Fall 10-24-2014

Physics: Rethinking the Foundations

Kevin H. Knuth
University at Albany, State University of New York, kknuth@albany.edu

Physics:
Rethinking the Foundations

Kevin H. Knuth
Depts. Physics and Informatics
University at Albany

with Prof. Newshaw Bahreyni and James Walsh

Familiarity Breeds the Illusion of Understanding

anonymous

Breaking through the Illusion

The Laws of Physics

Breaking through the Illusion

The Laws of Physics

Decreed by Nature? (Prescribe - Ontology)

> The Laws of Nature are but the mathematical thoughts of God
>
> \- Euclid

Breaking through the Illusion

The Laws of Physics

Decreed by Nature? (Prescribe - Ontology)
Observer-Based Rules for Information Processing? (Describe - Epistemology)

> Observations not only disturb what is to be measured, they produce it.
> \- Pasqual Jordan

> How can it be that mathematics, being after all a product of human thought which is independent of experience, is so admirably appropriate to the objects of reality?
> \- Albert Einstein

> … all things physical are information-theoretic in origin and … this is a participatory universe
> \- John Archibald Wheeler

Breaking through the Illusion

The Laws of Physics

Decreed by Nature? (Prescribe - Ontology)
Observer-Based Rules for Information Processing? (Describe - Epistemology)

Relevant Variables

Breaking through the Illusion

The Laws of Physics

Decreed by Nature? (Prescribe - Ontology)
Observer-Based Rules for Information Processing? (Describe - Epistemology)

Relevant Variables

Breaking through the Illusion

The Laws of Physics

Decreed by Nature? (Prescribe - Ontology)
Observer-Based Rules for Information Processing? (Describe - Epistemology)

Relevant Variables

Foundational?
Convenient?

Breaking through the Illusion

The Laws of Physics

Decreed by Nature? (Prescribe - Ontology)
Observer-Based Rules for Information Processing? (Describe - Epistemology)

Relevant Variables

Foundational?
Convenient?

Motion

Breaking through the Illusion

The Laws of Physics

Decreed by Nature? (Prescribe - Ontology)
Observer-Based Rules for Information Processing? (Describe - Epistemology)

Relevant Variables

Foundational?
Convenient?

Motion

Doctrine of Parmenides?
Zeno's Paradoxes?

Breaking through the Illusion

The Laws of Physics

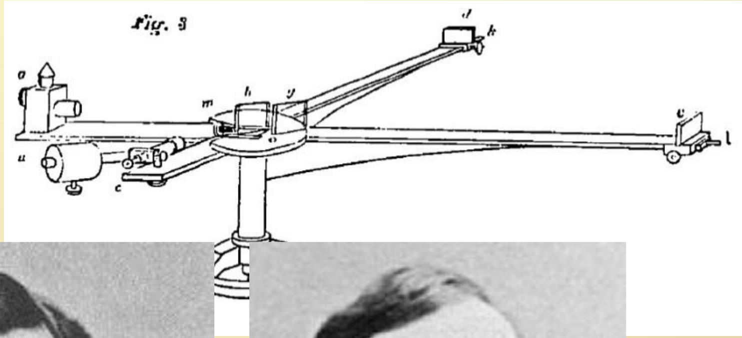
Decreed by Nature? (Prescribe - Ontology)
Observer-Based Rules for Information Processing? (Describe - Epistemology)

Relevant Variables

Foundational?
Convenient?

Motion

Doctrine of Parmenides?
Zeno's Paradoxes?
Constant Speed of Light?



A. A. Michaelson

E. W. Morley

Breaking through the Illusion

The Laws of Physics

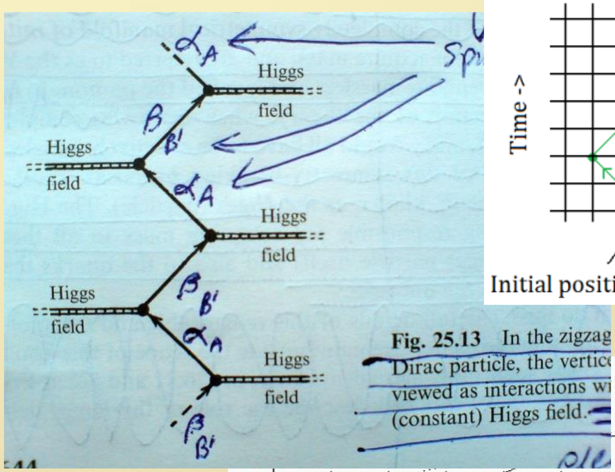
Decreed by Nature? (Prescribe - Ontology)
Observer-Based Rules for Information Processing? (Describe - Epistemology)

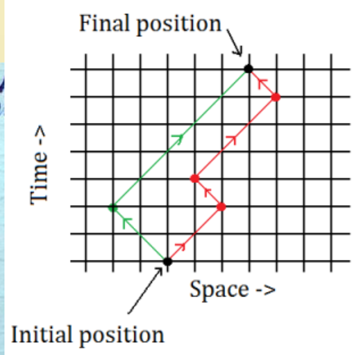
Relevant Variables

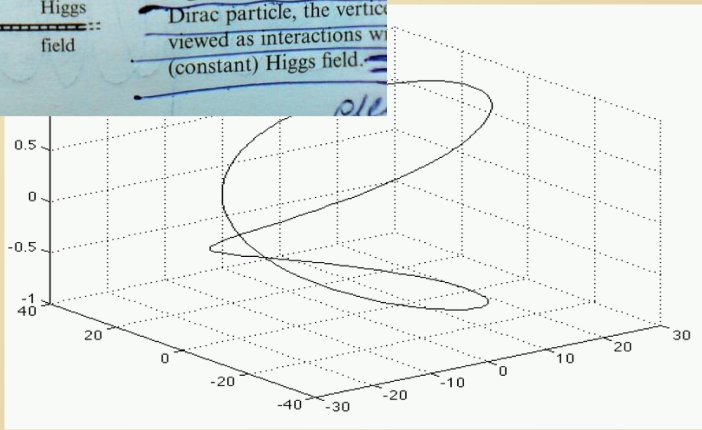
Foundational?
Convenient?

Motion

Doctrine of Parmenides?
Zeno's Paradoxes?
Constant Speed of Light?
Zitterbewegung?







Breaking through the Illusion

The Laws of Physics

Decreed by Nature? (Prescribe - Ontology)
Observer-Based Rules for Information Processing? (Describe - Epistemology)

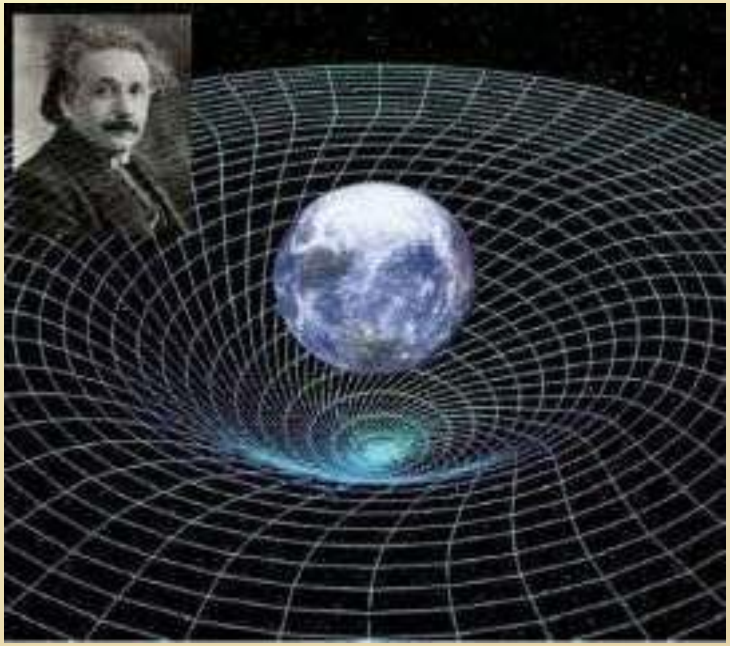
Relevant Variables

Foundational?
Convenient?

Motion

Doctrine of Parmenides?
Zeno's Paradoxes?
Constant Speed of Light?
Zitterbewegung?

Space-Time



Breaking through the Illusion

The Laws of Physics

Decreed by Nature? (Prescribe - Ontology)
Observer-Based Rules for Information Processing? (Describe - Epistemology)

Relevant Variables

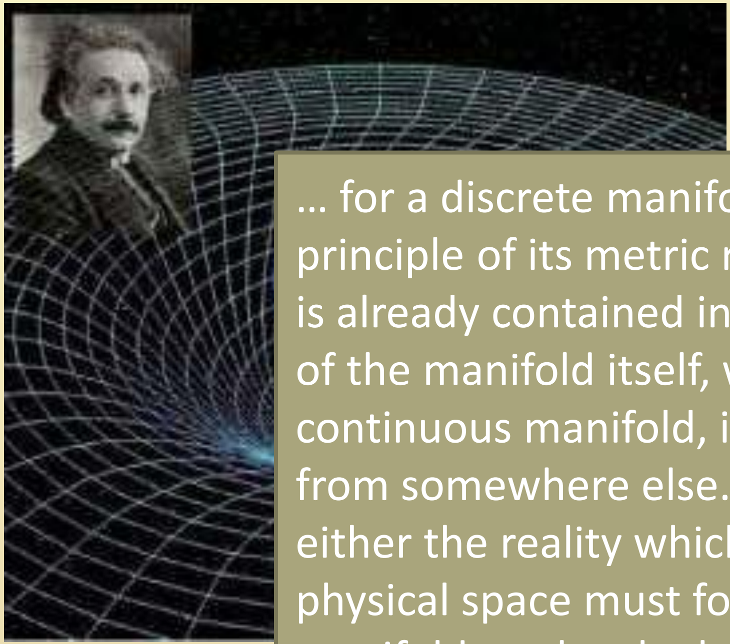
Foundational?
Convenient?

Motion

Doctrine of Parmenides?
Zeno's Paradoxes?
Constant Speed of Light?
Zitterbewegung?

Space-Time

Continuous Manifold?



… for a discrete manifold, the principle of its metric relationships is already contained in the concept of the manifold itself, whereas for a continuous manifold, it must come from somewhere else. Therefore, either the reality which underlies physical space must form a discrete manifold or else the basis of its metric relationships should be sought for outside i t

-Bernard Riemann 1854

Breaking through the Illusion

The Laws of Physics

Decreed by Nature? (Prescribe - Ontology)
Observer-Based Rules for Information Processing? (Describe - Epistemology)

Relevant Variables

Foundational?
Convenient?

Motion

Doctrine of Parmenides?
Zeno's Paradoxes?
Constant Speed of Light?
Zitterbewegung?

Space-Time

Continuous Manifold?
Testable?

> Science … is the most reliable form of knowledge because it is based on testable hypotheses.
>
> \- Paul Davies

Breaking through the Illusion

The Laws of Physics

Decreed by Nature? (Prescribe - Ontology)
Observer-Based Rules for Information Processing? (Describe - Epistemology)

Relevant Variables

Foundational?
Convenient?

Motion

Doctrine of Parmenides?
Zeno's Paradoxes?
Constant Speed of Light?
Zitterbewegung?

Space-Time

Continuous Manifold?
Testable?
Properties?

> I hold that space cannot be curved, for the simple reason that it can have no properties.
>
> Nikolai Tesla, 1932

Breaking through the Illusion

The Laws of Physics

Decreed by Nature? (Prescribe - Ontology)
Observer-Based Rules for Information Processing? (Describe - Epistemology)

Relevant Variables

Foundational?
Convenient?

Motion

Doctrine of Parmenides?
Zeno's Paradoxes?
Constant Speed of Light?
Zitterbewegung?

Space-Time

Continuous Manifold?
Testable?
Properties?
Change vs. Distinguishability?


Spacetime
By Kyle Haller

Starting Over

Electrons

Many of us feel that we have experienced electrons directly.

They seem to be bright crackly sorts of things.

But what are they really?

Electrons

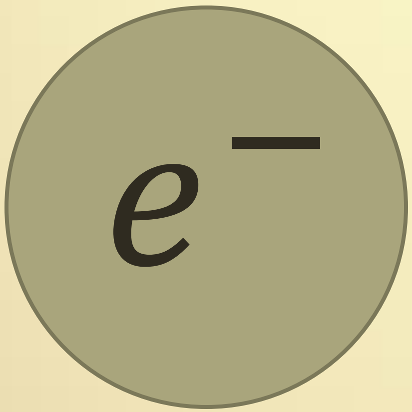
Imagine that electrons might be pink and fuzzy.

Maybe they smell like watermelon.

Whatever properties or attributes they may possess, we can only know about such qualities if they affect how electrons influence us or our equipment.

Knuth 2013, Knuth 2014

An Operational Perspective



The **only** properties that we can know about are those that affect how an electron influences others.

Operational Viewpoint:
Define electron properties based on how they influence others

Since we cannot know what an electron is, perhaps it is best to simply focus on what an electron does.

Influence

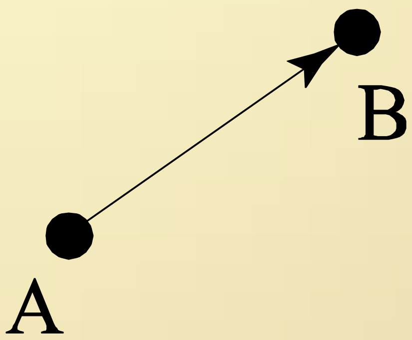
The observer, when he seems to himself to be observing a stone, is really, if physics is to be believed, observing the effects of the stone upon himself.

- Bertrand Russell

Influence and Events

We consider that all we can know is that particles (entities) **influence** one another.

Both an ***act of influence*** and an ***act of being influenced*** are considered to be ***events***.



Notes

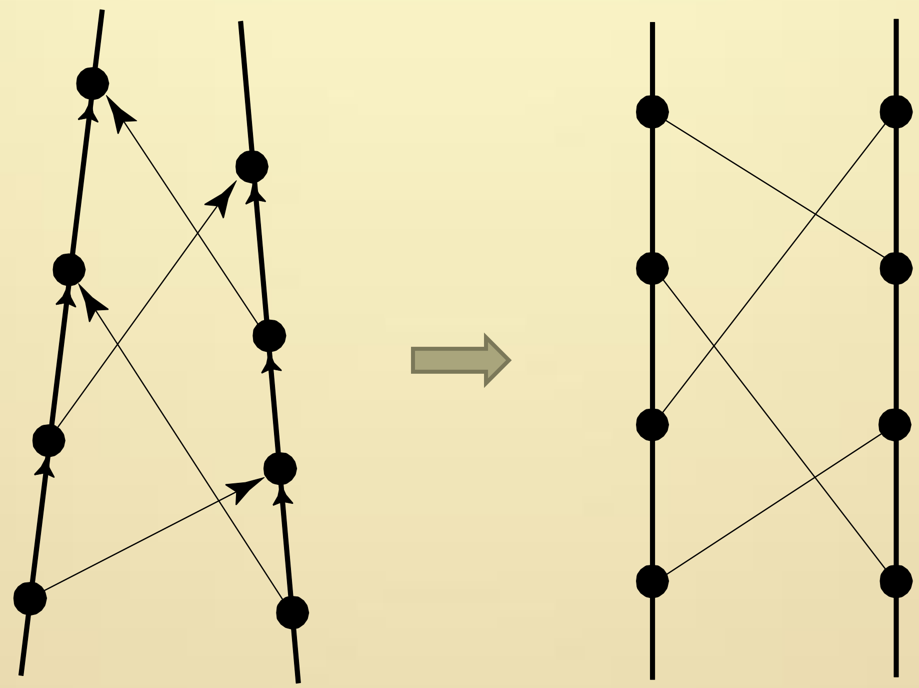
Events occur in pairs

Each event is associated with a different particle

The asymmetry of influence allows these two events to be ordered

Partially-Ordered Set Model

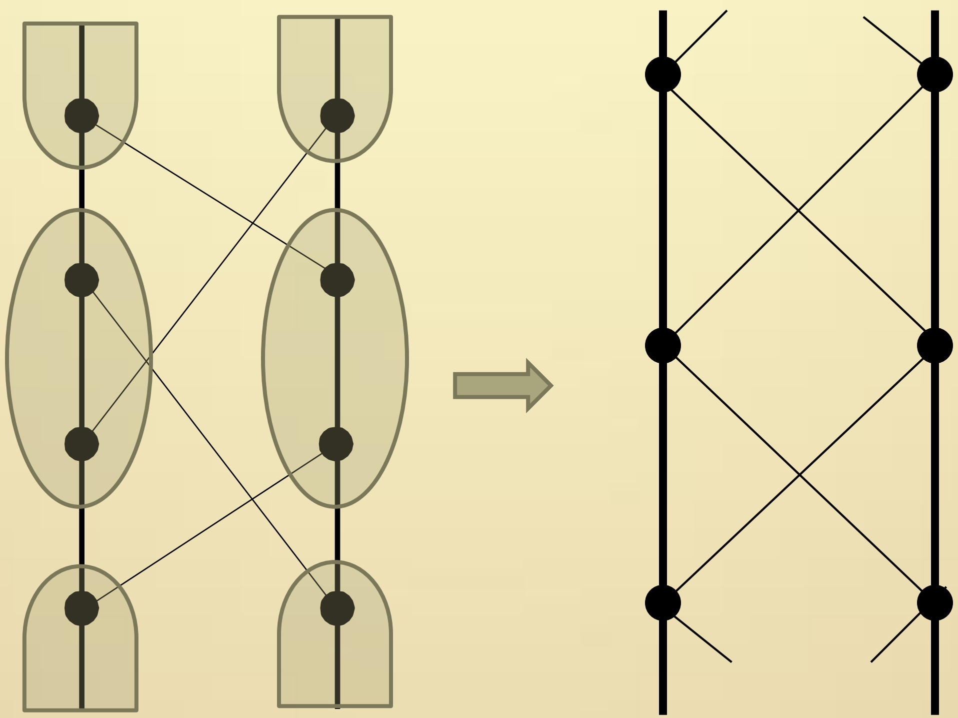
Particles are represented by an ordered sequence of states (nodes connected by thick lines with little arrows) with each state being determined in part by directed interactions with another particle (thin lines with big arrows)



Remove arrows and straighten chains
Focus on nodes (elements) and ignore states

Coarse Graining

Influence relates one element on one particle chain to one element on another particle chain. Here we consider coarse graining.



Note that connectivity depends on the ability to resolve events.

Quantification

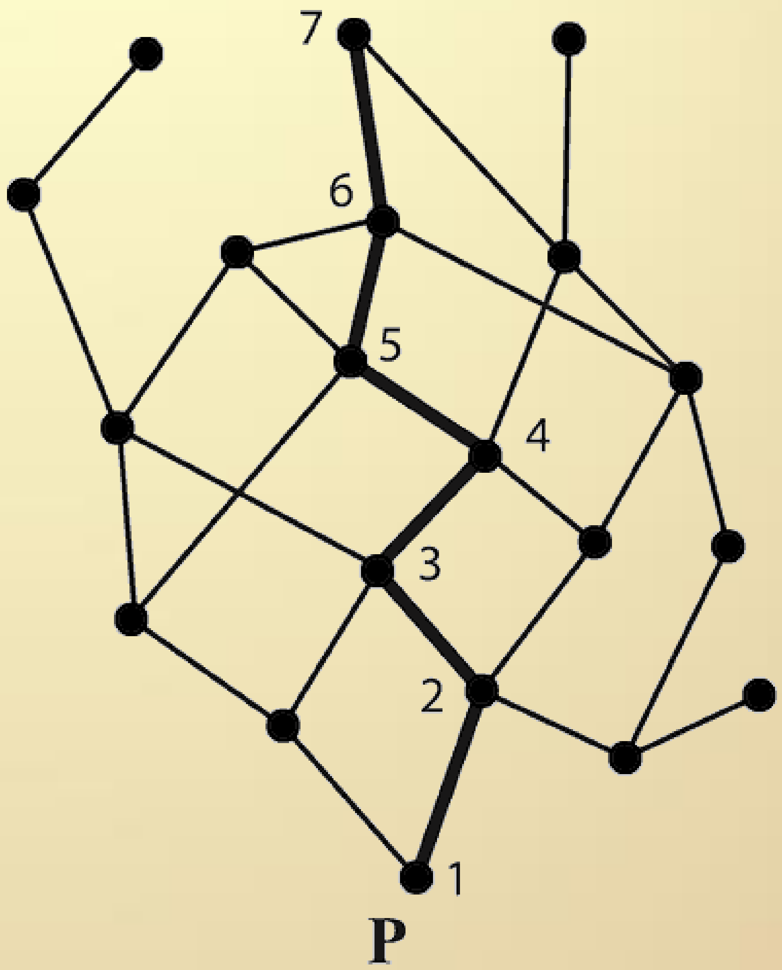
Measure that which is measurable
and make measurable that which is not so

Galieo Galilei

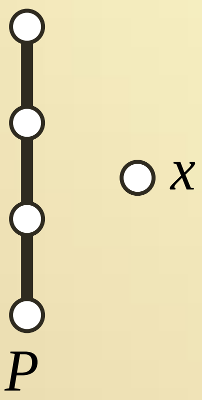
Quantifying a Chain

Chains are easily quantified by a **monotonic valuation** assigning to each element a number

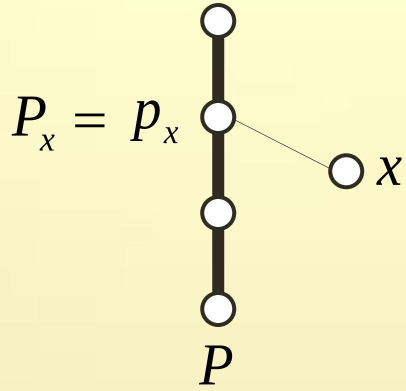
Both particles and observers are modeled by chains



Chain Projection

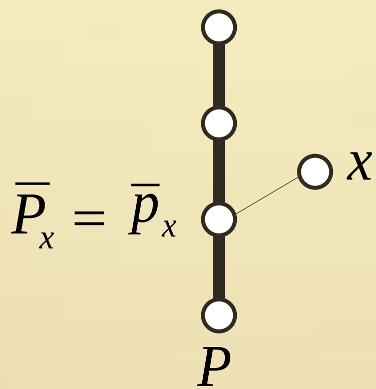


$p_i \,||\, x$ for all p_i



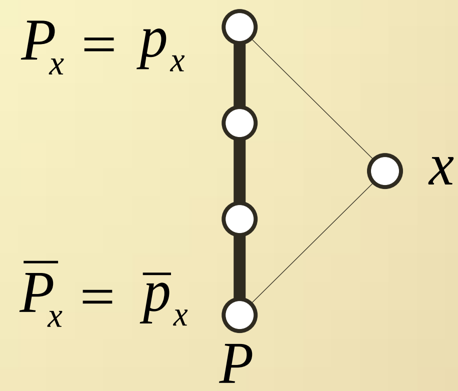
$p_i \geq x$ for all $p_i \geq p_x$

$p_i \,||\, x$ for all $p_i < p_x$



$p_i \leq x$ for all $p_i \leq \overline{p}_x$

$p_i \,||\, x$ for all $p_i > \overline{p}_x$

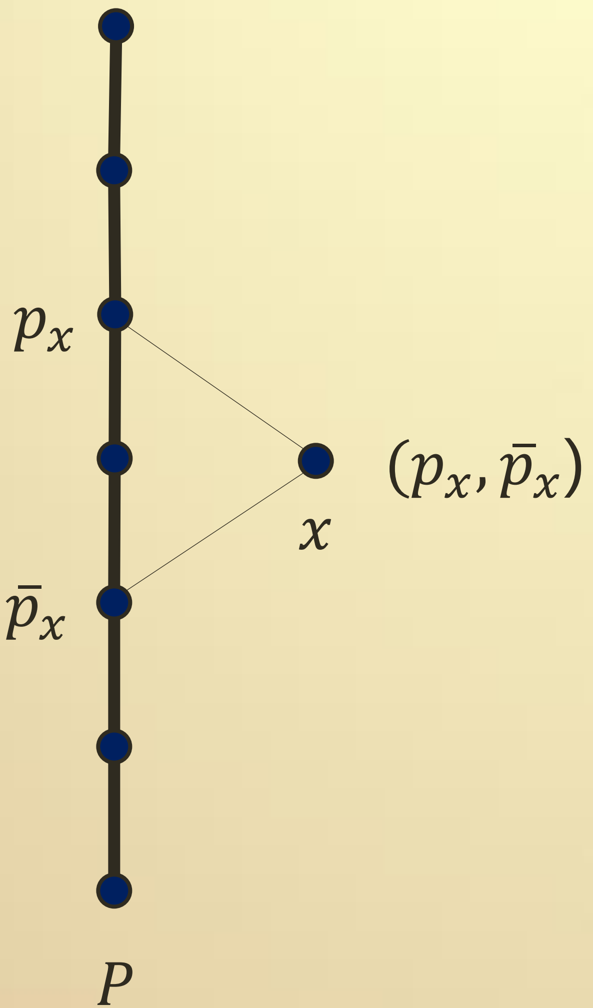


$p_i \leq x$ for all $p_i \leq \overline{p}_x$

$p_i \,||\, x$ for all $\overline{p}_x < p_i < p_x$

$p_i \geq x$ for all $p_i \geq p_x$

Quantification via Chain Projection

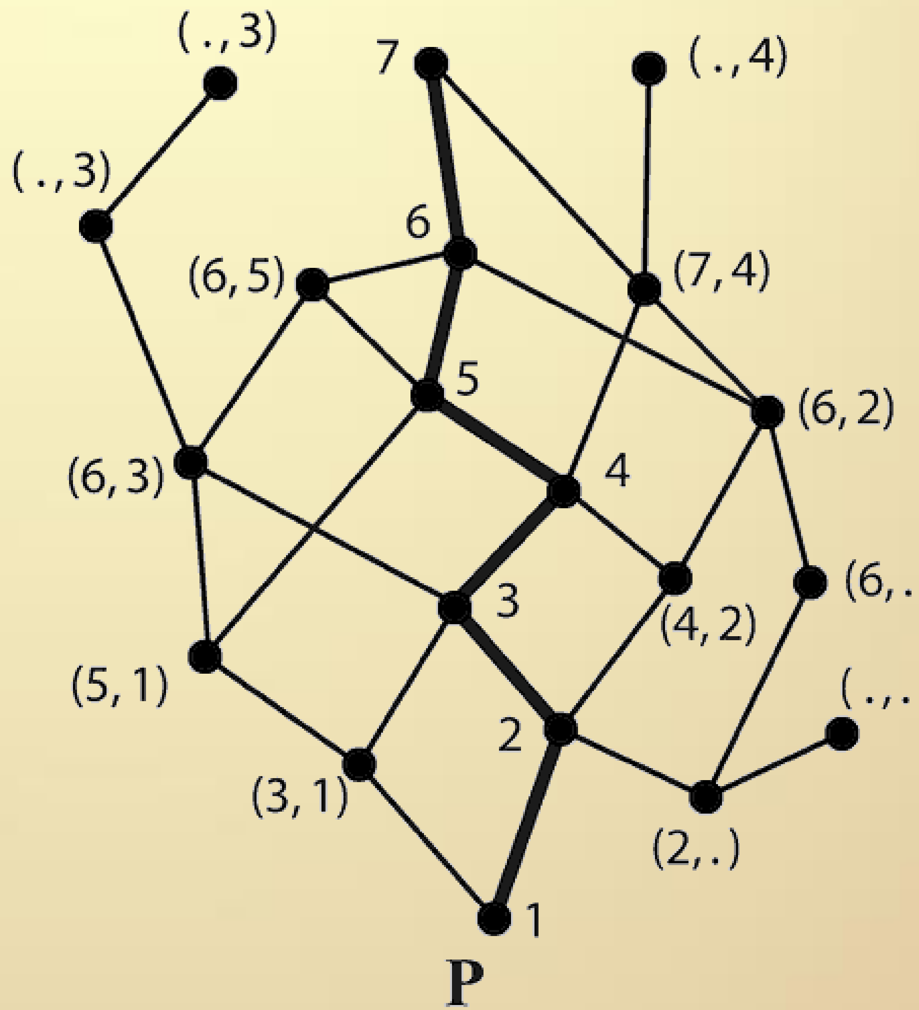


Quantification can be extended by relating poset elements to the embedded chain via **chain projection.**

For an element x, there is the potential to be quantified by a pair of numbers

Quantification via Chain Projection

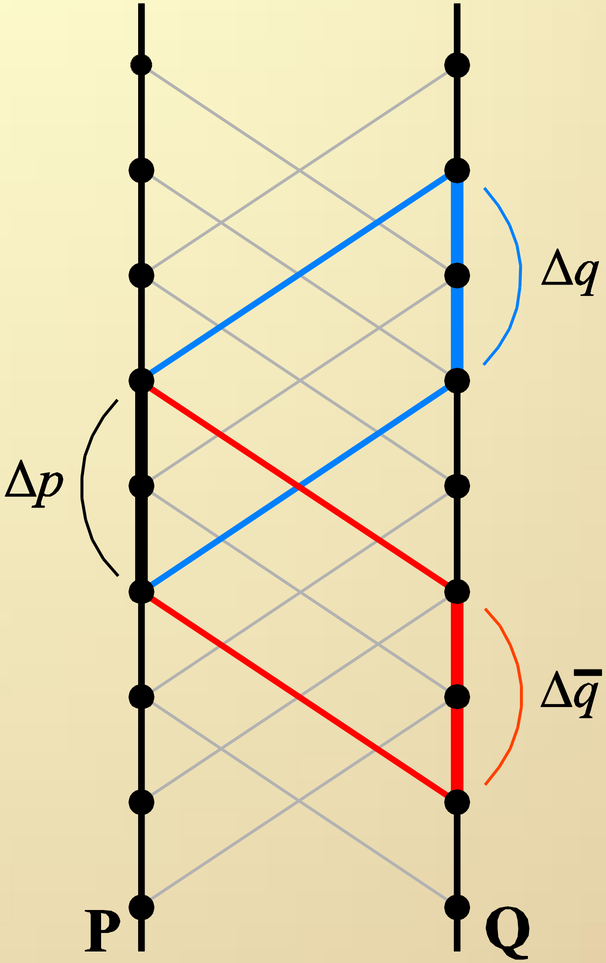
Quantifying the poset with respect to the chain **P** results in a rather strange chain-based coordinate system.



Coordinated Observers

Here we have two observers who Influence one another in a constant fashion so that the length of an interval along one chain equals the length of its projection onto the other chain.

$$\Delta p = \Delta q = \Delta \bar{q}$$

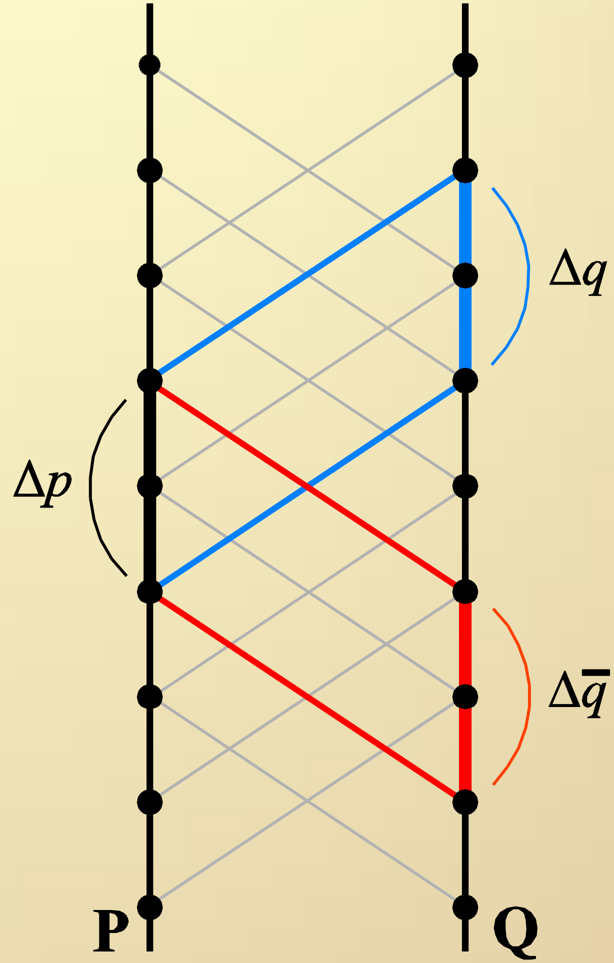


Along a Chain

Consider two coordinated observers, and consider an interval that spans the two chains.

The length of this interval is consistently quantified by

$$\frac{\Delta p + \Delta q}{2}$$

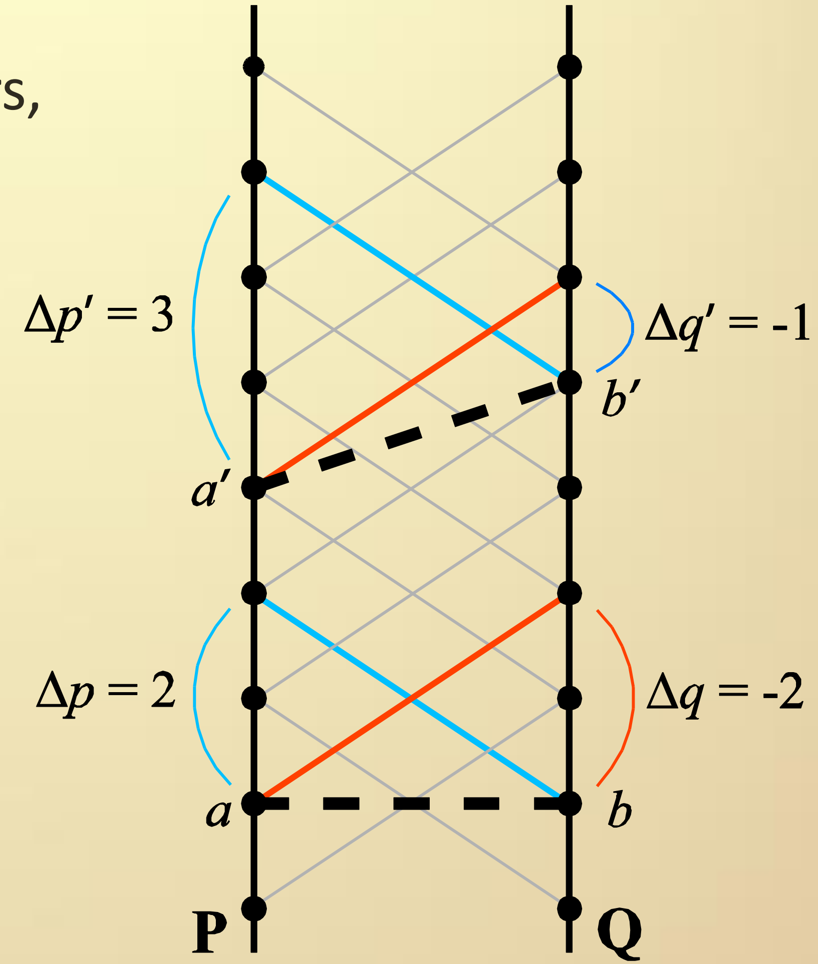


Between Chains

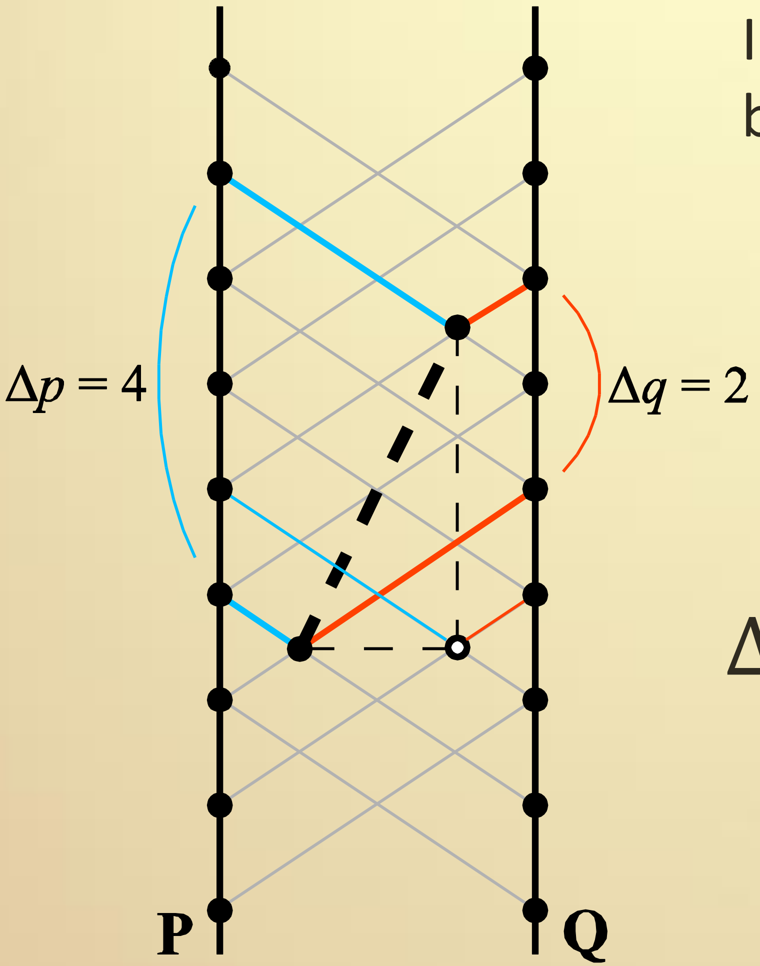
Consider two coordinated observers, and consider quantifying the relationship between these two chains.

We call this the **distance** between chains

$$\frac{\Delta p - \Delta q}{2}$$



Quantifying Intervals



Intervals are consistently quantified by

$$\Delta s^2 = \Delta p \Delta q$$

where

$$\Delta p \Delta q = \left(\frac{\Delta p + \Delta q}{2}\right)^2 - \left(\frac{\Delta p - \Delta q}{2}\right)^2$$

Emergence

Individual events. Events beyond law. Events so numerous and so uncoordinated that, flaunting their freedom from formula, they yet fabricate firm form.

- John Archibald Wheeler

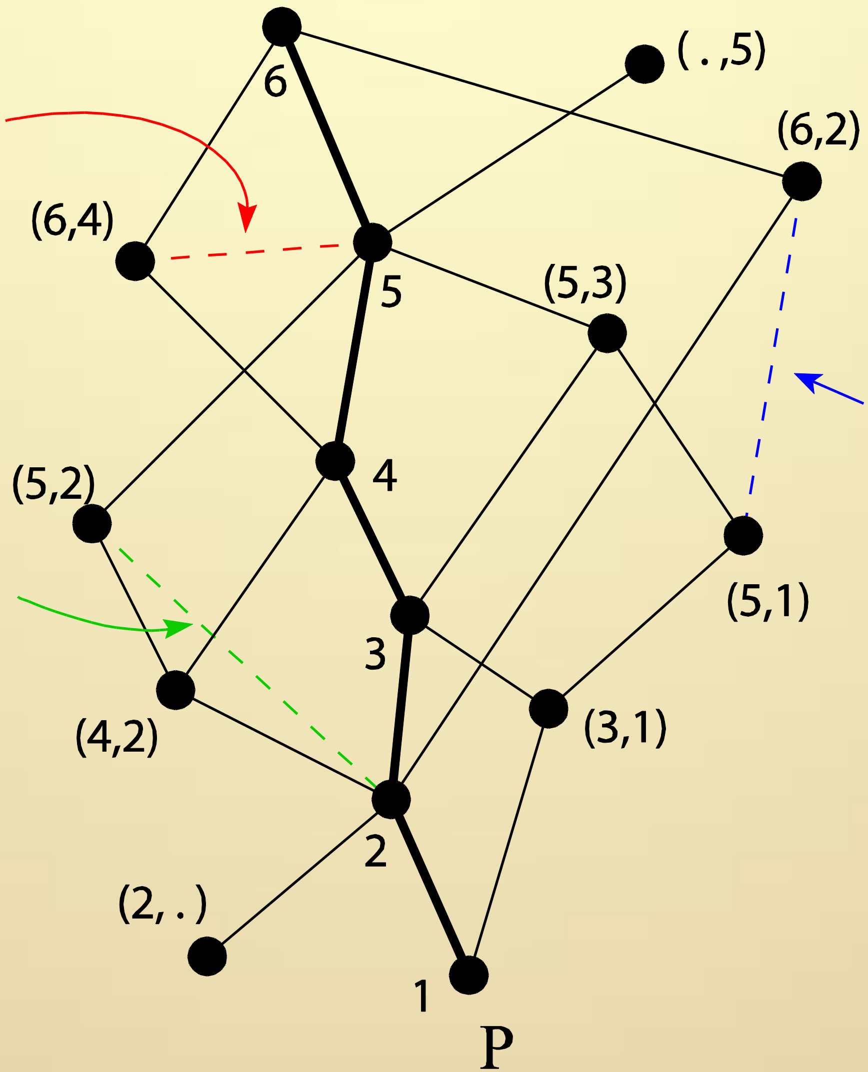
Quantifying a Poset

Antichain-like Interval

4-tuple: (5,5 ; 6,4)

pair: (6-5 , 4-5) = (1,-1)

scalar: (1)(-1) = -1



Chain-like Interval

4-tuple: (5,1 ; 6,2)

pair: (6-5 , 2-1) = (1,1)

scalar: (1)(1) = 1

Projection-like Interval

4-tuple: (2,2 ; 5,2)

pair: (5-2 , 2-2) = (3,0)

scalar: (3)(0) = 0

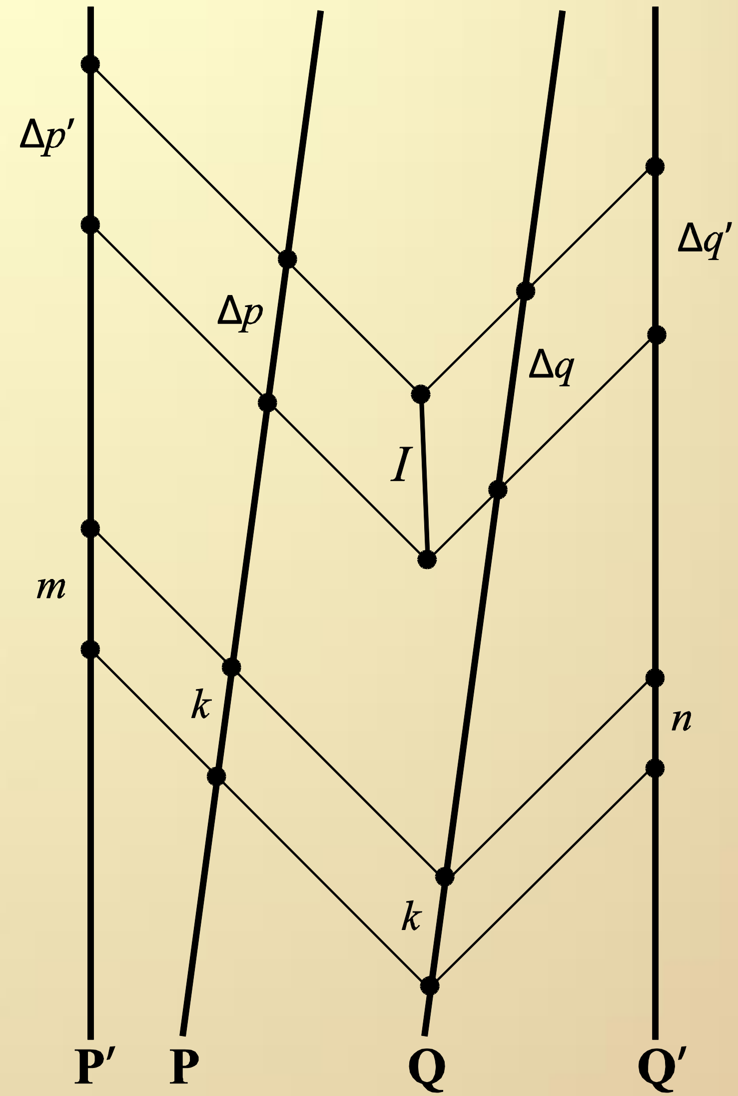
Pair Transformation

Coordinated observers **P** and **Q** quantify the interval $\boldsymbol{I}$ with the pair of numbers $(\Delta p, \Delta q)$

Coordinated observers **P'** and **Q'** quantify the interval $\boldsymbol{I}$ with the pair of numbers $(\Delta p', \Delta q')$

Intervals along **P** and **Q** of length k are quantified by **P'** and **Q'** by (m, n) which implies

$$(\Delta p', \Delta q') = \left(\sqrt{\frac{m}{n}} \Delta p, \sqrt{\frac{n}{m}} \Delta q \right)$$



Minkowski Metric

Writing $\Delta t = \frac{\Delta p + \Delta q}{2}$ $\Delta x = \frac{\Delta p - \Delta q}{2}$

The metric $\Delta s^2 = \left(\frac{\Delta p + \Delta q}{2}\right)^2 - \left(\frac{\Delta p - \Delta q}{2}\right)^2$

becomes

$$\Delta s^2 = \Delta t^2 - \Delta x^2$$

Speed

Writing $$\Delta t = \frac{\Delta p + \Delta q}{2} \qquad \Delta x = \frac{\Delta p - \Delta q}{2}$$

We define

$$\beta = \frac{\Delta x}{\Delta t} = \frac{\Delta p - \Delta q}{\Delta p + \Delta q}$$

As well as

$$\gamma = \frac{1}{\sqrt{1 - \beta^2}}$$

Lorentz Transformations

Relating one observer pair to the other

$$\beta = \frac{m-n}{m+n}$$

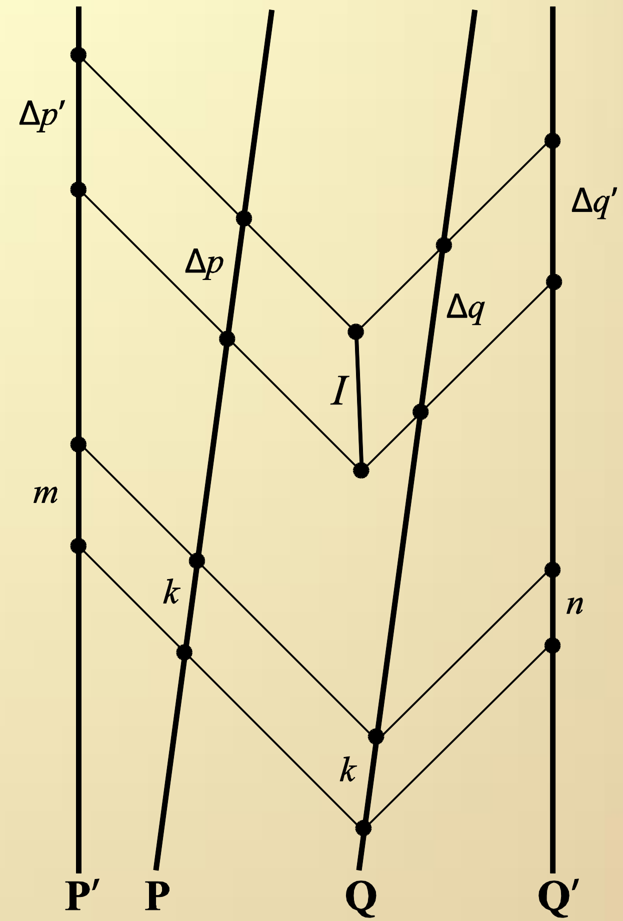
Recall $\Delta t = \frac{\Delta p + \Delta q}{2}$ $\qquad \Delta x = \frac{\Delta p - \Delta q}{2}$

The pair transformation

$$(\Delta p', \Delta q') = \left(\sqrt{\frac{m}{n}}\Delta p, \sqrt{\frac{n}{m}}\Delta q\right)$$

becomes

$$\Delta t' = \gamma \Delta t - \beta\gamma\Delta x$$
$$\Delta x' = -\beta\gamma\Delta t + \gamma\Delta x$$

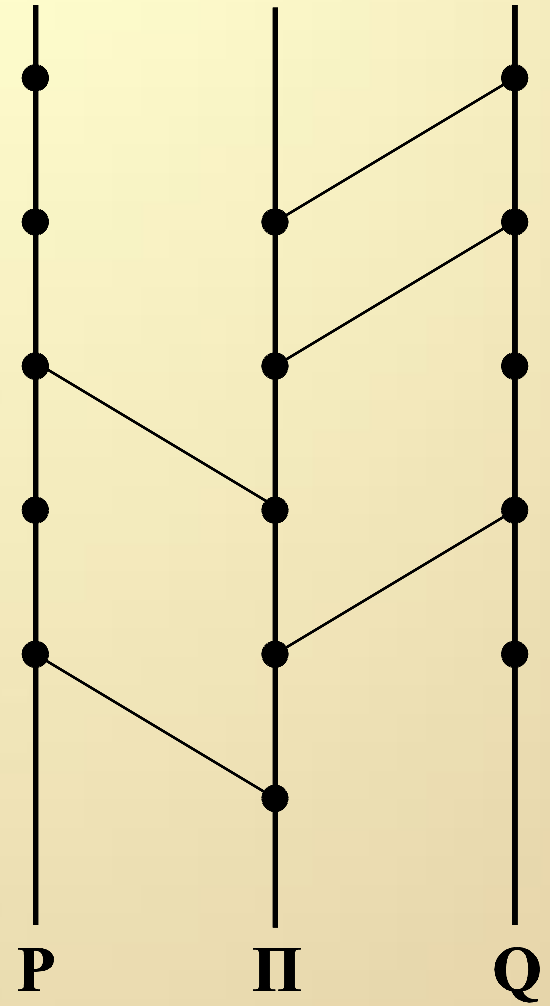


The Free Particle

Free Particle Model

Define a **Free Particle** as a particle that influences, but is not influenced.

This is an idealization that enables us to develop some useful concepts.



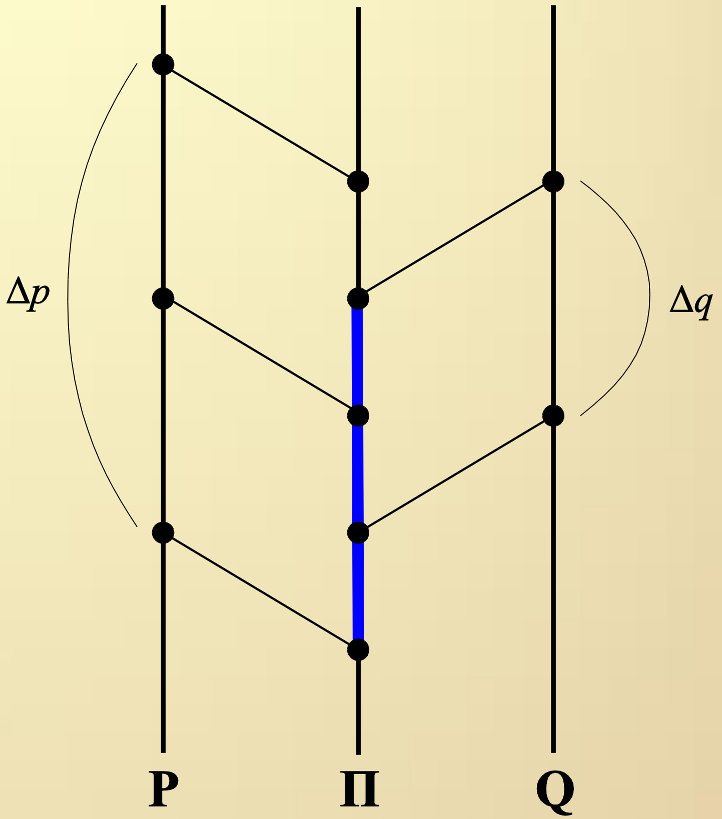
Rates v. Intervals

Instead of focusing on intervals, we could equivalently choose to quantify rates.

Rates and intervals are related by Fourier transforms.

Define

$$r_P = \frac{N}{\Delta p} \qquad r_Q = \frac{N}{\Delta q}$$



Rates are consistent only as coarse-grained averages!

Mass, Energy and Momentum

The product of rates is invariant

$$r_P r_Q = \frac{N^2}{\Delta p \Delta q}$$

Note

$$r_P r_Q = \left(\frac{r_P + r_Q}{2}\right)^2 - \left(\frac{r_Q - r_P}{2}\right)^2$$

which one might imagine to be analogous to

$$M^2 = E^2 - p^2$$

Speed in Terms of Rates

Recall

$$\beta = \frac{\Delta x}{\Delta t} = \frac{\Delta p - \Delta q}{\Delta p + \Delta q}$$

$$\frac{p}{E} = \frac{r_Q - r_P}{r_P + r_Q} = \frac{\frac{N}{\Delta q} - \frac{N}{\Delta p}}{\frac{N}{\Delta p} + \frac{N}{\Delta q}} = \frac{\frac{\Delta p}{\Delta p \Delta q} - \frac{\Delta q}{\Delta p \Delta q}}{\frac{\Delta q}{\Delta p \Delta q} + \frac{\Delta p}{\Delta p \Delta q}} = \frac{\Delta p - \Delta q}{\Delta p + \Delta q} = \frac{\Delta x}{\Delta t} = \beta$$

$$\beta = \frac{p}{E}$$

Knuth 2013, Knuth 2014a

Lorentz Transform and Rates

Rates transform as $r_P{}' = \sqrt{\frac{n}{m}} r_P$ $r_Q{}' = \sqrt{\frac{m}{n}} r_Q$

We can rewrite the Energy and Momentum as

$$E' = \frac{1}{2}\left(\sqrt{\frac{n}{m}} r_P + \sqrt{\frac{m}{n}} r_Q\right) \qquad p' = \frac{1}{2}\left(\sqrt{\frac{m}{n}} r_Q - \sqrt{\frac{n}{m}} r_{QP}\right)$$

becomes

$$E' = \gamma E + \gamma\beta p \qquad p' = \gamma\beta E + \gamma p$$

Given $p = 0$, which implies $E = M$

$$E' = \gamma M \qquad p' = \gamma\beta M$$

Knuth 2013, Knuth 2014a

Complementarity

Position, Δx, and momentum, p, are Fourier Transform duals as are time, Δt, and Energy E

Momentum and Energy only make sense as long-term averages. That they cannot be defined at an event.

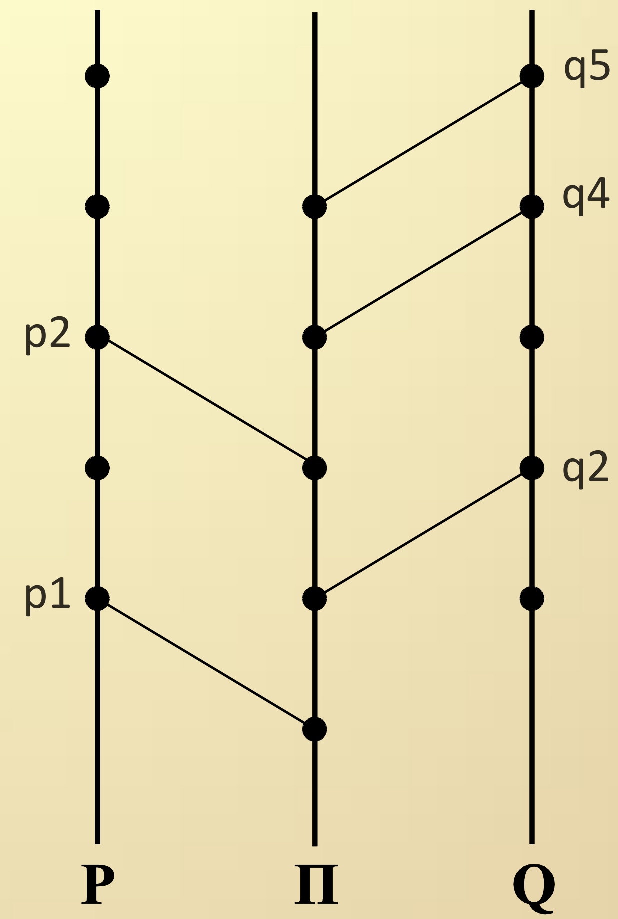
A particle *possesses* neither position nor momentum. These quantities describe the behavior of the particle.

Un-Orderable Influence Sequences

Observers **P** and **Q** both record detections.

However, the detections made by chain **P** cannot be ordered with respect to the detections made by chain **Q**.

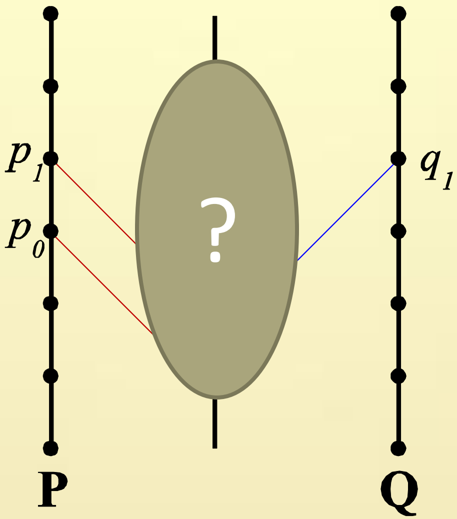
The particle's behavior is **informationally isolated** from the rest of the universe!
To make inferences, all possible orderings must be considered.



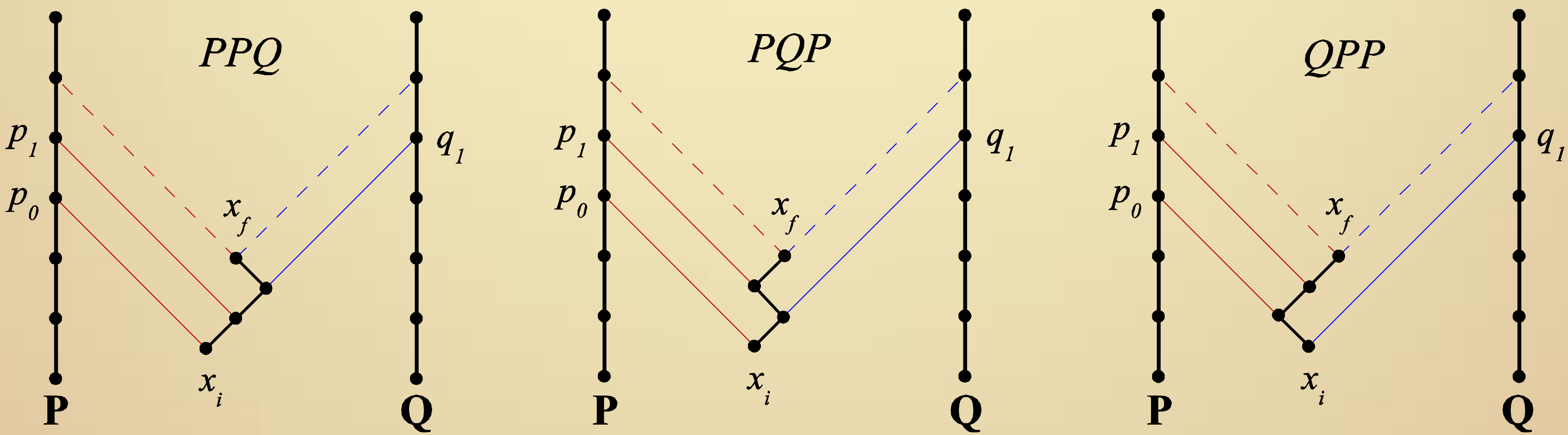
Information Isolation

Influence Sequences Correspond to Paths

Considering all possible sequences corresponds to considering all possible paths



(PPQ)



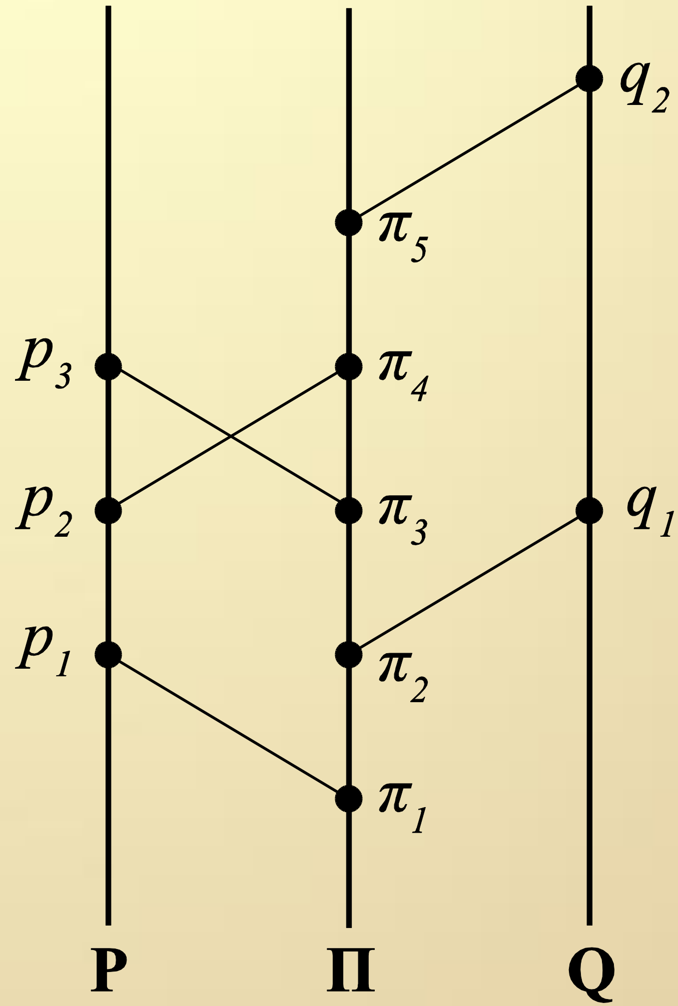
Measurement allows Ordering

Influencing the particle (measurement) allows one to order events thus breaking the informational isolation

In this example one is able to say that

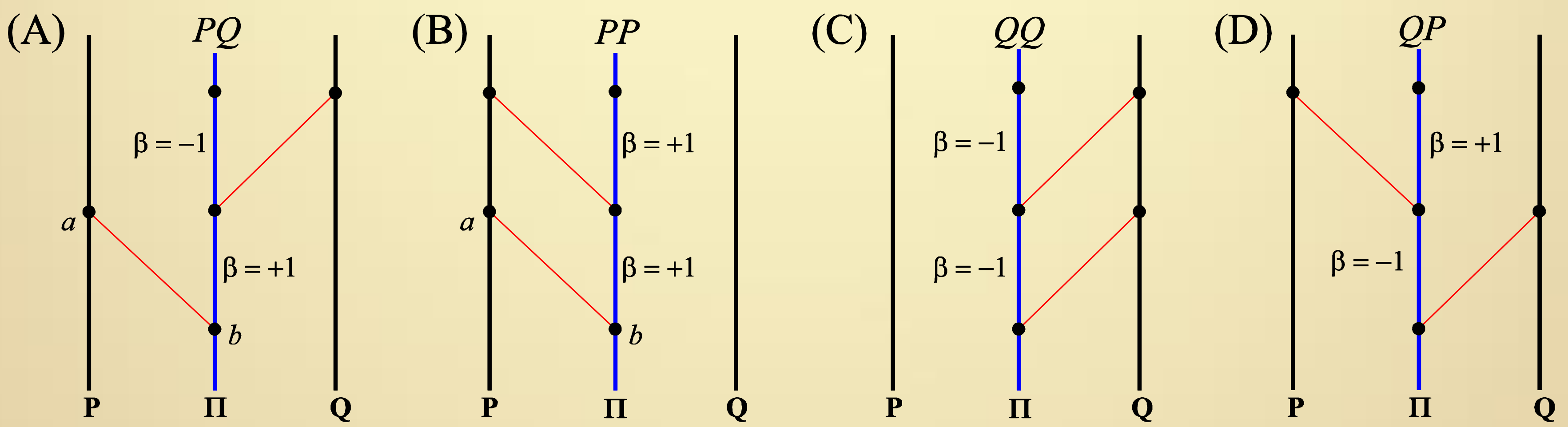
$$p_1 < p_2 < q_2$$

We have not yet fully explored the consequences in such cases.



Zitterbewegung

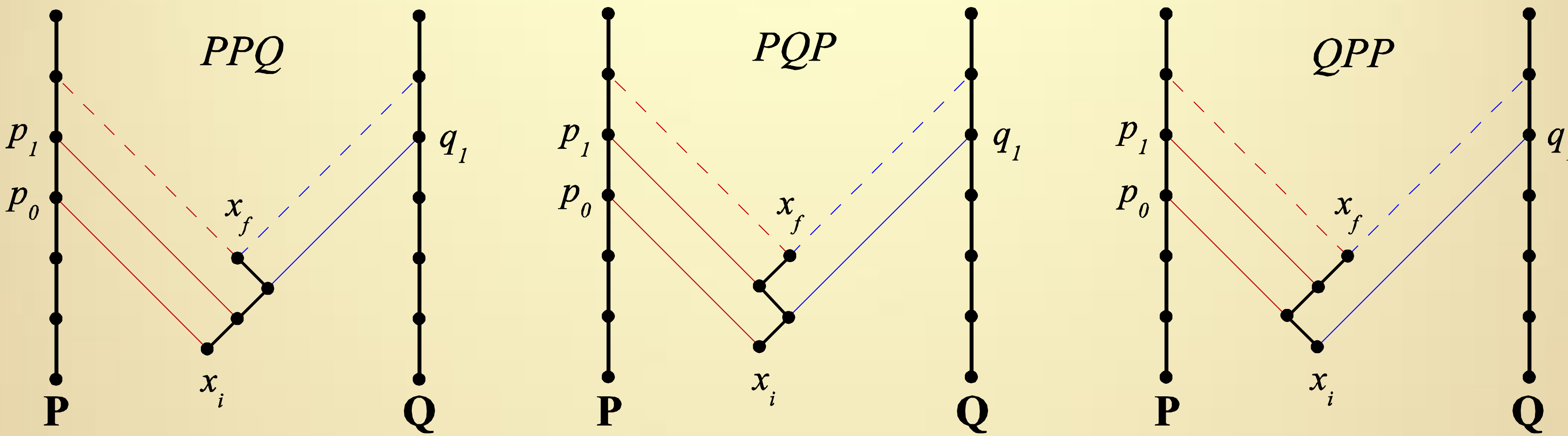
Intervals along a free particle chain have only one of two speeds, $\beta = \pm 1$, determined by the previous influence direction.



This effect was predicted by Schrodinger by considering the speed eigenvalues of the Dirac equation. He called it Zitterbewegung. It is thought to be closely related to spin and mass, and perhaps related to scattering off the Higg's field.

Knuth 2013, Knuth 2014a

Feynman Checkerboard Model of the Electron



We have shown that this problem is the same as the Feynman checkerboard problem (Feynman & Hibbs, 1965) where the electron is described as making Bishop moves on a chess board at the speed of light. Feynman made a quantum amplitude assignment to the two moves (continuation and reversal) that is known to lead to the Dirac equation. We have been able to derive these amplitudes using this framework and probability theory.

Statistical Mechanics of Motion

Average Speed

Since influence in the P-direction results in $\beta = +1$
and influence in the Q-direction results in $\beta = -1$
we can find the average speed by

$$\begin{aligned}\langle\beta\rangle &= (+1)\Pr(P) + (-1)\Pr(Q) \\ &= \Pr(P) - \Pr(Q)\end{aligned}$$

Since $\Pr(P) + \Pr(Q) = 1$, we have that

$$\Pr(P) = \frac{1 + \langle\beta\rangle}{2}$$

$$\Pr(Q) = \frac{1 - \langle\beta\rangle}{2}$$

Entropy of a Free Particle

Since motion to the left and right is probabilistic, we can compute the entropy of a particle with average speed β

$$S = -\Pr(P)\log\Pr(P) - \Pr(Q)\log\Pr(Q)$$

which in terms of the speed β:

$$S = -\frac{1+\beta}{2}\log\frac{1+\beta}{2} - \frac{1-\beta}{2}\log\frac{1-\beta}{2}$$

Minimum at $\beta = \pm 1$ and maximum at rest $\beta = 0$

Doing work on an object reduces its entropy thus making it move

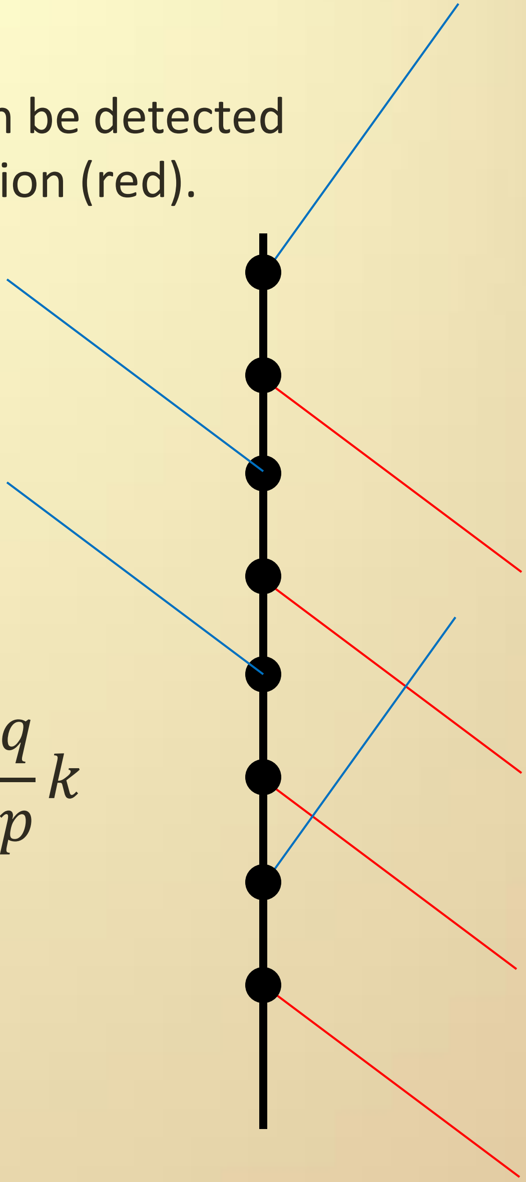
Forces

Acts of influence clearly affect rates of influence in one direction or another.

This affects the momentum, which means that influence must also give rise to forces.

Constant Rate of Incoming Influence

Consider a particle that influences others (blue) so it can be detected and also is influenced at a constant rate from one direction (red). How do coordinated observers interpret this?



For each incoming influence event, Δp is incremented: $\Delta\tilde{p} = \Delta p + k$

where $k = \sqrt{\frac{m}{n}}$

We then have that (for $\Delta p \gg k$)

$$\Delta\tilde{p} = \Delta p + k$$

$$\Delta\tilde{q} = \Delta q \frac{\Delta p}{\Delta p + k} \approx \Delta q - \frac{\Delta q}{\Delta p} k$$

So that

$$\delta\Delta p = \Delta\tilde{p} - \Delta p = k$$

$$\delta\Delta q = \Delta\tilde{q} - \Delta q = -\frac{\Delta q}{\Delta p} k$$

Constant Rate of Incoming Influence

So for one incoming influence, we have

$$\delta\Delta p = \Delta\tilde{p} - \Delta p = k$$

$$\delta\Delta q = \Delta\tilde{q} - \Delta q = -\frac{\Delta q}{\Delta p}k$$

For many influence events, we define the rate as

$$r \doteq \frac{N_r}{N_P \Delta\tau}$$

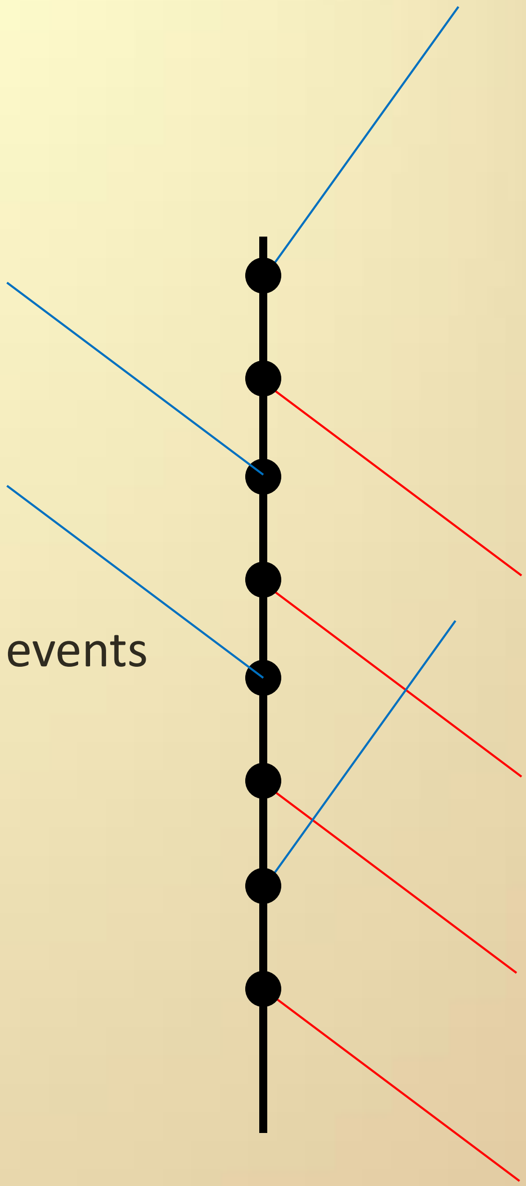
Where N_r and N_P are the number of incoming r-events and outgoing P events

We then have

$$d\Delta p = N_r \delta\Delta p = N_r k = r N_P k \Delta\tau = r\Delta p \Delta\tau$$

$$d\Delta q = N_r \delta\Delta q = -N_r \frac{\Delta q}{\Delta p} k = -r\, N_P k \frac{\Delta q}{\Delta p} \Delta\tau$$

$$= -r\Delta q \Delta\tau$$



Constant Rate of Incoming Influence

The incoming influences increment by

$$d\Delta p = r\Delta p\Delta\tau$$

$$d\Delta q = -r\Delta q\Delta\tau$$

Together with the outgoing influences, we have

$$\frac{d\Delta p}{d\tau} = \left(r + \frac{1}{\tau}\right)\Delta p$$

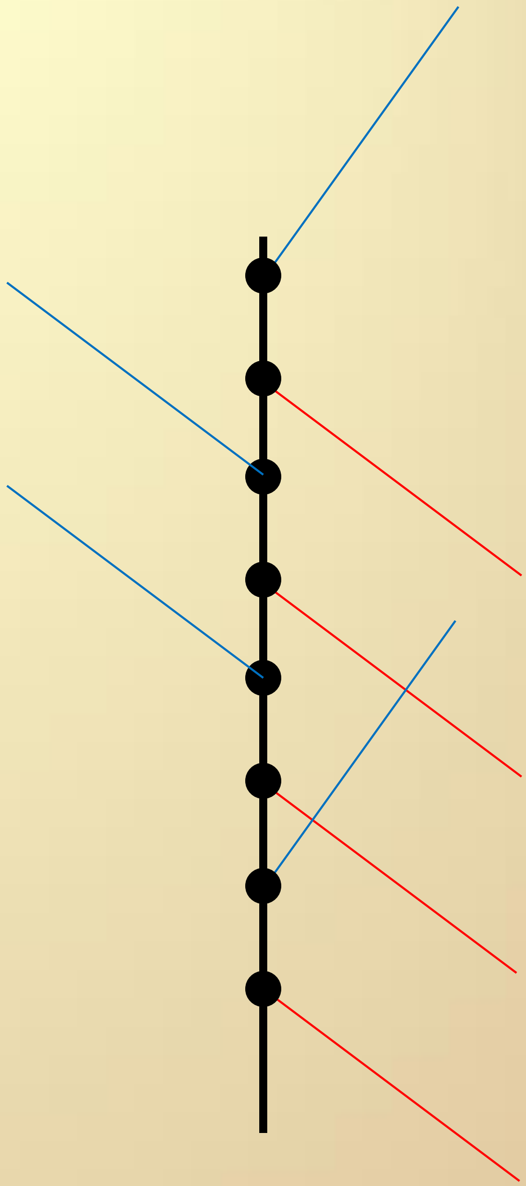
$$\frac{d\Delta q}{d\tau} = \left(-r + \frac{1}{\tau}\right)\Delta q$$

Which have as a solution:

$$\Delta p = A\tau e^{r\tau}$$

$$\Delta q = B\tau e^{-r\tau}$$

Since $\Delta p\Delta q$ is invariant, $A = B^{-1}$. Writing $A = e^{\varphi_0}$ we have…



Constant Rate of Incoming Influence

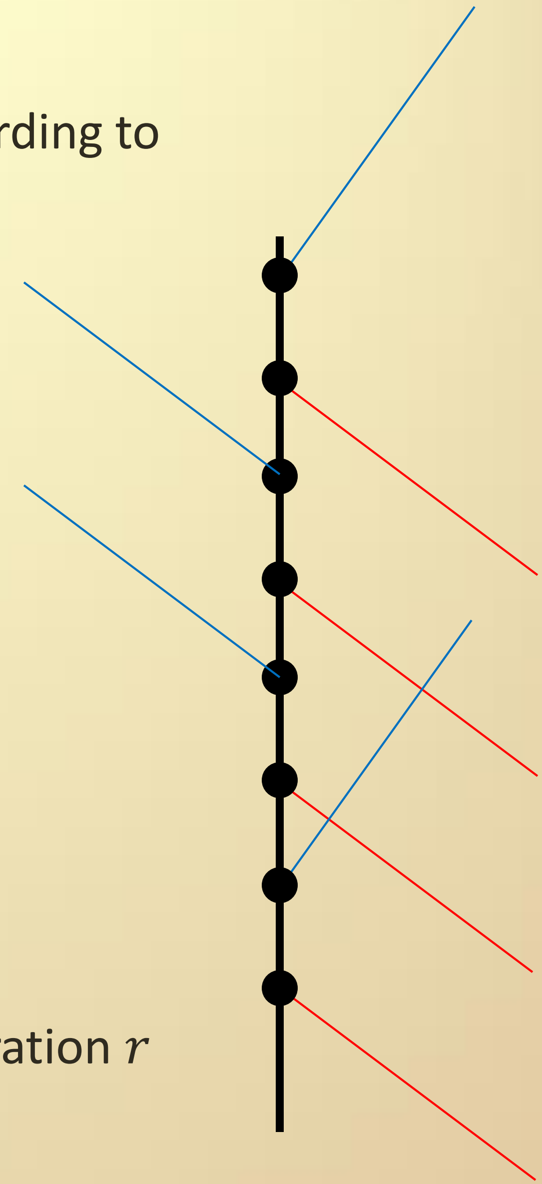
The intervals change as a function of proper time according to

$$\Delta p = \tau e^{r\tau + \varphi_0}$$
$$\Delta q = \tau e^{-r\tau - \varphi_0}$$

The speed becomes:

$$\beta = \frac{\Delta p - \Delta q}{\Delta p + \Delta q} = \frac{e^{r\tau + \varphi_0} - e^{-r\tau - \varphi_0}}{e^{r\tau + \varphi_0} + e^{-r\tau - \varphi_0}}$$

$$\beta = \tanh(r\tau + \varphi_0)$$



Which is RELATIVISTIC ACCELERATION with an acceleration r and initial rapidity φ_0 !

Forces

The average influence rate results in the following changes

$$d\Delta p = (r_{\bar{q}} - r_{\bar{p}})\Delta p d\tau$$

$$d\Delta q = (r_{\bar{p}} - r_{\bar{q}})\Delta q d\tau$$

Writing $r = r_{\bar{q}} - r_{\bar{p}}$ we can write the momentum as

$$dP = \frac{N}{2}\left[\frac{\Delta p(1 + r d\tau) - \Delta q(1 - r d\tau)}{\Delta p \Delta q} - \frac{\Delta p - \Delta q}{\Delta p \Delta q}\right]$$

$$\frac{dP}{d\tau} = \frac{N}{\sqrt{\Delta p \Delta q}} \frac{\Delta p + \Delta q}{2\sqrt{\Delta p \Delta q}} r$$

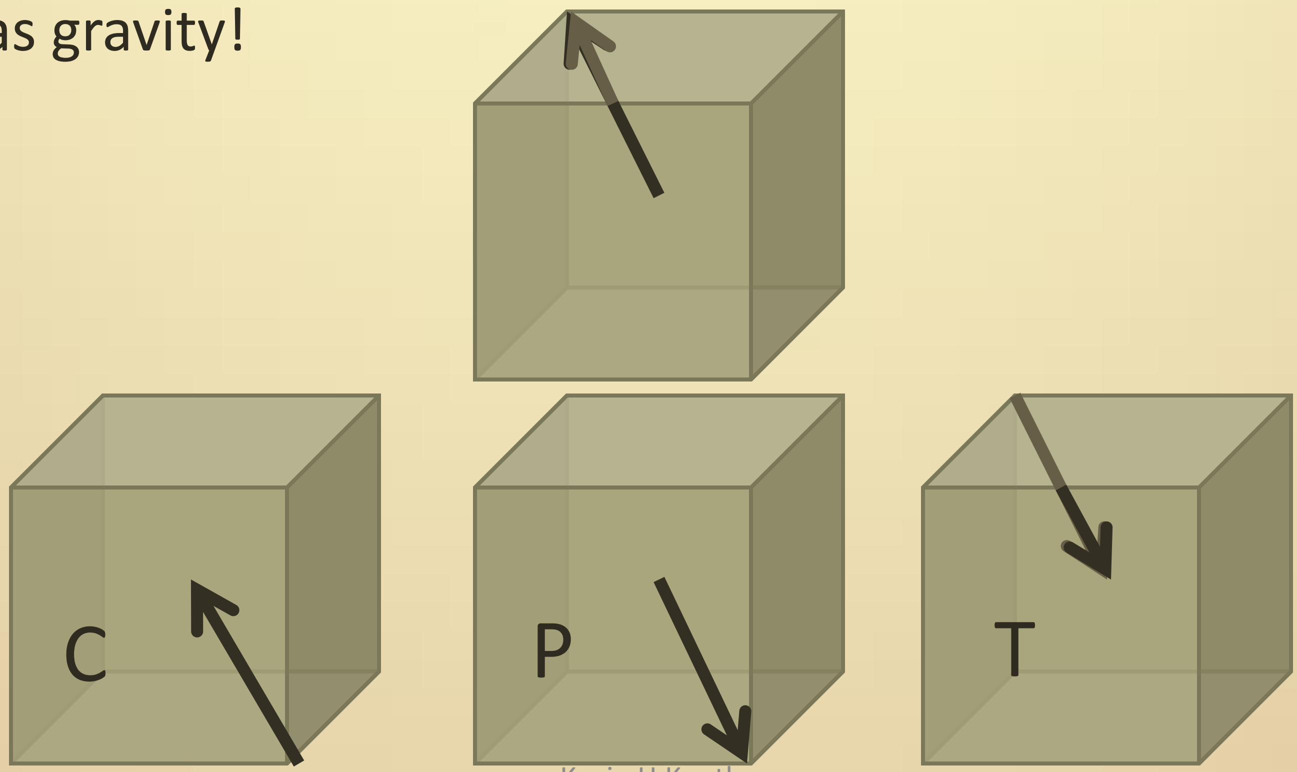
$$\frac{dP}{d\tau} = M\gamma r$$

Which is Newton's Second Law!

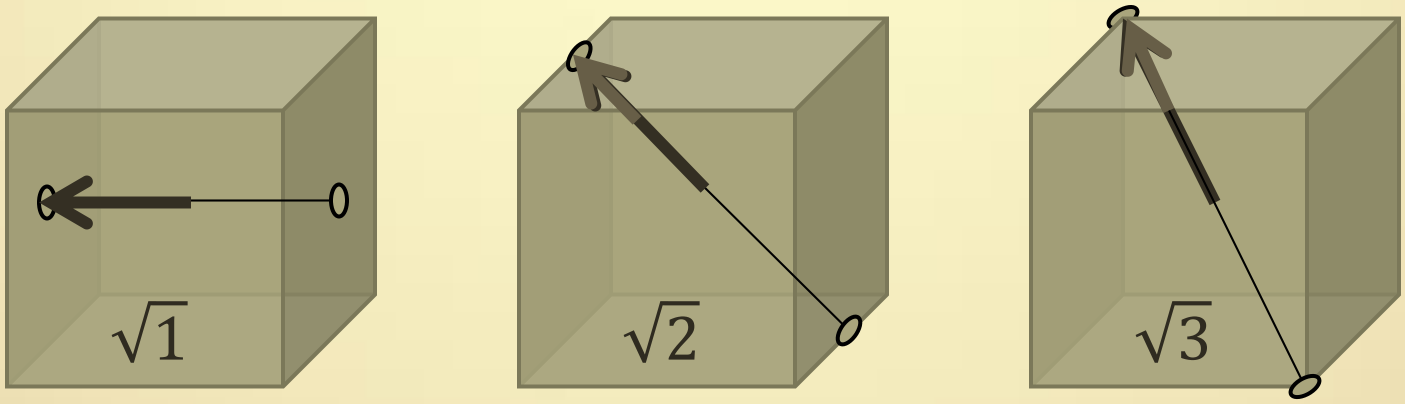
What Next?

Three-Dimensions and CPT

We can interpret time-reversal and parity in the poset.
However, we know that CPT is the invariant.
Could it be that Charge Conjugation is supported by the poset?
If so, these influence events may give rise to electromagnetism as well as gravity!



Fine Structure Constant?



$$e = \frac{1}{3}\left(\frac{19}{45}\sqrt{1} + \frac{2}{45}\sqrt{2} + \frac{11}{45}\sqrt{3}\right)$$
$$= 0.302822118577806$$
$$= 0.302822120882(961) \text{ (accepted)}$$

$$1/\alpha = 137.0360011601577$$
$$= 137.035999173(35) \text{ (accepted)}$$

It from bit symbolizes the idea that every item of the physical world has at bottom — at a very deep bottom, in most instances — an immaterial source and explanation; that which we call reality arises in the last analysis from the posing of yes-no questions and the registering of equipment-evoked responses; in short, that all things physical are information-theoretic in origin and that this is a participatory universe.

- John Archibald Wheeler

Thank You

This talk represents work from the following papers:

Knuth K.H., Bahreyni N. 2014. A Potential Foundation for Emergent Space-Time.
In press J. Math. Phys. arXiv:1209.0881 [math-ph]. http://arxiv.org/abs/1209.0881

Knuth K.H. 2013. Information-based physics and the influence network. 2013 FQXi Essay Entry (http://fqxi.org/community/forum/topic/1831)

Knuth K.H. 2014a. Information-Based Physics: An Observer-Centric Foundation.
Contemporary Physics, 55(1):12-32. arXiv:1310.1667 [quant-ph]. http://arxiv.org/abs/1310.1667

Knuth K.H. 2014b. **The problem of motion: the statistical mechanics of Zitterbewegung**. MaxEnt 2014, Amboise, France, AIP Conference Proceedings.

Walsh J., Knuth K.H. 2014. **Information-Based Physics, Influence and Forces**. MaxEnt 2014, Amboise, France, AIP Conference Proceedings.

I would like to thank **Newshaw Bahreyni, James Walsh**, Ariel Caticha, Keith Earle, Oleg Lunin, and John Skilling for lively discussions, insights and comments.
This work was supported by a grant from the Templeton Foundation.

Entropy of a Free Particle

Since motion to the left and right is probabilistic, we can compute the entropy of a particle with average speed β

$$S = -\Pr(P)\log\Pr(P) - \Pr(Q)\log\Pr(Q)$$

which in terms of the speed β:

$$S = -\frac{1+\beta}{2}\log\frac{1+\beta}{2} - \frac{1-\beta}{2}\log\frac{1-\beta}{2}$$

which simplifies to

$$\mathrm{S} = -\log\frac{1}{2} + \log\gamma - \beta\log(z+1)$$

Knuth, unpublished

Entropy in Terms of Energy

Recall that $\beta = \frac{p}{E}$ and that $p^2 = E^2 - m^2$

This allows us to write the Entropy of a Free Particle as

$$S = -\frac{1}{2}\log M^2 + \log 2E + \frac{p}{2E}\log\left(\frac{E-p}{E+p}\right)$$

One can define a temperature by taking the derivative of the entropy with respect to the energy

$$T = \left(\frac{dS}{dE}\right)^{-1} = \frac{M}{pE^2}\log\left(\frac{E-p}{E+p}\right)$$

$$= \frac{(1-\beta^2)^{\frac{3}{2}}}{M\beta}\log\left(\frac{1-\beta}{1+\beta}\right)$$