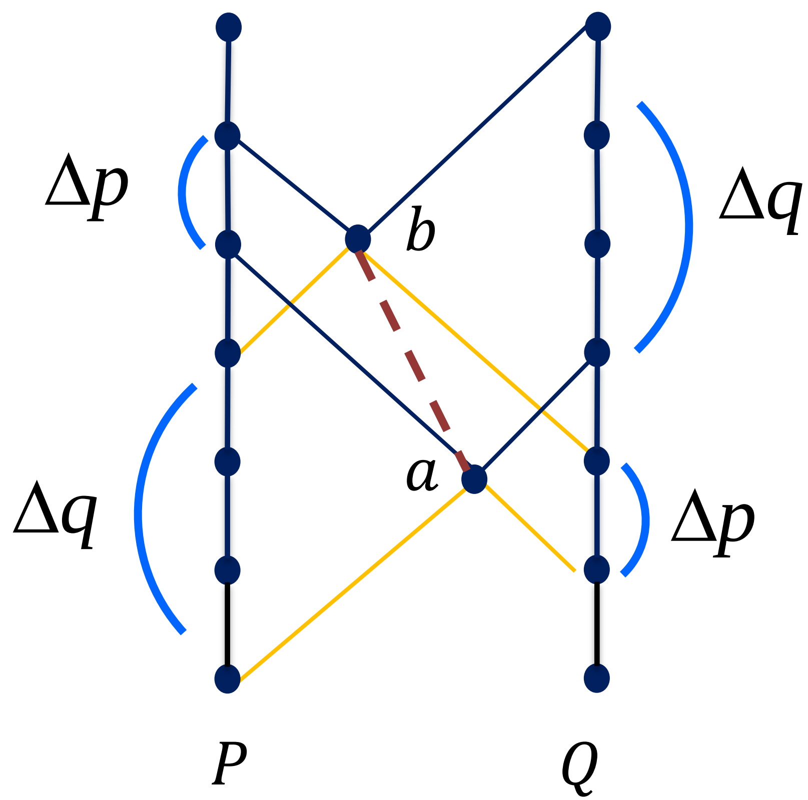
Coordinated Chains



Coordinated Chains



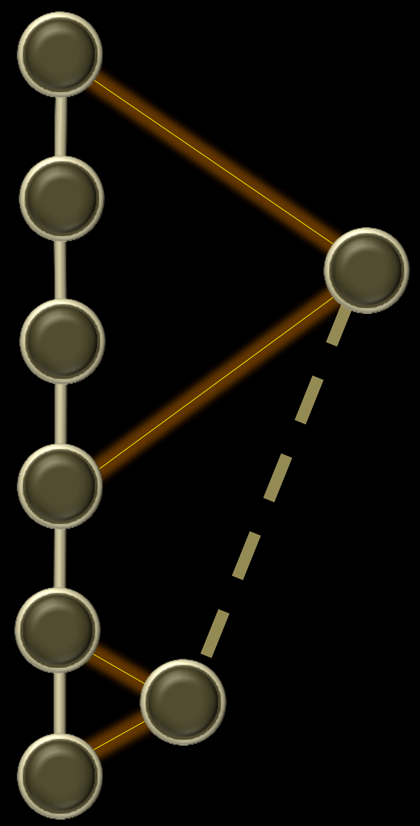
Quantification via Coordinated Chains



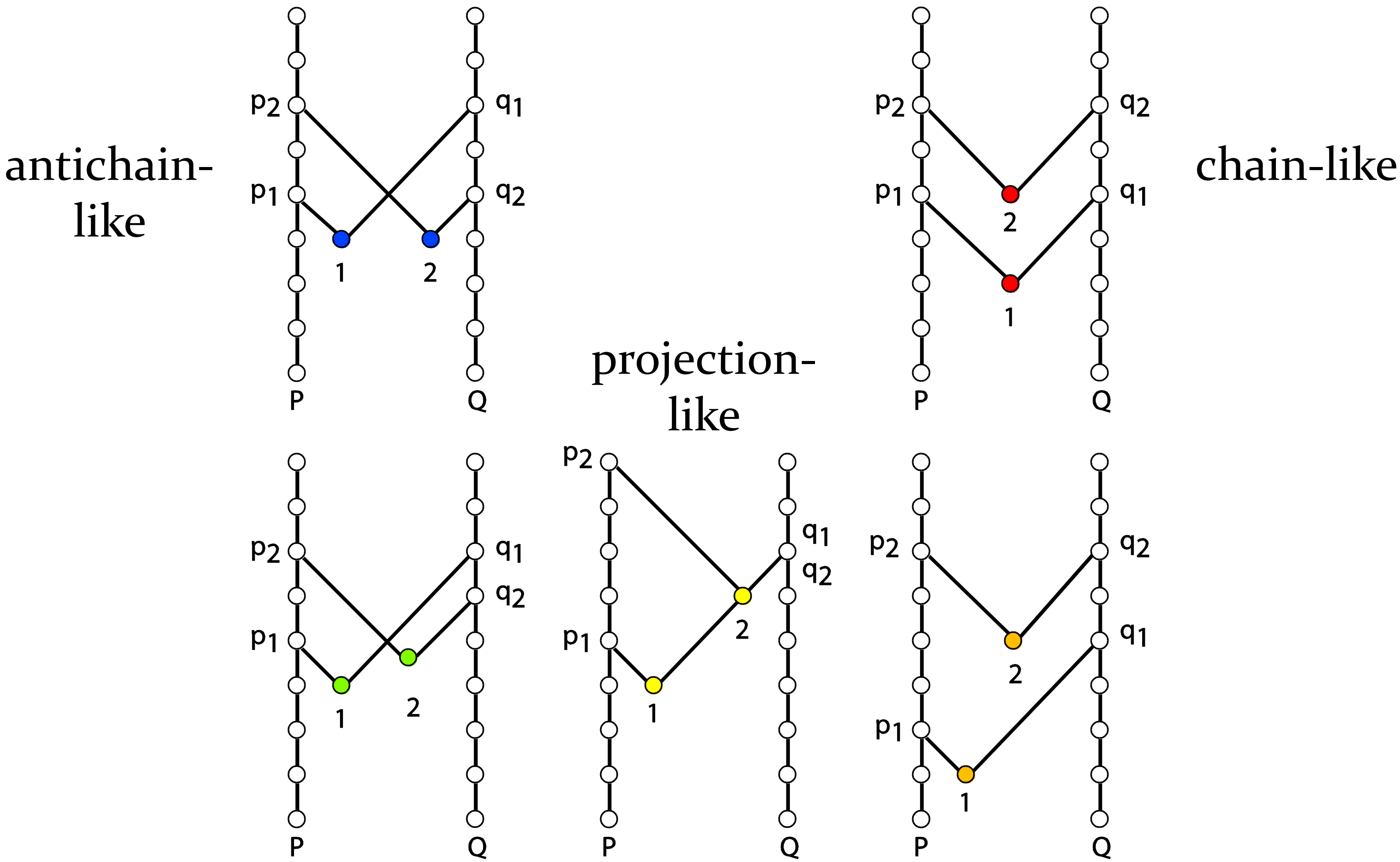
Interval Pair $(\Delta p, \Delta q) = \left(p_b - p_a, q_b - q_a\right)$

Interval Scalar $\Delta p \Delta q = \left(p_a - p_a\right)\left(q_b - q_a\right)$

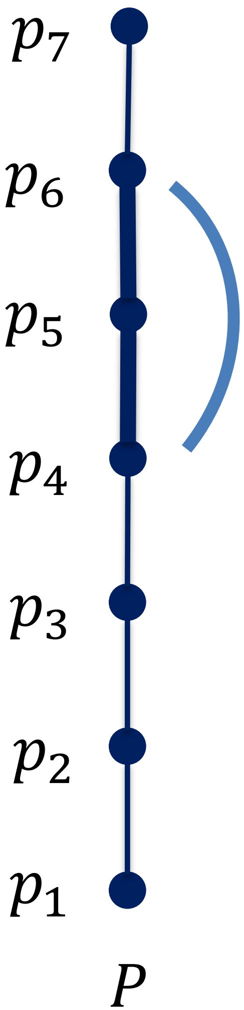
Distance



Interval Classes



Closed Intervals Reside on Chains



$$[p_6, p_4] = \{p_6, p_5, p_4\}$$

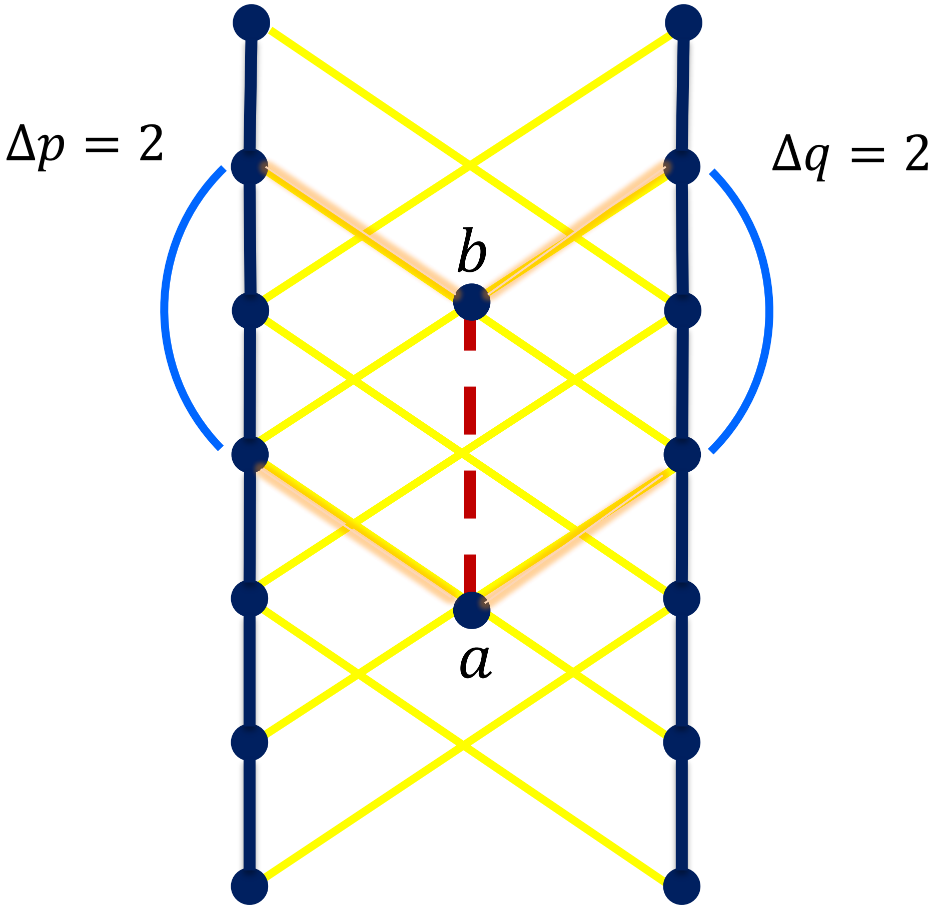
Closed intervals can be quantified by pairs: (p_6, p_4)

Or by scalars (theorem): $p_6 - p_4$

Distance Along Chains

The length of a purely chain-like interval can be written as:

$$d(a,b) = \frac{\Delta p + \Delta q}{2}$$

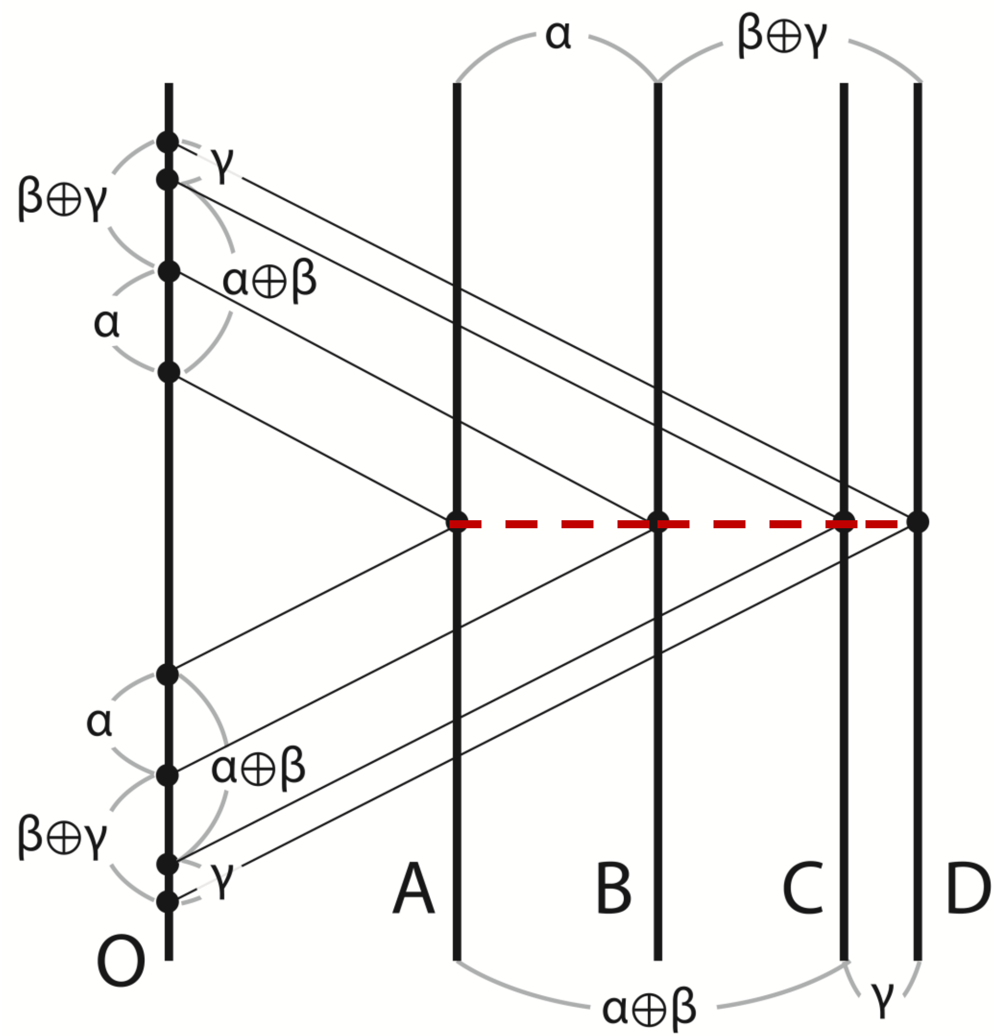


Distance Between Coordinated Chains

Collinear chains can be ordered.

Intervals defined between chains when joined obey associativity.

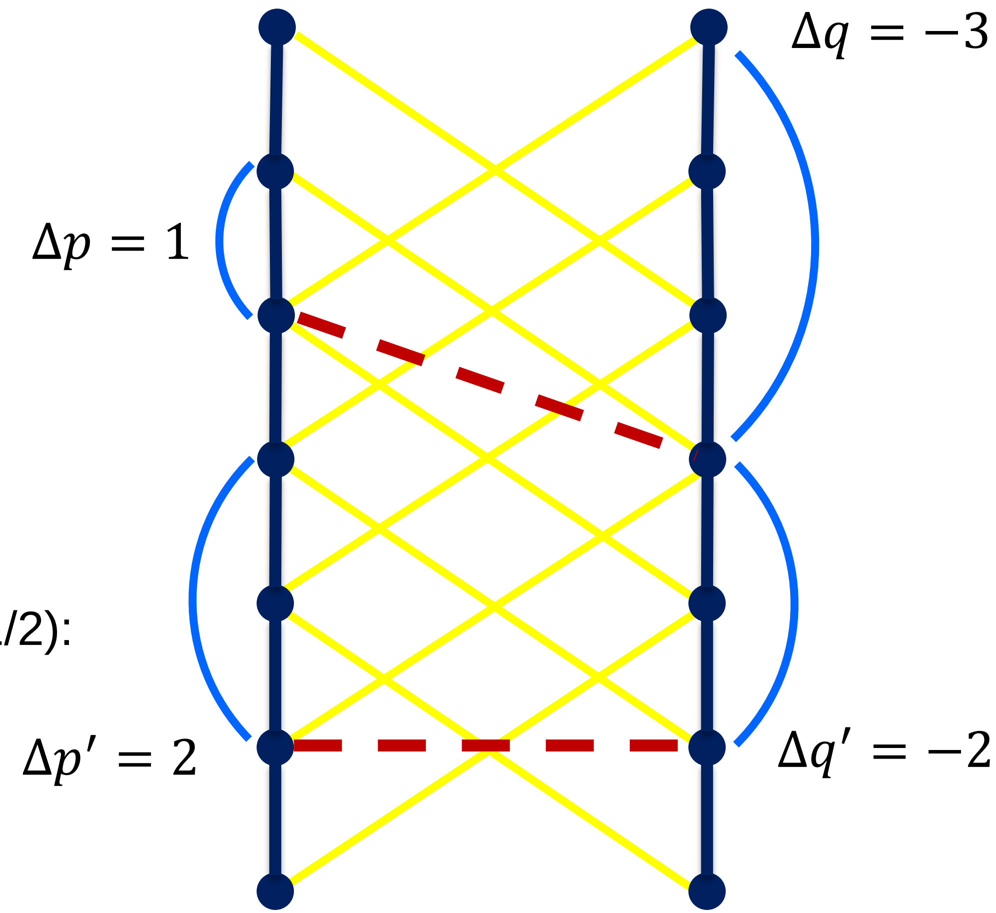
This implies that the quantification of the distance between coordinated chains must be additive (linear).



Distance Between Chains

The distance cannot depend on which elements are used.

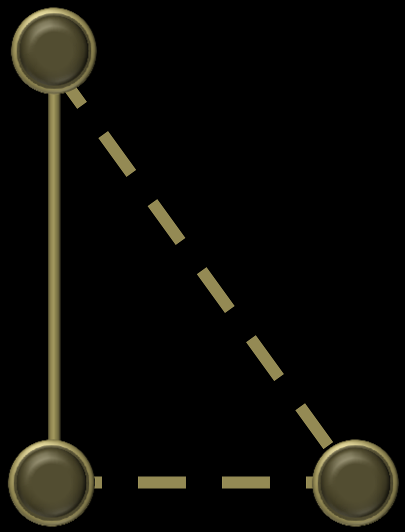
$$D[\![P,Q]\!] = a\Delta p + b\Delta q$$
$$= a\Delta p' + b\Delta q'$$



Solution (setting arb constant to 1/2):

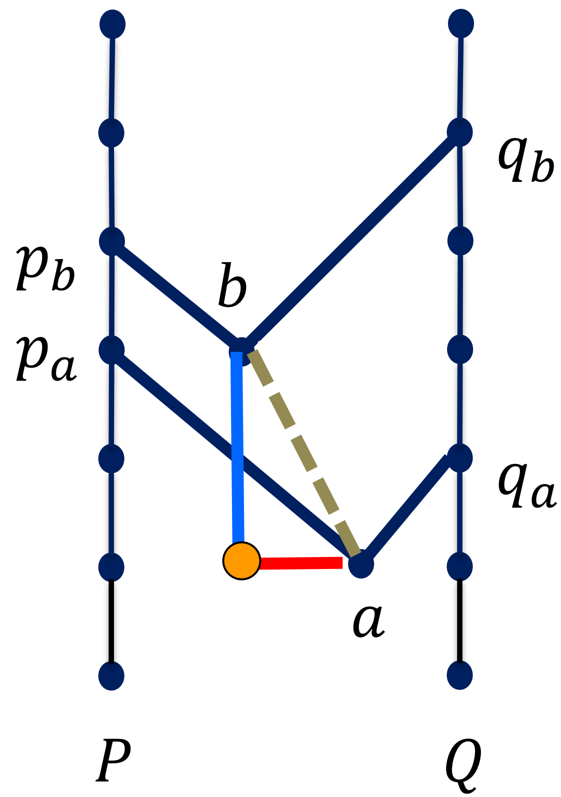
$$D[\![P,Q]\!] = \frac{\Delta p - \Delta q}{2}$$

Symmetric-Antisymmetric Decomposition



Decomposition

$$\Delta p = p_b - p_a$$



$$\Delta q = q_b - q_a$$

$$(\Delta p, \Delta q) = \left(\frac{\Delta p + \Delta q}{2}, \frac{\Delta p + \Delta q}{2}\right) + \left(\frac{(\Delta p - \Delta q)}{2}, \frac{-(\Delta p - \Delta q)}{2}\right)$$

symmetric antisymmetric

Minkowskian Form

THEOREM: Minkowskian Form

The pair when decomposed into symmetric and anti-symmetric pairs

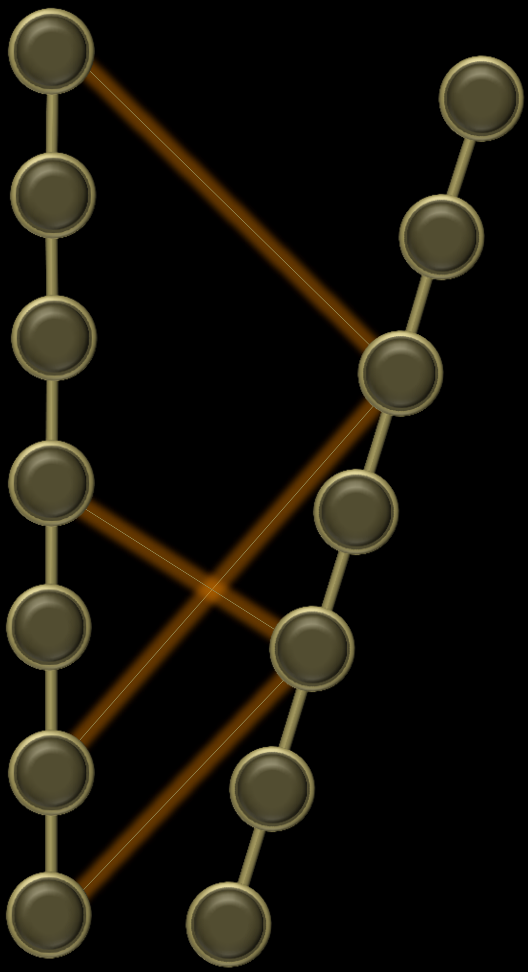
$$(\Delta p, \Delta q) = \left(\frac{\Delta p + \Delta q}{2}, \frac{\Delta p + \Delta q}{2} \right) + \left(\frac{(\Delta p - \Delta q)}{2}, \frac{-(\Delta p - \Delta q)}{2} \right)$$

defines the scalar

$$\Delta p \Delta q = \left(\frac{\Delta p + \Delta q}{2} \right)^2 - \left(\frac{\Delta p - \Delta q}{2} \right)^2$$

which is the sum of the scalars defined by the pairs resulting from the decomposition.

Transformation



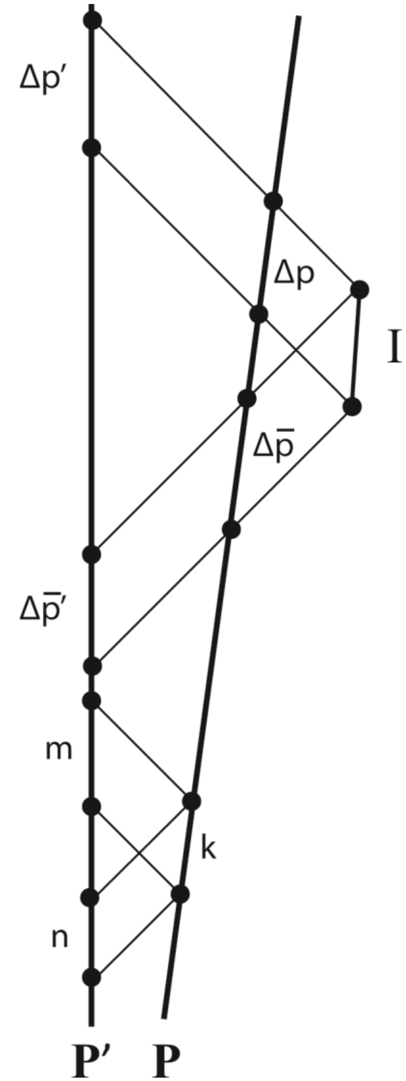
Linearly Related Chains

Consider two chains that project onto one another in a constant fashion:

$(ak, bk)_P = (am, bn)_{P'}$

so that the scalar

$k^2 = mn$



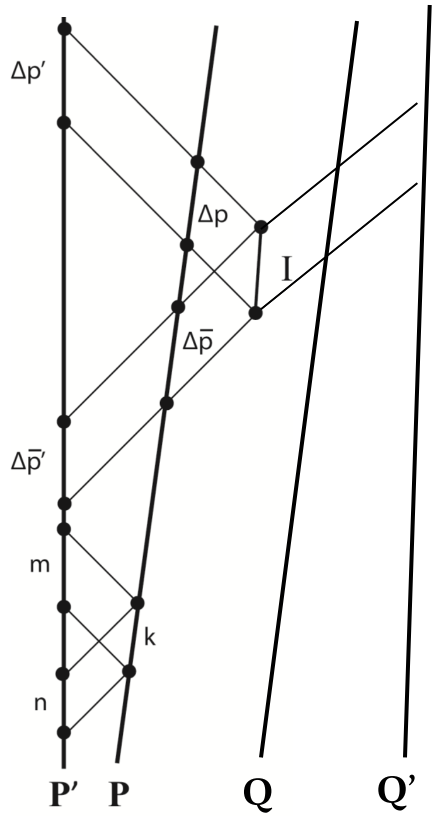
Pair Transformation

THEOREM: Generalized Lorentz Transformation

A pair quantifying an interval in the frame PQ is related to the pair quantifying an interval in the frame P'Q' by

$$L_{PQ \to P'Q'}\,(\Delta p, \Delta q)_{PQ} = \left(\Delta p\,\sqrt{\frac{m}{n}}, \Delta q\sqrt{\frac{n}{m}}\right)_{P'Q'}$$

where the chain-like interval in PQ quantified by $(\sqrt{mn}, \sqrt{mn})$ projects to (m, n) in P'Q'



SPACE-TIME PICTURE

Spacetime
By Kyle Haller

Time reflects the fact that everything does not happen at once

Space reflects the fact that everything does not happen to you

Susan Sontag

Space-Time Picture

$$\Delta t = \frac{\Delta p + \Delta q}{2}$$

$$\Delta x = \frac{\Delta p - \Delta q}{2}$$

Minkowski Metric

$$\Delta p \Delta q = \left(\frac{\Delta p + \Delta q}{2}\right)^2 - \left(\frac{\Delta p - \Delta q}{2}\right)^2$$

$$\Delta s^2 = \Delta t^2 - \Delta x^2$$

Lorentz Transformation

The pair transformation

$$(\Delta p', \Delta q') = (\Delta p\sqrt{\frac{m}{n}}, \Delta q\sqrt{\frac{n}{m}})$$

Can be rewritten as

$$(\frac{\Delta t' + \Delta x'}{2}, \frac{\Delta t' - \Delta x'}{2}) = (\frac{\Delta t + \Delta x}{2}\sqrt{\frac{m}{n}}, \frac{\Delta t - \Delta x}{2}\sqrt{\frac{n}{m}})$$

Lorentz Transformation

Solving for $\Delta t'$ and $\Delta x'$

$$\Delta t' = \frac{\sqrt{\frac{n}{m}} + \sqrt{\frac{m}{n}}}{2}\,\Delta t + \frac{\sqrt{\frac{n}{m}} - \sqrt{\frac{m}{n}}}{2}\,\Delta x$$

$$\Delta x' = \frac{\sqrt{\frac{n}{m}} - \sqrt{\frac{m}{n}}}{2}\,\Delta t + \frac{\sqrt{\frac{n}{m}} + \sqrt{\frac{m}{n}}}{2}\,\Delta x$$

By defining

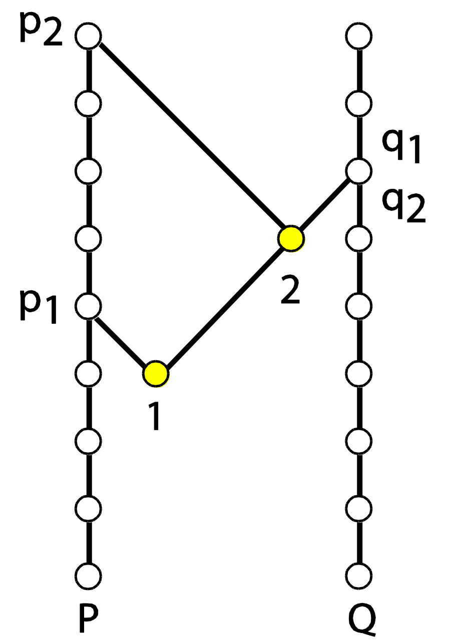
$$\beta = \frac{m-n}{m+n} = \frac{\Delta x}{\Delta t}$$

we have

$$\Delta t' = \frac{1}{\sqrt{1-\beta^2}}\,\Delta t + \frac{-\beta}{\sqrt{1-\beta^2}}\,\Delta x$$

$$\Delta x' = \frac{-\beta}{\sqrt{1-\beta^2}}\,\Delta t + \frac{1}{\sqrt{1-\beta^2}}\,\Delta x$$

Natural Speed Limit



Maximum speed occurs when $n = \Delta q = 0$

$$\beta = \frac{m-n}{m+n} = +1$$

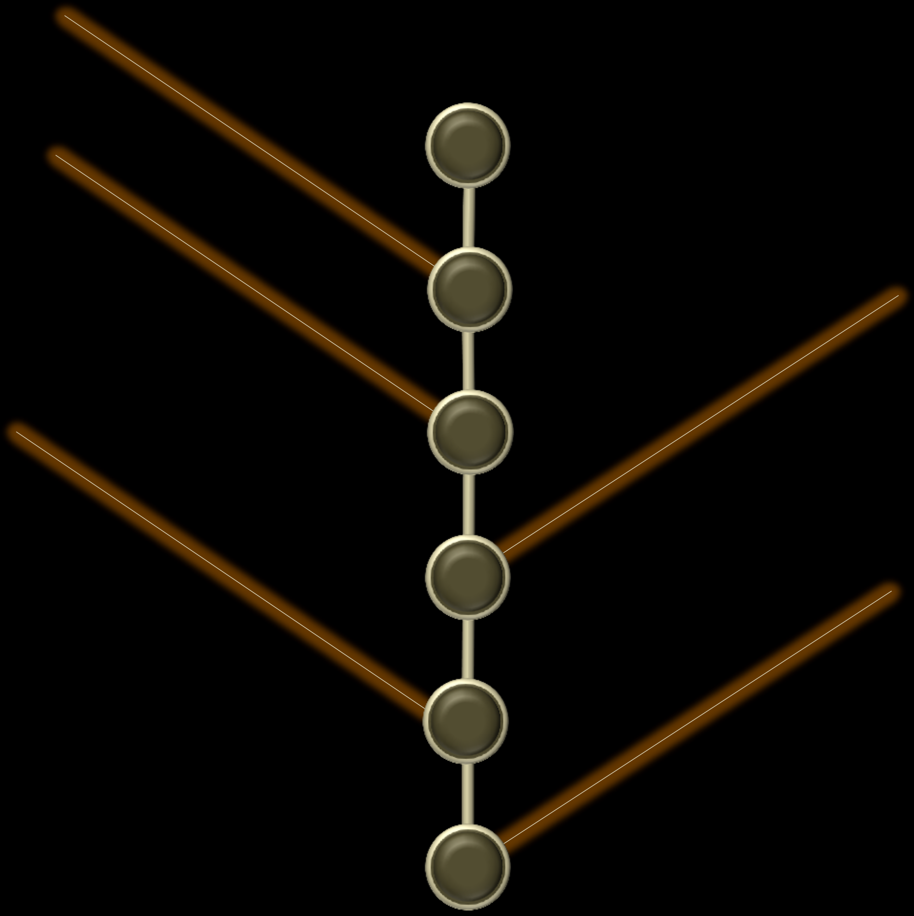
or when $m = \Delta p = 0$ so that

$$\beta = \frac{m-n}{m+n} = -1$$

Such intervals have the same speed $\beta = \pm 1$ in all frames

$$(\Delta p', \Delta q') = \left(0\,\frac{\sqrt{m}}{\sqrt{n}}, \Delta q\,\frac{\sqrt{n}}{\sqrt{m}}\right) = (0, \Delta q')$$

The Free Particle



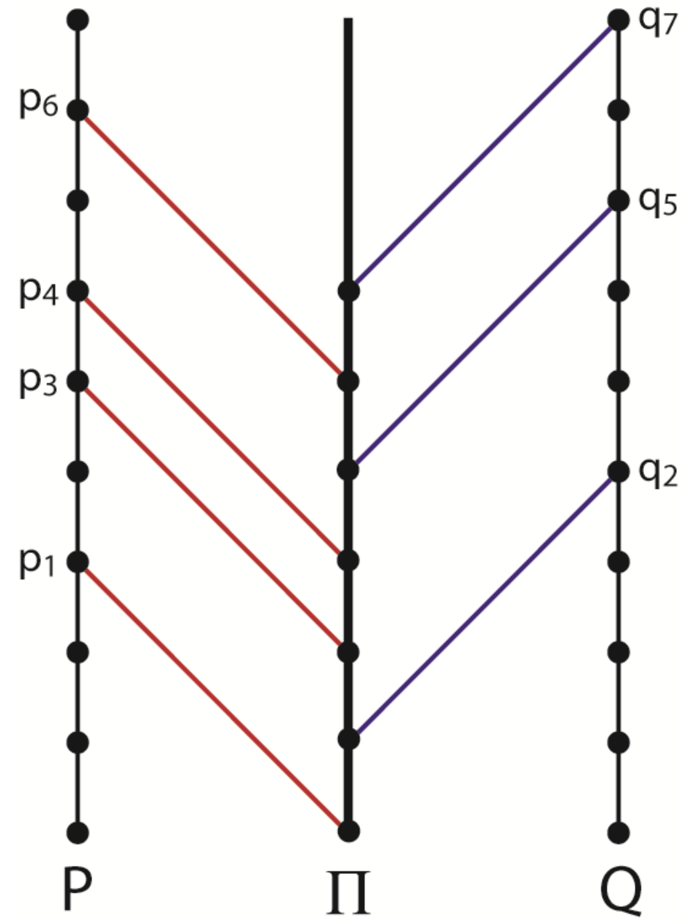
Free Particle

A **free particle** influences but is not influenced

Coordinated chains define a 1+1 dimensional subspace

P and Q : Observer Chains
Π : "Particle"

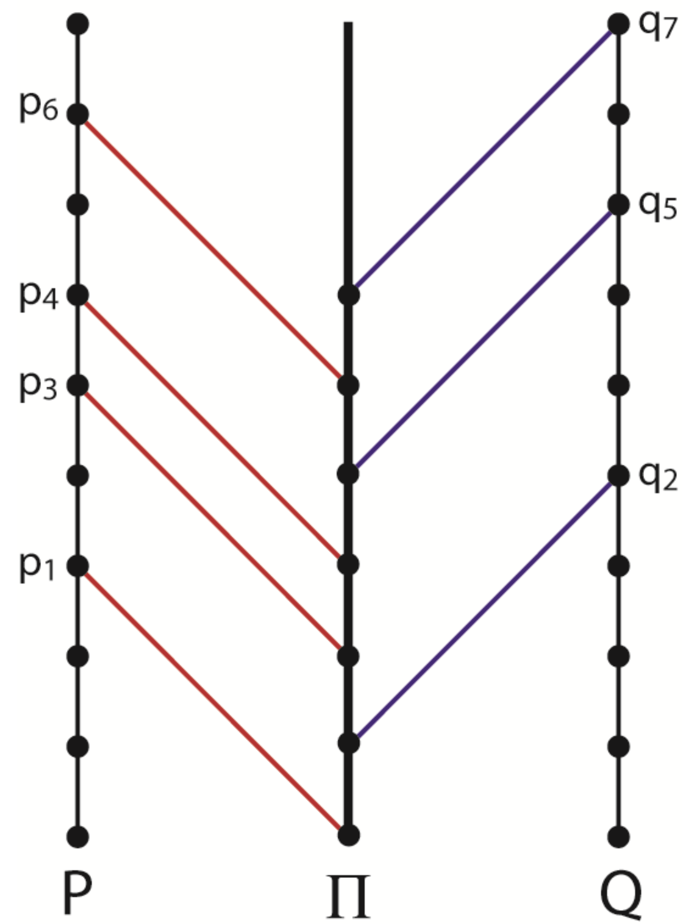
Poset connections represent **direct particle-particle interactions** where Π can influence one observer at a time.



Observer Detections

Observer chain P is influenced at p1, p3, p4, p6

Observer chain Q is influenced at q2, q5, q7

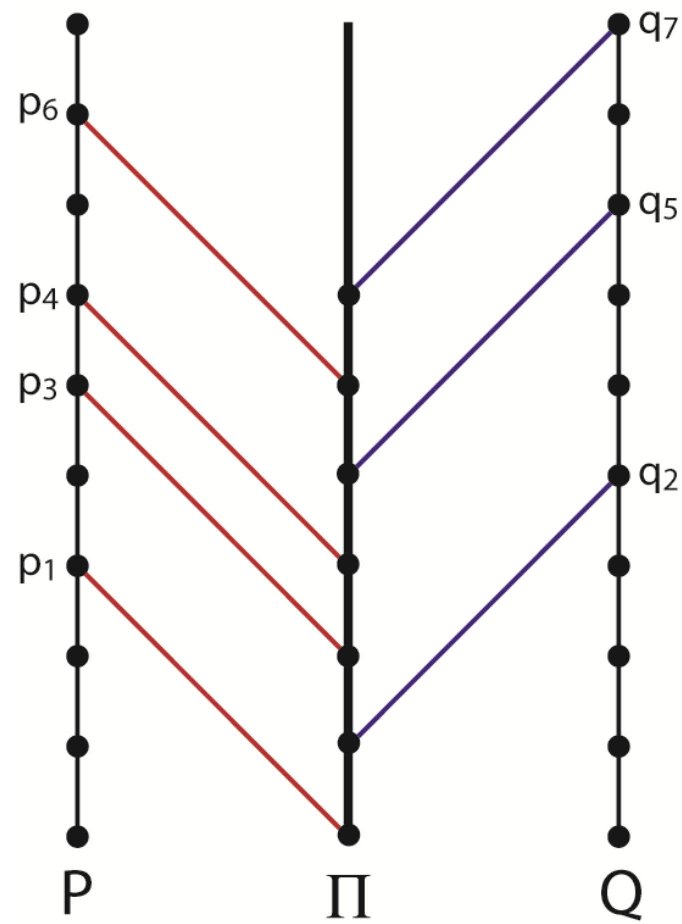


Incomplete Information

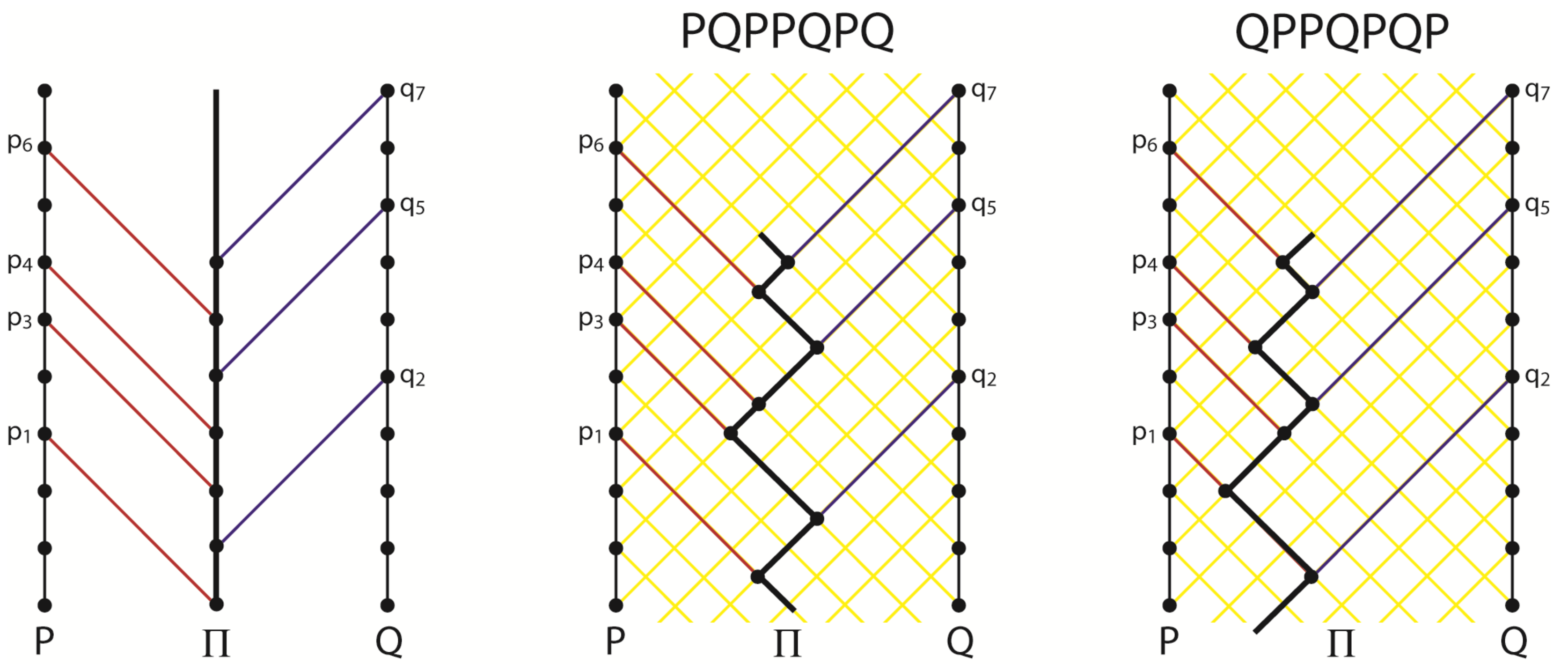
Observer chain P is influenced at p1, p3, p4, p6

Observer chain Q is influenced at q2, q5, q7

However, even by combining detections, the particle interaction pattern cannot be uniquely reconstructed

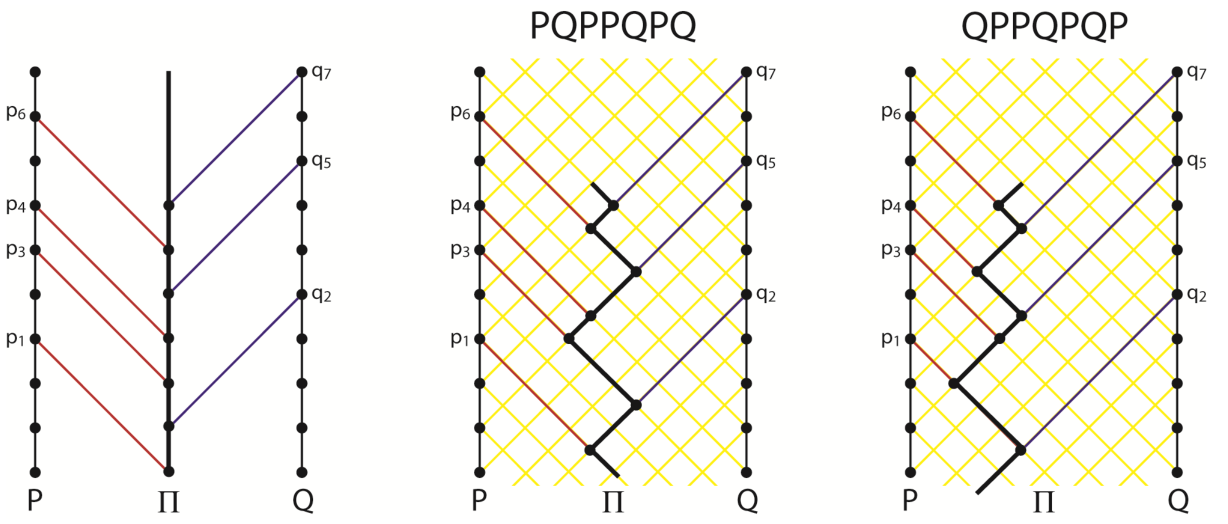


Reconstruction Attempts



There are $\binom{3+4}{3} = \binom{3+4}{4} = 35$ possible reconstructions!

BIT from IT

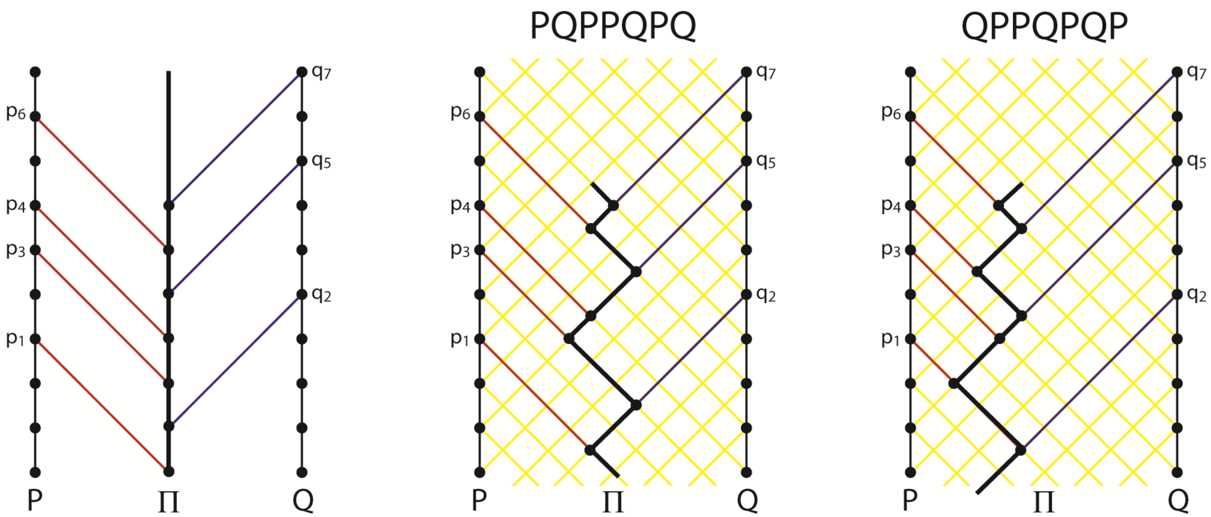


PPPPQQQ
PPPQPQQ
…
PQPPQPQ
…
QPPQPQP
QQQPPPP

The detected interactions result in 35 possible bit strings.

Poset Picture: 35 possible interaction patterns
Space-Time Picture: 35 possible space-time paths

Zitterbewegung

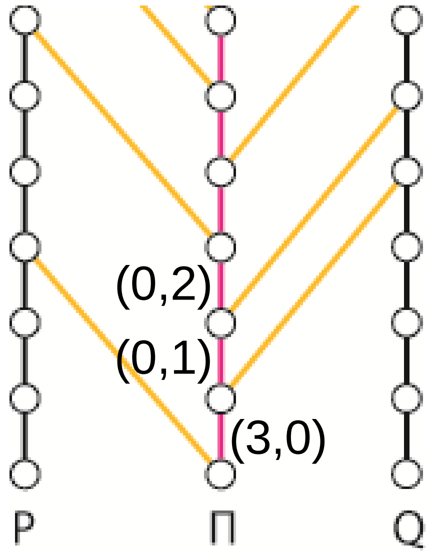


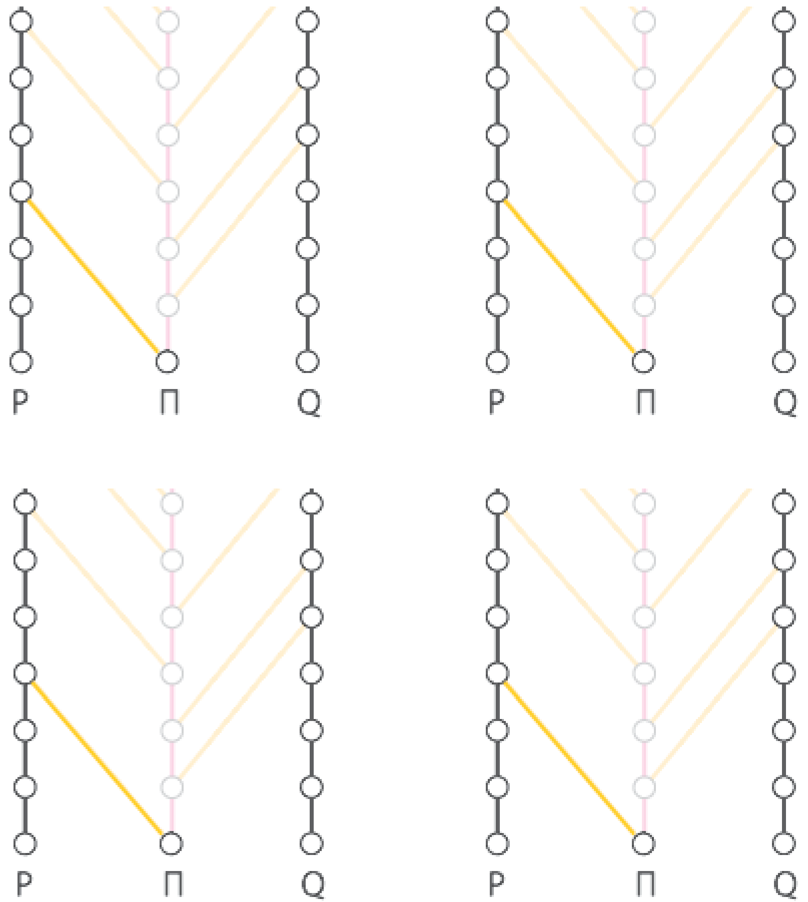
The particle is interpreted as zig-zagging back-and-forth at the maximum speed, $\beta = \pm 1$

$$\beta = \frac{\Delta p - \Delta q}{\Delta p + \Delta q}$$

It acts as if making bishop-moves on a chessboard

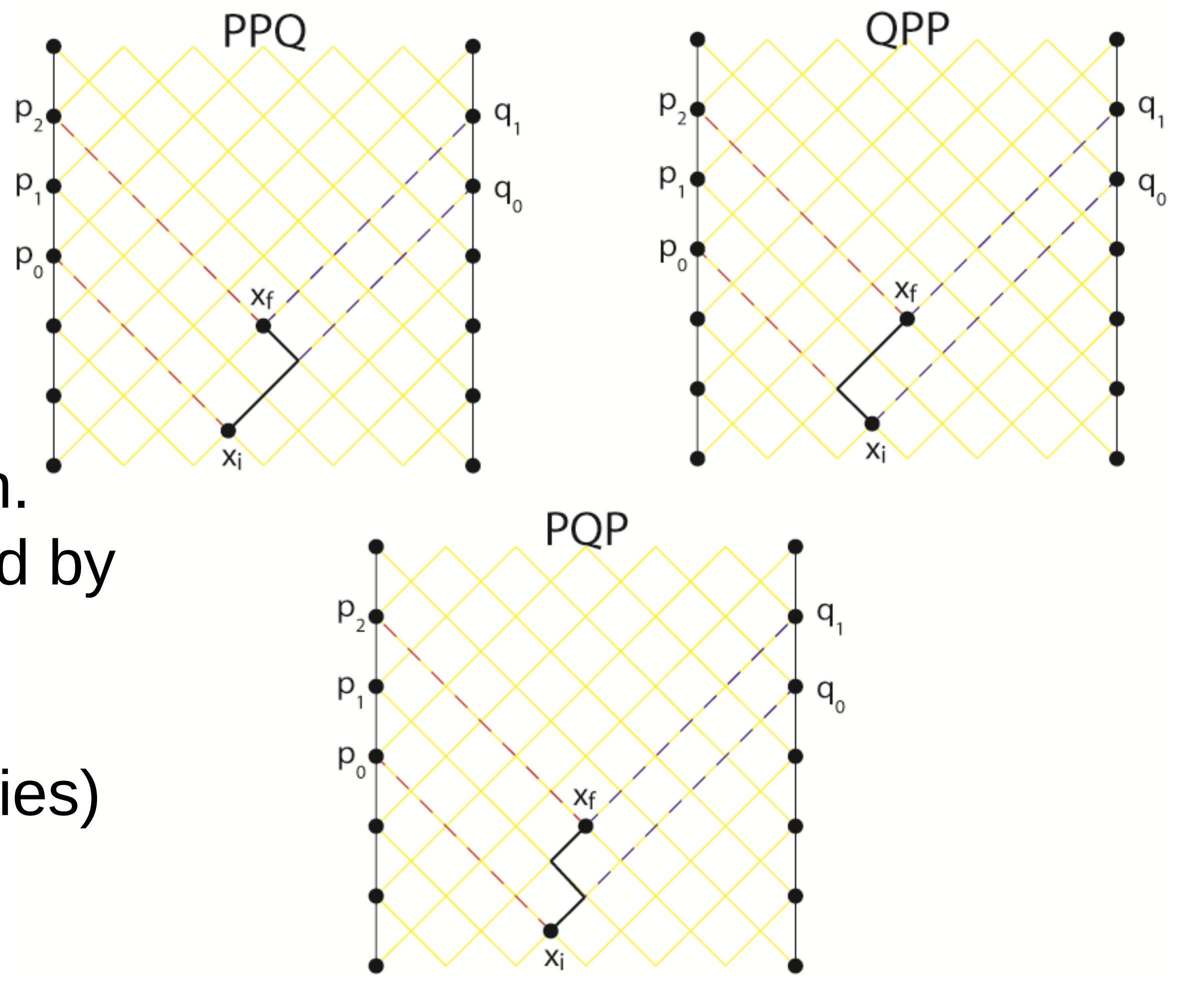
Uncertainty in Position and Speed





Nothing moves in this picture.
Particles "transition" based
on their interactions.

Space-Time Picture (IT from BIT)

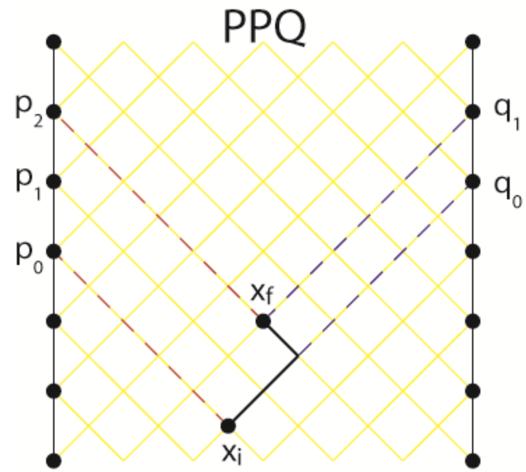


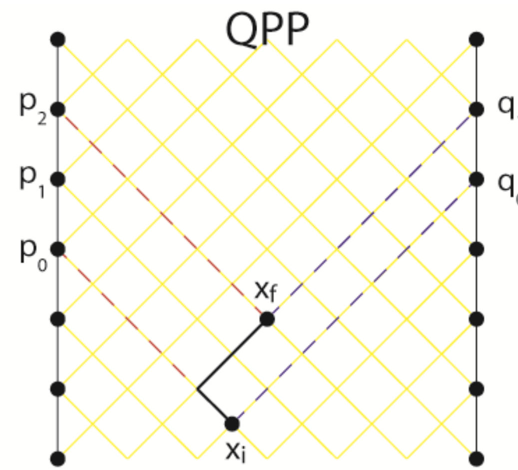
No continuous motion.
Only transition defined by interaction.

Positions (and velocities) are not well-defined.

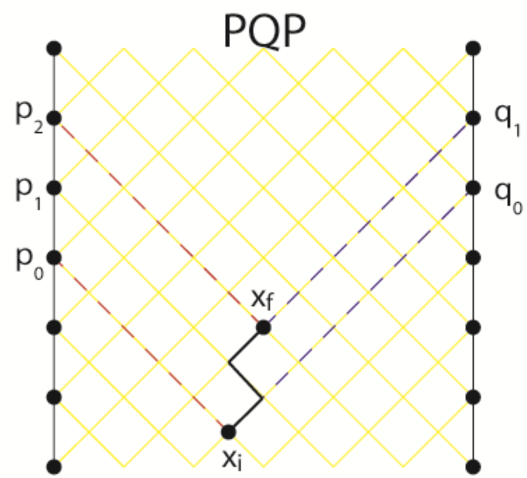
All possible paths must be considered

Space-Time Picture (IT from BIT)



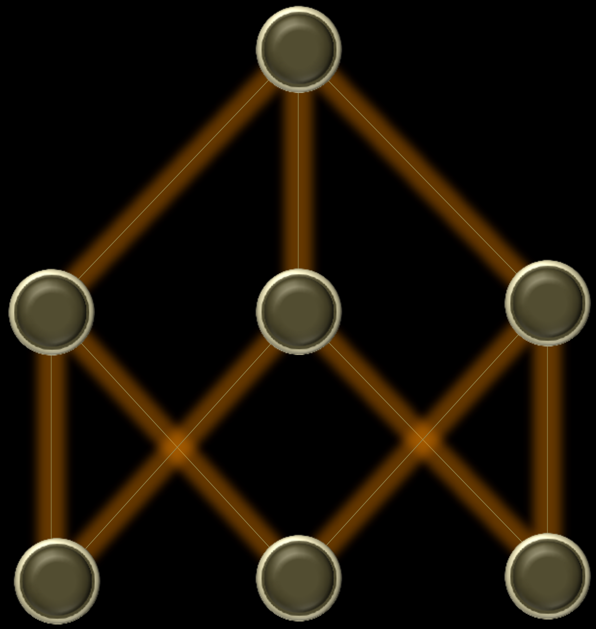


Feynman Checkerboard Model of the Dirac Eqn.
Feynman & Hibbs, 1965



Investigated by many others
eg: Gaveau, Schulman, McKeon, Ord, Gersch, Plavchan, Earle

Inference



Entropy of a Bit Sequence

$$Prob(P) = \frac{\Delta P}{\Delta P + \Delta Q}$$

$$Prob(Q) = \frac{\Delta Q}{\Delta P + \Delta Q}$$

$$\Delta P = \Delta t + \Delta x$$
$$\Delta Q = \Delta t - \Delta x$$

$$\Delta P + \Delta Q = 2\,\Delta t$$

$$\frac{\Delta P}{\Delta P + \Delta Q} = \frac{\Delta t + \Delta x}{2\Delta t} = \frac{1}{2}(1+\beta)$$

$$\frac{\Delta Q}{\Delta P + \Delta Q} = \frac{\Delta t - \Delta x}{2\Delta t} = \frac{1}{2}(1-\beta)$$

$$S = \frac{\Delta P}{\Delta P + \Delta Q}\log\frac{\Delta P}{\Delta P + \Delta Q} + \frac{\Delta Q}{\Delta P + \Delta Q}\log\frac{\Delta Q}{\Delta P + \Delta Q}$$

$$S = \frac{1}{2}(1+\beta)\log\frac{1}{2}(1+\beta) + \frac{1}{2}(1-\beta)\log\frac{1}{2}(1-\beta)$$

.
.
.

Entropy of a Bit Sequence

$$S = -\log\frac{1}{2} + \log\gamma - \beta\log(z+1)$$

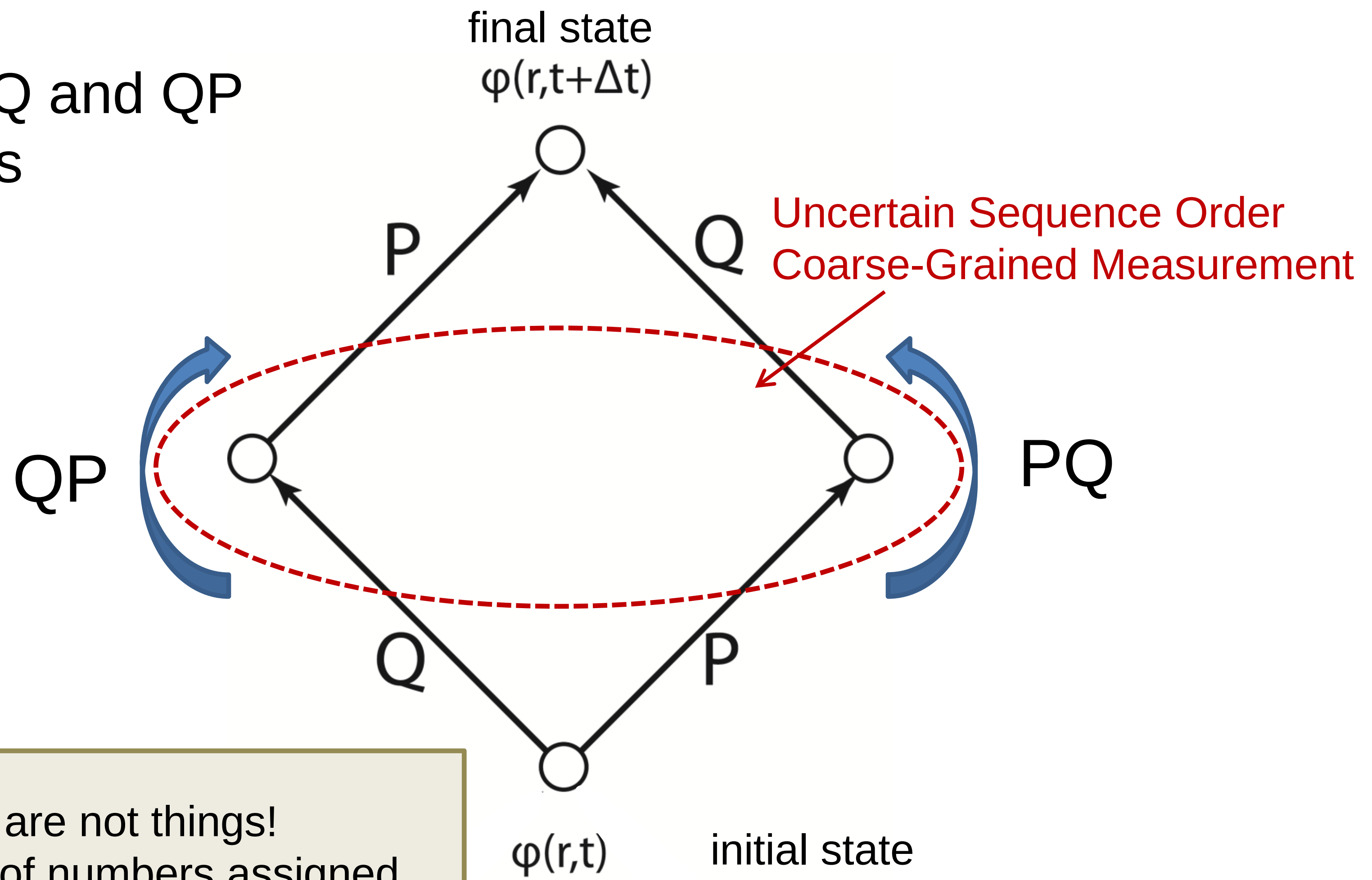
Lorentz factor!

Velocity!

Red-shift!

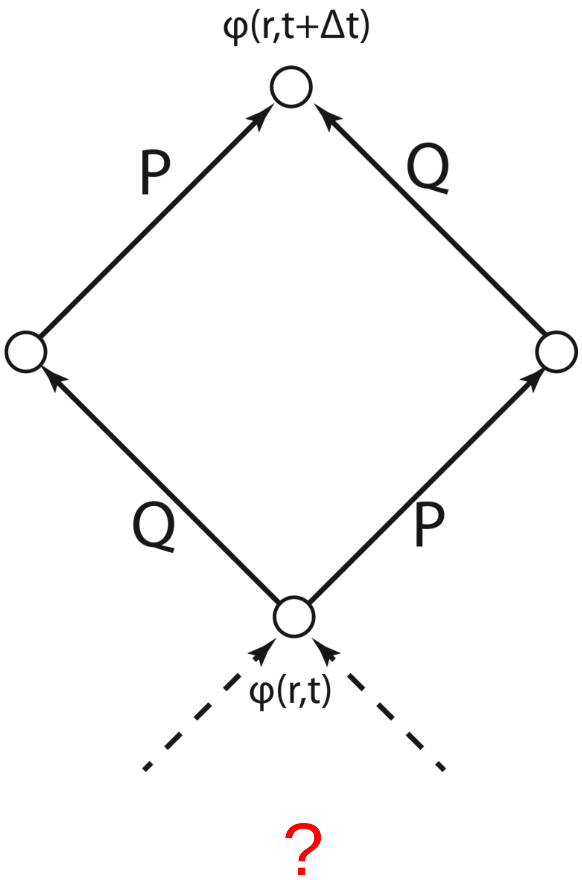
Inferences about Sequences

Look at PQ and QP sequences



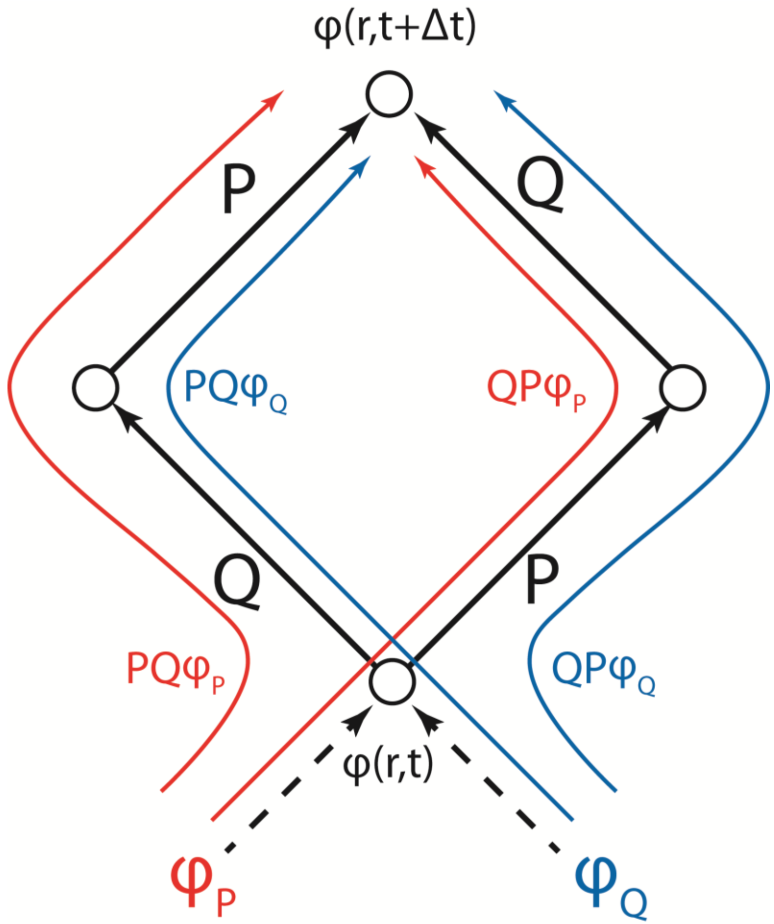
Wavefunctions are not things!
They are pairs of numbers assigned to perform inference.

Initial State is Uncertain!



Two Components

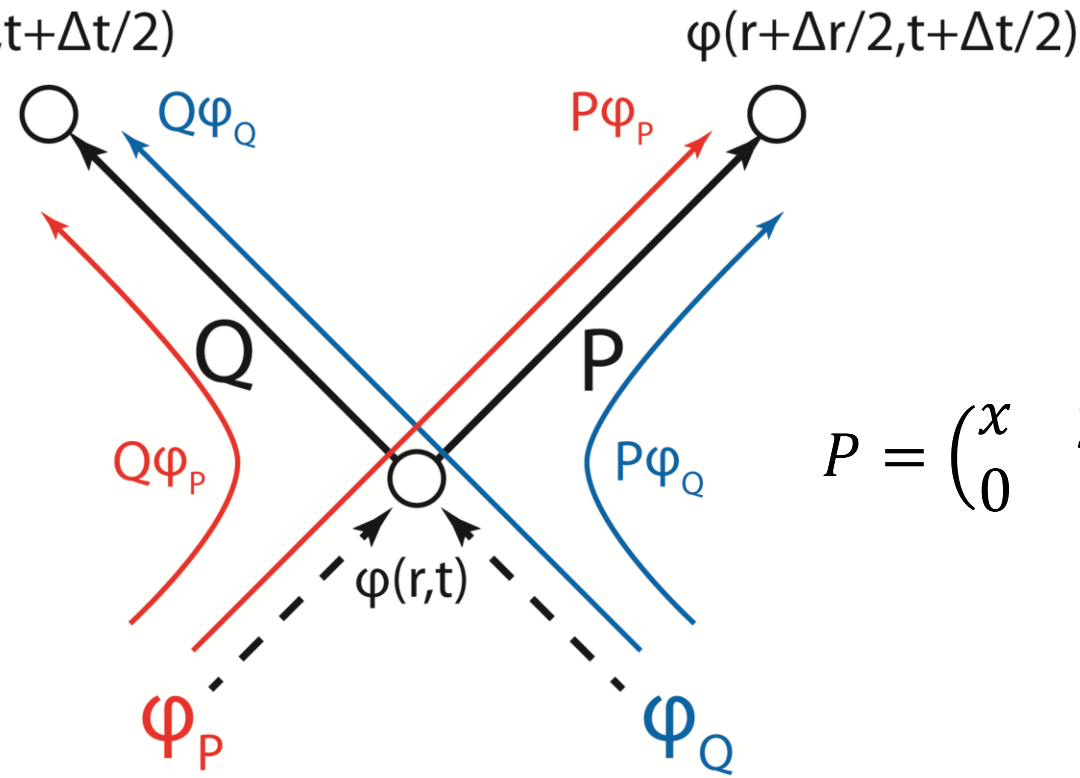
$$\varphi(r,t) = \begin{pmatrix} \varphi_P \\ \varphi_Q \end{pmatrix}$$



Must sum over four sequences!

Handled by summing over two paths per component

Moves = Matrices



$$Q = \begin{pmatrix} 0 & 0 \\ y & x \end{pmatrix}$$

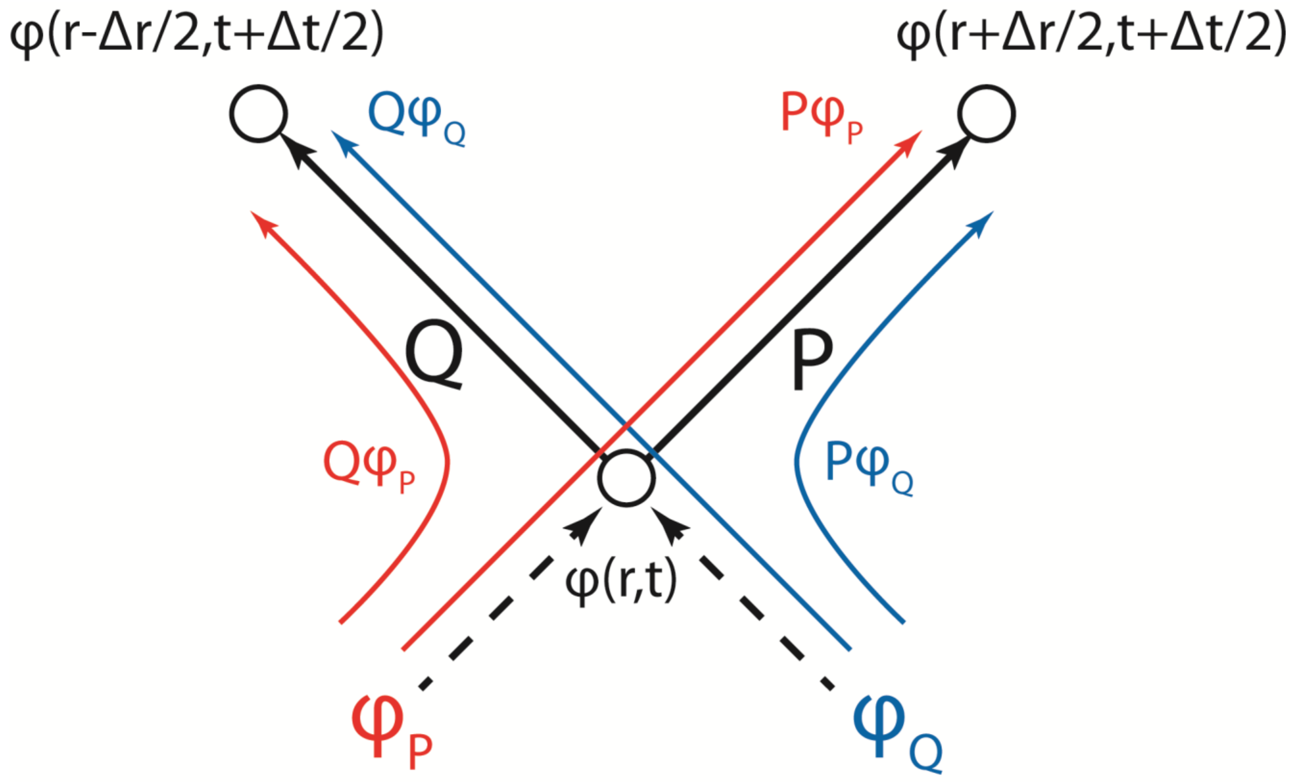
$$P = \begin{pmatrix} x & y \\ 0 & 0 \end{pmatrix}$$

$$P\begin{pmatrix} \varphi_P \\ \varphi_Q \end{pmatrix} = \begin{pmatrix} x & y \\ 0 & 0 \end{pmatrix}\begin{pmatrix} \varphi_P \\ \varphi_Q \end{pmatrix} = \begin{pmatrix} x\varphi_P + y\varphi_Q \\ 0 \end{pmatrix}$$

$$Q\begin{pmatrix} \varphi_P \\ \varphi_Q \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ y & x \end{pmatrix}\begin{pmatrix} \varphi_P \\ \varphi_Q \end{pmatrix} = \begin{pmatrix} 0 \\ y\varphi_P + x\varphi_Q \end{pmatrix}$$

Only Two Possibilities

$$Prob\left(\left(r-\frac{\Delta r}{2},t+\frac{\Delta t}{2}\right)\middle|(r,t)\right)+Prob\left(\left(r+\frac{\Delta r}{2},t+\frac{\Delta t}{2}\right)\middle|(r,t)\right)=1$$



Matrix Constraints

$$Prob\left(\left(r-\frac{\Delta r}{2},t+\frac{\Delta t}{2}\right)\middle|(r,t)\right)+Prob\left(\left(r+\frac{\Delta r}{2},t+\frac{\Delta t}{2}\right)\middle|(r,t)\right)=1$$

$$(Q\varphi\ )^{\dagger}(Q\varphi\ )+\ (P\varphi\ )^{\dagger}(P\varphi\ )=1$$

$$\varphi^{\dagger}\left(Q^{\dagger}Q+\ P^{\dagger}P\right)\varphi=1$$

$$Q^{\dagger}Q+\ P^{\dagger}P=I$$

Matrix Constraints

Since $P = \begin{pmatrix} x & y \\ 0 & 0 \end{pmatrix}$ $Q = \begin{pmatrix} 0 & 0 \\ y & x \end{pmatrix}$

$$Q^{\dagger}Q + P^{\dagger}P = I$$

is

$$\begin{pmatrix} 0 & y^* \\ 0 & x^* \end{pmatrix}\begin{pmatrix} 0 & 0 \\ y & x \end{pmatrix} + \begin{pmatrix} x^* & 0 \\ y^* & 0 \end{pmatrix}\begin{pmatrix} x & y \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

which implies

$$x^*x + y^*y = 1$$

$$x^*y + y^*x = 0$$

Solving Constraints

Write $x = ae^{i\alpha}$ $y = be^{i\beta}$

the constraints

$$x^*x + y^*y = 1$$

$$x^*y + y^*x = 0$$

become

$$a^*a + b^*b = 1$$

$$e^{i\theta} + e^{-i\theta} = 0$$

Where $\theta = \alpha - \beta$

Solution

$$a^* a + b^* b = 1$$

$$e^{i\theta} + e^{-i\theta} = 0$$

The relative phase angle θ must be $\frac{\pi}{2}$ or $\frac{3\pi}{2}$
(need complex numbers)

The amplitudes describe the relative probability of changing direction.
Consider the case where these are equal:

$$a = b = \frac{1}{\sqrt{2}}$$

Transfer Matrices

Choosing x to be real, we have

$$P = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & i\\0 & 0\end{pmatrix} \qquad Q = \frac{1}{\sqrt{2}}\begin{pmatrix}0 & 0\\i & 1\end{pmatrix}$$

So that

$$P\begin{pmatrix}\varphi_P\\0\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}\varphi_P\\0\end{pmatrix}$$

$$P\begin{pmatrix}0\\\varphi_Q\end{pmatrix} = \frac{i}{\sqrt{2}}\begin{pmatrix}\varphi_Q\\0\end{pmatrix}$$

$$Q\begin{pmatrix}\varphi_P\\0\end{pmatrix} = \frac{i}{\sqrt{2}}\begin{pmatrix}0\\\varphi_P\end{pmatrix}$$

$$Q\begin{pmatrix}0\\\varphi_Q\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\\varphi_Q\end{pmatrix}$$

Transfer Matrices

Choosing x to be real, we have

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So that

$$P\begin{pmatrix} \varphi_P \\ 0 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} \varphi_P \\ 0 \end{pmatrix}$$

$$P\begin{pmatrix} 0 \\ \varphi_Q \end{pmatrix} = \frac{i}{\sqrt{2}}\begin{pmatrix} \varphi_Q \\ 0 \end{pmatrix}$$

Factor of i on reversal

$$Q\begin{pmatrix} \varphi_P \\ 0 \end{pmatrix} = \frac{i}{\sqrt{2}}\begin{pmatrix} 0 \\ \varphi_P \end{pmatrix}$$

Factor of i on reversal

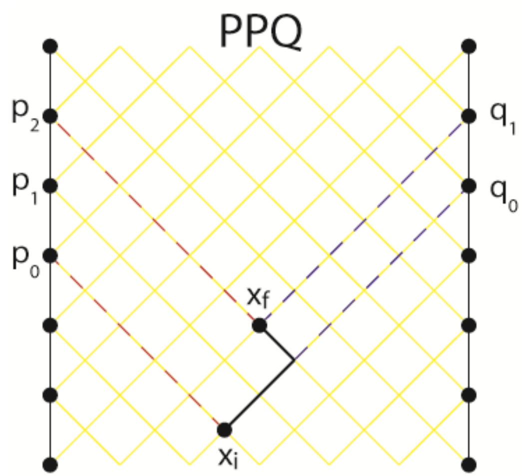
$$Q\begin{pmatrix} 0 \\ \varphi_Q \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \varphi_Q \end{pmatrix}$$

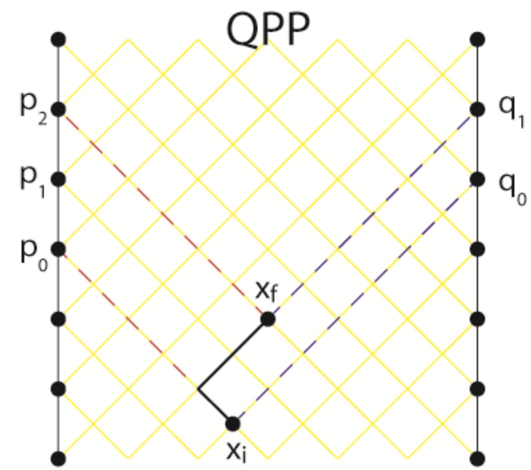
Space-Time Picture (IT from BIT)

Assign an $i\epsilon$ for every reversal.

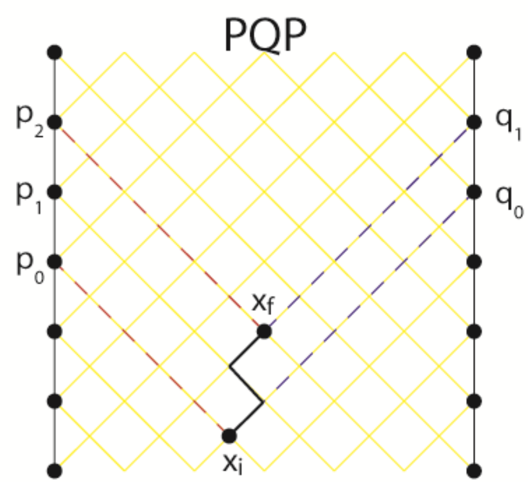
Sum over all possible paths.

Yields Dirac Equation for 1+1 dimensions





Feynman Checkerboard Model of the Dirac Eqn.
Feynman & Hibbs, 1965



Investigated by many others
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THANK YOU

Templeton Foundation
“Quantifying Relations as a Foundation for Physics”

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Philip Goyal
Oleg Lunin
Margaret May
John Skilling

Publications

Knuth K.H., Bahreyni N. 2012. The Physics of Events: A Potential Foundation for Emergent Space-Time. arXiv:1209.0881v1 [math-ph]

Knuth K.H. 2012. Inferences about Interactions: Fermions and the Dirac Equation. MaxEnt 2012. arXiv:1212.2332 [quant-ph]

Knuth K.H., Skilling J. 2012. Foundations of Inference. Axioms 1(1), 38-73; doi:10.3390/axioms1010038

Goyal P., Knuth K.H., Skilling J. 2010. Why Quantum Theory is Complex, Phys. Rev. A 81, 022109. arXiv:0907.0909v3 [quant-ph]

Goyal P., Knuth K.H. 2011. Quantum theory and probability theory: their relationship and origin in symmetry, Symmetry 3(2):171-206. http://www.mdpi.com/2073-8994/3/2/171